## GENERAL COMMENTS

As in 2005, the majority of Further Mathematics students appeared to be reasonably well prepared for examination 1 in 2006, with the average marks for the Core - Data analysis and each of the modules close to, or exceeding, 50 per cent. The number of students that sat for Further Mathematics Examination 1 in 2006 was 22 868, compared to 21815 in 2005.

## SPECIFIC INFORMATION

The tables below indicate the percentage of students who chose each option. The correct answer is indicated by shading.

## Section A

Core - Data analysis

| Question | \% A | \% B | \% C | \% D | \% E | \% No <br> Answer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 5 | 13 | 7 | 75 | 1 | 0 |
| $\mathbf{2}$ | 1 | 74 | 3 | 1 | 22 | 0 |
| $\mathbf{3}$ | 24 | 6 | 10 | 3 | 57 | 0 |
| $\mathbf{4}$ | 5 | 8 | 67 | 16 | 4 | 1 |
| $\mathbf{5}$ | 2 | 5 | 5 | 85 | 2 | 0 |
| $\mathbf{6}$ | 25 | 58 | 11 | 6 | 1 | 0 |
| $\mathbf{7}$ | 54 | 18 | 15 | 6 | 6 | 1 |
| $\mathbf{8}^{\boldsymbol{*}}$ | 6 | 10 | 33 | 10 | 41 | 0 |
| $\mathbf{9}$ | 42 | 26 | 18 | 7 | 5 | 1 |
| $\mathbf{1 0}$ | 3 | 7 | 64 | 13 | 12 | 0 |
| $\mathbf{1 1}$ | 4 | 6 | 25 | 61 | 4 | 1 |
| $\mathbf{1 2}$ | 17 | 10 | 66 | 5 | 1 | 0 |
| $\mathbf{1 3}$ | 29 | 59 | 5 | 3 | 4 | 0 |

*It should be noted that due to a misprint in the paper for Question 8, involving the units for the answers, all options were credited as correct.

The Core section was generally well done, with only Question 13 proving difficult for the majority of students.
In Question 13, students were given the equation of a trend line for predicting unemployment rates from the month of the year. The trendline had been derived using deseasonalised data. Monthly seasonal indices were also provided. Students were asked to use the trend line to predict the actual unemployment rate in June. The majority of students (59 per cent) were able to predict the deseasonalised unemployment rate from the trend line, but then failed to seasonalise the value they obtained to determine the actual unemployment rate. Only 29 per cent of students answered this question correctly, choosing option A.

## Section B

Module 1 - Number patterns and applications

| Question | \% A | \% B | \% C | \% D | \% E | \% No <br> Answer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 5 | 2 | 13 | 78 | 1 | 0 |
| $\mathbf{2}$ | 11 | 1 | 80 | 4 | 3 | 0 |
| $\mathbf{3}$ | 5 | 7 | 79 | 3 | 6 | 0 |
| $\mathbf{4}$ | 65 | 6 | 24 | 2 | 3 | 0 |
| $\mathbf{5}$ | 9 | 9 | 16 | 55 | 11 | 1 |
| $\mathbf{6}$ | 10 | 12 | 13 | 39 | 25 | 0 |
| $\mathbf{7}$ | 13 | 70 | 4 | 6 | 6 | 1 |
| $\mathbf{8}$ | 29 | 11 | 20 | 33 | 6 | 1 |
| $\mathbf{9}$ | 21 | 9 | 19 | 18 | 33 | 1 |

This module was generally well done, with the exceptions of Questions 6, 8 and 9.

Question 6 required students to determine the length of a crystal after 14 days growth. Only 25 per cent of students gave the correct response, option E. Option D, which was chosen by 39 per cent of students, corresponded to the length of the crystal at the start of the 14th day, which only allowed for 13 days growth.

Question 8 was an application of a Fibonacci-related sequence which required students to determine and then sum the first five terms in the sequence to give the answer 19 (option D). Option A, which was chosen by 29 per cent of students, corresponded to the value of the fifth term in the sequence.

Question 9 required students to model an athlete's weight gain over an extended period of time using an infinite geometric sequence. Only 33 per cent of students gave the correct response, option E. The relatively even distribution responses across the incorrect options A-D suggests that the majority of students guessed their answer to this question.

