GENERAL COMMENTS

The number of students presenting for Further Mathematics Examination 2 in 2002 was 20 402, an increase of 4.04% who sat in 2001. The selection of modules chosen by students in 2002 is shown in the table below.

MODULE	%
1 Number patterns and applications	48.6
2 Geometry and trigonometry	83.0
3 Graphs and relations	48.6
4 Business-related mathematics	66.1
5 Networks	39.6

The paper this year was of comparable difficulty to 2001 with Geometry and trigonometry the most challenging module. Some sections required careful reading of the information provided, a necessary feature of an *analysis* examination. Teachers could usefully guide students on suitable strategies, such as underlining important information that will assist in answering analysis questions.

Most students were able to complete the core questions and to make a good start on each of their three selected modules. A significant number of students made no attempt at the core questions but managed to obtain some marks on one or more modules.

Within each module, questions were designed to become increasingly more challenging in the latter stages. In some instances students had to rely on a previously calculated result for a consequential result. The marking scheme was structured to accommodate this.

There was evidence of students scanning, rather than carefully reading, questions. This is most often seen where there has been confusion between conditions, for instance between arithmetic or geometric sequences or simple and compound interest calculations.

In 2002, there seemed to be rather less reliance upon 'quick fix' formulas than observed in 2001. Students generally used the standard formulas and transposed as necessary.

Sometimes, answers were unreasonable and, upon checking, this should have been obvious to the student. Students should be encouraged to check that their solutions are suitable for the given information.

Areas of weakness

Parts of the core questions required a good understanding of residual analysis. Many students had difficulty explaining when a residual plot indicates that data has been reasonably linearised. Re-coding of years caused difficulties for some students. Concerns were noted in the modules as follows:

Module 1 – Number patterns and applications

- working with proportionality
- confusion between T_n, T_{n+1} and T_{n-1} in difference equations
- recognising a sequence as being geometric rather than arithmetic

Module 2 – Geometry and trigonometry

- students who consider that all triangles are right triangles
- rounding off to the nearest ten
- understanding bearings
- diagrams are not drawn to scale unless indicated

Module 3 – Graphs and relations

- finding the gradient of a line
- finding the equation to a given line
- solving simultaneous equations
- sketching a constraint where, for example, $x \geq 2$
- considering further points if two intersections are found to give the optimum solution

Module 4 – Business-related mathematics

- working with a decimal percentage
- understanding terms in relevant formulas
- flat rate depreciation
- reducing balance calculations
- the annual interest rate converted to a rate per payment for the annuities formula
- understanding input and output in the TVM solver – especially negative values

Module 5 – Networks

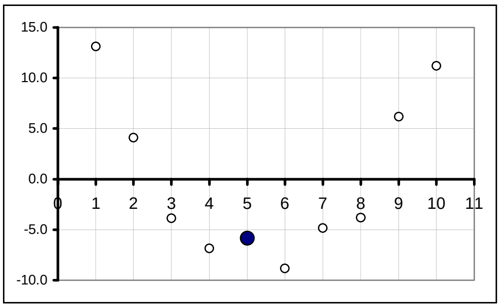
- full requirement for an Eulerian path
- spanning trees
- shortest circuit

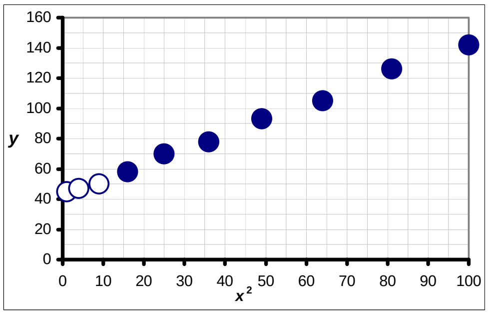
Many students did not show any working out but simply stated their answer. Where more than one mark is available an incorrect answer with no marking will not obtain any marks; however, an incorrect answer with some working may be awarded method marks.

Some students continue to write numerical answers only, without any working shown. This was particularly noted in the Business Related Mathematics module where graphics calculator programs can provide quick answers, whether applicable to the question or not. Clearly, method or consequential marks cannot be allocated if the stated answer alone is provided and is incorrect. This is especially of concern where a student may have correctly performed the mathematics but had given an answer with fewer decimal places than required. Such students scored 0 marks for their answer although a correct answer will score the full marks for a question. Teachers should continue to encourage students to clearly show their working out.