## Module 2 - Geometry and trigonometry

| Question | \% A | \% B | \% C | \% D | \% E | \% No <br> Answer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 5 | 77 | 7 | 8 | 3 | 0 |
| $\mathbf{2}$ | 3 | 5 | 4 | 82 | 5 | 0 |
| $\mathbf{3}$ | 83 | 4 | 5 | 3 | 5 | 0 |
| $\mathbf{4}$ | 3 | 5 | 65 | 21 | 5 | 0 |
| $\mathbf{5}$ | 13 | 2 | 80 | 2 | 2 | 0 |
| $\mathbf{6}$ | 4 | 9 | 8 | 68 | 11 | 1 |
| $\mathbf{7}$ | 2 | 10 | 5 | 13 | 69 | 0 |
| $\mathbf{8}$ | 22 | 11 | 39 | 22 | 4 | 1 |
| $\mathbf{9}$ | 12 | 31 | 19 | 31 | 7 | 0 |

This module was generally well done with the exceptions of Questions 8 and 9.
Question 8 required students to determine the depth of water in a circular pipe, given its surface width. Only 22 per cent of students were successful in choosing option A. Option C, which corresponded to the distance of the surface from the centre of the pipe, was given by 39 per cent of students. A further 22 per cent of students wrongly assumed that the cross-section of the surface was semi-circular and chose option D.

A possible solution strategy is as follows.

- Draw in a radius line that is a perpendicular bisector of the water surface line.
- Label points as shown on the diagram below.
- From right-angled triangle $B C D, C B=30 \mathrm{~cm}\left(=\sqrt{50^{2}-40^{2}} \mathrm{~cm}\right)$
- Then $d=C A-C B=50-30=20 \mathrm{~cm}$.


Assessment
Report

Correctly answering Question 9, a bearings problem, depended critically on being able to construct an appropriate diagram from the information given. This was clearly beyond most students, with only 31 per cent giving the correct response, option D.

A possible solution strategy is as follows.

- As $M$ and $P$ are the same distance from $O$, they must lie on the circumference of the same circle.
- Use this information to construct a diagram as shown below.
- A carefully drawn diagram that takes into account that the angle MON must be greater than SOP (from the bearings), shows that the bearing of $P$ from $M$ is between $180^{\circ}$ and $270^{\circ}$.

Note: For the bearing to be exactly $180^{\circ}$, the bearing of $P$ would have to be $142^{\circ}$. For the bearing to be between $090^{\circ}$ and $180^{\circ}$, the bearing of $P$ would have to be less than $142^{\circ}$.


Module 3 - Graphs and relations

| Question | \% A | \% B | \% C | \% D | \% E | \% No <br> Answer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 6 | 6 | 4 | 70 | 14 | 0 |
| $\mathbf{2}$ | 10 | 2 | 18 | 5 | 65 | 0 |
| $\mathbf{3}$ | 2 | 2 | 81 | 11 | 4 | 0 |
| $\mathbf{4}$ | 50 | 23 | 8 | 9 | 9 | 1 |
| $\mathbf{5}$ | 10 | 44 | 17 | 14 | 15 | 0 |
| $\mathbf{6}$ | 6 | 8 | 46 | 24 | 15 | 1 |
| $\mathbf{7}$ | 9 | 13 | 10 | 5 | 64 | 1 |
| $\mathbf{8}$ | 64 | 14 | 7 | 7 | 7 | 1 |
| $\mathbf{9}$ | 15 | 14 | 54 | 10 | 6 | 1 |

This module was reasonably well done, with no question causing particular difficulties.

Module 4 - Business-related mathematics

| Question | \% A | \% B | \% C | \% D | \% E | \% No <br> Answer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 6 | 91 | 1 | 1 | 1 | 0 |
| $\mathbf{2}$ | 4 | 8 | 62 | 19 | 5 | 1 |
| $\mathbf{3}$ | 14 | 41 | 9 | 11 | 23 | 1 |
| $\mathbf{4}$ | 7 | 42 | 47 | 3 | 0 | 0 |
| $\mathbf{5}$ | 8 | 5 | 5 | 14 | 67 | 1 |
| $\mathbf{6}$ | 9 | 14 | 44 | 18 | 13 | 1 |
| $\mathbf{7}$ | 10 | 33 | 7 | 47 | 2 | 1 |
| $\mathbf{8}$ | 37 | 20 | 14 | 18 | 10 | 1 |
| $\mathbf{9}$ | 23 | 27 | 19 | 11 | 20 | 1 |

This was the least well done of all the modules. Questions 3, 4, 8 and 9 proved more difficult than the remainder of questions.

Question 3 was unexpectedly challenging, with only 23 per cent of students giving the correct response, option E. A large number of students, 41 per cent, clearly failed to take into account that the pension was paid monthly, not annually, and incorrectly chose option B.

In Question 4, students were given the price of an item that included GST and asked to determine the amount of GST included in that price. Only 42 per cent of students gave the correct response, option B. Almost half of the students incorrectly responded with option C, which showed they had incorrectly equated the amount of GST included in a price to 10 per cent of the total price.

Question 8 concerned the growth of an investment, but was formulated graphically rather than numerically. The aim was to test conceptual understanding of the different forms of investment growth. Only 37 per cent of students were successful in answering this question, choosing option A. Teachers and students are reminded that graphical analysis of business and financial situations is a requirement of the study design.

Question 9 was expected to be challenging. Students needed to realise that the future value of a loan represented the remaining principal. A TVM solver (or its equivalent) was needed to assist with the computations. Only 27 per cent of students correctly chose option B. The most common error, made by 23 per cent of students, was to calculate the monthly repayment required to pay off the loan in 10 payments (option A). A further 20 per cent chose option E, erroneously equating the total amount paid off the loan after the tenth repayment to the amount paid off the principal, forgetting that these repayments also included interest.

A possible solution strategy is as follows.

- Use a TVM solver to determine the monthly repayments required to pay off the loan in five years ( $Q=\$ 375.40$ )
- Use a TVM solver to determine the future value of the loan after making 10 payments of $\$ 375.40$ per month ( $F V=\$ 15$ 542.40)
- Use the fact that the amount paid off the principal after 10 payments $=$ Principal - Future value $=\$ 2457.60$

Module 5 - Networks and decision mathematics

| Question | \% A | \% B | \% C | \% D | \% E | \% No <br> Answer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 1 | 85 | 6 | 1 | 7 | 0 |
| $\mathbf{2}$ | 3 | 87 | 4 | 3 | 3 | 0 |
| $\mathbf{3}$ | 18 | 13 | 16 | 43 | 10 | 0 |
| $\mathbf{4}$ | 8 | 6 | 9 | 70 | 7 | 0 |
| $\mathbf{5}$ | 3 | 8 | 6 | 3 | 80 | 0 |
| $\mathbf{6}$ | 13 | 3 | 63 | 16 | 4 | 0 |
| $\mathbf{7}$ | 14 | 12 | 59 | 11 | 3 | 0 |
| $\mathbf{8}$ | 37 | 13 | 11 | 28 | 11 | 1 |
| $\mathbf{9}$ | 28 | 26 | 19 | 17 | 9 | 1 |

This module was generally well done, with the exception of Questions 8 and 9.

Only 28 per cent of students correctly answered Question 8, choosing option D. The key to answering this question correctly was that only planar graphs satisfied Euler's formula. Of the five graphs presented, only graph D could not be redrawn as a planar graph. Interestingly, 37 per cent of students incorrectly chose option A, perhaps unaware of the fact that any graph with four or less vertices is planar.

Only 17 per cent of students correctly answered Question 9, choosing option D. To correctly answer this question, students had to first identify $B D C E H J$ as the critical path. When choosing an activity to be crashed on the critical path, care needed to be taken to ensure that crashing this activity did not create a new critical path. This restricted the amounts by which critical path activities $C, E, H$ or $J$ could be crashed to one hour. However, inspection shows that the remaining activity, $B$, could be crashed by a maximum of four hours without creating a new critical path. Note that $D$ is a dummy activity, so it could not be crashed.

Module 6 - Matrices

| Question | \% A | \% B | \% C | \% D | \% E | \% No <br> Answer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 6 | 84 | 4 | 3 | 2 | 0 |
| $\mathbf{2}$ | 5 | 9 | 5 | 75 | 5 | 0 |
| $\mathbf{3}$ | 84 | 5 | 4 | 5 | 1 | 0 |
| $\mathbf{4}$ | 3 | 88 | 6 | 2 | 1 | 0 |
| $\mathbf{5}$ | 3 | 2 | 11 | 78 | 4 | 1 |
| $\mathbf{6}$ | 26 | 17 | 5 | 29 | 23 | 1 |
| $\mathbf{7}$ | 14 | 27 | 42 | 9 | 7 | 1 |
| $\mathbf{8}$ | 12 | 5 | 6 | 22 | 56 | 1 |
| $\mathbf{9}$ | 11 | 40 | 12 | 14 | 22 | 1 |

This module was generally well done, with the exception of Question 6, for which only 23 per cent of students correctly chose option E. The relatively even distribution of responses across the remaining options suggests that the majority of students guessed their answer to this question.

A possible solution strategy is as follows.
Recognise that:

- matrix $X$ must have three rows for the product matrix $X A=\left[\begin{array}{ll}4 & 1 \\ 1 & 4 \\ 3 & 5\end{array}\right]$ to have three rows
- matrix $X$ must have three columns for the product $X A$ to be defined.

Thus $X$ must be a $(3 \times 3)$ matrix.