SPECIFIC INFORMATION

Core – data analysis

Question	Marks	%	Response
Question 1	**a**		**[Year]** 'Time' was accepted here but most students obtained the correct answer.
	bi		**[$10 990]** One mark for 10.99 Many students found an answer of 10.99, not realising this indicated **thousands** of **dollars**.
	bii		**[$185 750]** One mark for 185.75
	ci		**[-5.85], +5.85 and 5.9 were also accepted.** Teachers should use the value convention where residual = actual value – predicted value. This convention gives a residual plot that can be visually related to the data and regression line plot. For 2002, the sign was ignored in the calculation but a negative value was required in the diagram for 1cii below.
	cii		 Students who correctly plotted their residual value from 1ci in the **negative** region gained a consequential mark here.
	di		**year^2 (x^2)** | **1** | **4** | **9** | **16** | **25** | **36** | **49** | **64** | **81** | **100**

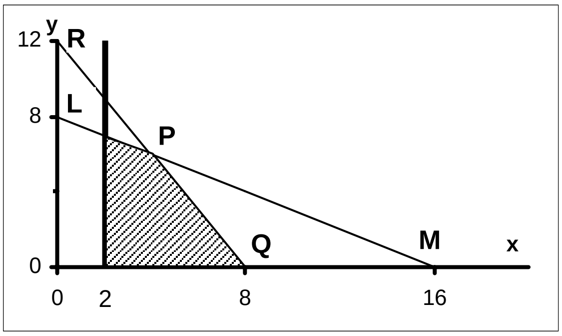
	dii	 One mark for a generally correct linear pattern; 2 marks for all points correct. The most common error was the incorrect plotting of the point for (81, 126) which was often shown around (81, 132) One mark for each correct number and correct to two decimal places, although 42.89 + 1*x* was accepted. One mark total was awarded if the correct numbers were in the incorrect answer boxes. Many students still do not make effective use of a graphics calculator to find the equation for the regression line. There was one mark for 267.89; and consequential marks were available for correctly substituting 5 or 15 into their equation from 1diii but which had to involve the square of *x*.
	cv	**There is no pattern in this new residual plot, the points appear to be randomly distributed with respect to the horizontal axis.** This question was not well answered. Many students indicated that there were ‘an equal number of points above and below the horizontal axis’. This was not accepted as a pattern could still exist under such a condition.

Module 1– Number patterns and applications		
Question 1	**a**	**[8 new spells]** Most students got this answer.
	b	**[155 spells in total]** Many students misread this question and found 29, the number of spells learned in Week 10 rather than the total in the ten weeks. a = first term = 2 and d = common difference = 3 so: $S_{10} = \frac{10}{2}(2\times 2+(10-1)\times 3)$ $= 5\times(4+9\times 3)$ $= 5\times 31$ $= 155$ Students may be expecting questions in the order that the concepts are usually taught but there should be no such presumption for any examination.
	c	**[Week 17]** Generally well done.
Question 2		**[6 grams]** One mark for a clear recognition of 9 parts required for the recipe. This question caused many difficulties for a simple calculation of $\frac{2}{9}\times 27$.
Question 3	**a**	**[T_1 = 19]** One mark for substituting 27 for T_{n+1} in the difference equation. Many were unable to use the difference equation backwards and found T_3 instead by substituting 27 for T_n rather than for T_{n+1}.
	b	**[267]** Students may have developed a table to find T_6 but this was not very evident.

	c	**[Neither – there is no common difference and no common ratio]** One mark for stating 'Neither'. Generally not well done as many incorrectly justified either arithmetic or geometric. Some students correctly justified 'a combination of arithmetic and geometric' and gained full marks.
Question 4	**a**	**[5 km^2]** Most students got this.
	b	**[157.5 km^2]** One mark for substituting their values of *a* and *r* into the geometric series formula. Most students got this.
	c	**[390 km^2]** One mark for 160 as the sum of the infinite series. Many did not identify this as an infinite geometric series question. Of those who did, a number had $a = 230$. Of those who correctly used $a = 80$ and $r = 0.5$, most forgot to add the initial 230 km^2 to the 160 km^2 from the infinite series.

Module 2 – Geometry and trigonometry		
Question 1	**a**	**[563 336 m^2]** An answer in the range [563 330, 563 340] was accepted in recognition of different ways this question could have been tackled. One mark for substituting relevant dimensions into $Area = \frac{1}{2}bc\sin A$. Generally well done although there continue to be many who insist on using $A = \frac{1}{2}bh$ for all triangles.
	b	**[1398 m]** One mark for an appropriate use of the sine or cosine rules toward finding the length PR. Many did not consider, or understand, that the length of any one side of a triangle cannot exceed the sum of the other two lengths. Consequently, some impossible lengths for PR were offered. Several students tried to apply Pythagoras' Theorem to triangle QPR.
	c	**[048°T], 48° was also accepted** One mark for calculating angle QPR = 33°. Many did not seem to fully understand bearings and did not know what to do with the 015°T given for the bearing from P to S. A number calculated angle PRQ and gave this answer as the required bearing. Others assumed angle SPQ was a right angle. A common incorrect answer was 42°.
	d	**[398 980 m^2]** An answer within the range of [398 200, 399 100] was accepted. This allowed for students who used rounded off figures through their calculations or other extended methods that might have included breaking up into two right triangles. There was one consequential mark for finding angle SPR by 90 – 15 (their angle QPR from within 1c) and another for using their angle SPR in $Area = \frac{1}{2}bc\sin A$ to get a final answer. Rounding off to 'the nearest 10m^2' caused some difficulties with many simply dropping off the units column, for example 563 336 became 56 333 or 56 334.
	e	**[962 320 m^2]** Most students got this consequential mark as the sum of their answers for 1a and 1d.
Question 2	**ai**	**(BC = 291m)** Generally well done.
	aii	**[CD = 421m]** One mark for using the length of 400 m in a Pythagoras substitution. Most students got this answer but a significant number showed no calculation. Many students clearly *assumed* that CD bisected the 130 m between the offsets to C and to D. This led to several adding $\sqrt{160^2 + 65^2} + \sqrt{240^2 + 65^2}$ which gave the correct answer but with an incorrect method. Some students added the lengths of the existing straight sections of road from C to D.

	b	**[Yes – the pine tree is on the line CD. This must be supported by relevant calculations and analysis]** One mark for a calculation to find the location where CD cuts AX. This was most often done by proportion although some students used trigonometry. One consequential mark for comparing their clearly calculated position of the tree with the stated position of the tree at 450m north of A. This included any argument that incorporated an incorrect assumption about CD bisecting the 130 m section. Others measured the diagram and assumed it was drawn to scale. This did not qualify for the first mark above but could have applied to the consequential mark. A statement that the tree was on the line was not awarded any marks if it was not supported by calculations and interpretation of results. On the other hand, a student could gain two of the three available marks if their incorrect calculations had justified their assertion that the tree was not on the line. Unacceptable arguments were those that considered only the existing sections of road between C and D and found that the tree would be along this path.

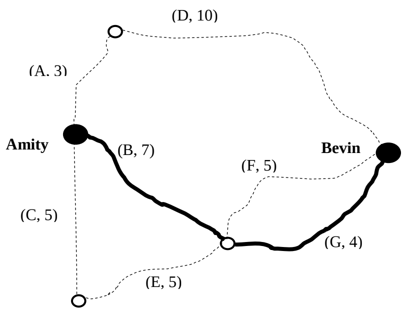
Module 3 – Graphs and relations		
Question 1	**a**	**[3.5 minutes]** Reasonably done with 3 minutes being a common incorrect answer.
	bi	**[Section E-F]** Generally well done. Other answers accepted included ‘11–14 minute mark’ or ‘between 800 m and 1400 m from Frog Hollow’.
	bii	**[200 m/min], also accept 12 km/h** One mark for the correct quantity and 1 mark for the correct unit of speed.
Question 2	**a**	**[$3x + 2y = 24$], or similar** Most commonly, students left answers as $30x + 20y \leq 240$. The inequality sign was ignored in this instance despite the question asking for an equation to a line.
	b	**[$x = 4$, $y = 6$]** One mark for each value allocated correctly. An ordered pair was accepted if brackets were used. One mark for an attempt at simultaneous equations if no correct answers were shown.
	ci	**[$T = 5x + 7y$]** Generally well done.
	cii	**[$T_{max} = 62$]** One mark for substituting **two** relevant points into their equation from 2ci. One consequential mark for their correct conclusion after having also substituted their point from 2b.
	d	 One mark for the correct line at $x = 2$. One consequential mark for shading a region above their line and below the two given lines LM and RQ. A number of students were not able to answer this question fully. Many showed no line or an inclined line for the given constraint of $x \geq 2$.

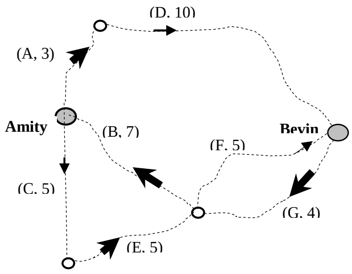
	e	**[Solutions at (2, 7) and (4, 6). There is no solution at $x = 3$ since this gives y as a non-integer decimal]** One mark for finding the point (2, 7). One mark for finding the two points (2, 7) and (4, 6) as valid points along the line LM. One mark for identifying that $2 \le x \le 4$ and that x=3 would give a decimal value for y which must represent only an integral number of rides. Therefore, there are only two integral solution pairs. Many students only produced the solution (4, 6) without finding (2, 7) as being valid. Some students checked points that did not match their feasible region. Almost all students failed to discuss whether $x = 3$ was valid. Students seemed to consider only the points at the corners of the feasible region rather than any possibilities where intervening points might also exist.

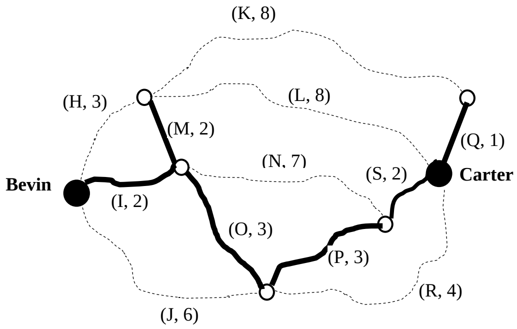
Module 4 – Business-related mathematics		
Question 1	**ai**	**[$600.00]** Generally well done with many answers given as $600 which was accepted.
	aii	**[$3085.50]** Generally well done. A common answer offered was $3085.5 which was accepted.
	b	**[$7.20]** Many students gave incorrect responses. The decimal percentage (0.3%) confused some while others did not read that this rate was per month rather than annually. A common incorrect answer was $10.80.
Question 2	**a**	**[5.2%]** Generally well done.
	b	**[1.048]** Poorly answered by many students showing little understanding of the significance of c. Common incorrect answers were 0.048, 4.8, 1.05 and 1.048^2
	c	**[5.07%]** One mark for writing $4416 = 4000 \times R^2$ or equivalent. Some incorrectly wrote $416 = 4000 \times R^2$ while others used the SI formula. A difficulty was evident with solving the correct equation for R. Some students showed no working and wrote an answer of 5.1%. As the question required two decimal places, these students scored 0 marks since there was no working to indicate that a simple rounding-off error may have occurred.
Question 3	**a**	**[r = 14.1%]** One mark for finding depreciation is $3320 per year. The poor setting out of student work for this question sometimes made method marks difficult to allocate.
	b	**[r = 21.6%]** One mark for $7000 = 23\,600 \times R^5$ or equivalent. This question caused difficulties with setting up the relevant equation. Several used the straight line method here. Many could not solve $R^5 = \frac{7000}{23\,600}$, which can be done numerically using $\sqrt[x]{y}$ function or the equation solver on a graphics calculator.
Question 4	**ai–ii**	**ai. [1.02]** **aii. [16]** One mark for each answer. Students continue to misrepresent the variables in the annuities formula despite similar questions in previous years. Common incorrect values for R were 1.08, 0.08 and 0.02 and for n were 4 and 12.

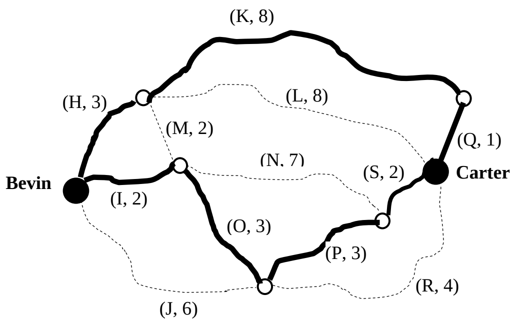
	b	**[Yes, the loan will be repaid within 4 years as a minimum required payment of $1473 is less than the proposed $1500.]** One mark for correctly finding a balance or for the minimum repayment for a loan over four years. An appropriate justification for their conclusion was required for any mark allocation. Many students had difficulties with interpreting their calculations. Many achieved a figure of -$503.21 for the Future Value with the time-value-money (TVM) Solver or for ***A*** in the annuities formula. However, the negative property of this result was often not understood. A number of students multiplied $1500 by 16 to get a total of $24 000 paid over four years. They then ignored any interest calculation and simply stated that, as this exceeded the $20 000 borrowed, ‘the loan must be repaid within the four years’. This argument was not accepted. The annuities formula is provided on the formula sheet and students should be familiar with all the terms in this formula. Method marks may be available if the student shows a full substitution into this formula. It was evident, however, that many did not appreciate the significance of ***A***, especially where this is negative. The TVM Solver is a complementary application of graphics calculator technology that can quickly produce the required results. Students who choose only this method are advised to list their input data (including relevant negative signs) as their ‘working out’ if potential method marks are to be accessed in the event theirs is not the correct answer.

Module 5 – Networks and decision mathematics

Question 1	**a**	**[11 km]** Many chose the roads B and F as the shortest path and wrote 12 km. One consequential mark was awarded if path B-F was indicated on the graph for 1b.
	b	 B-F was the most common incorrect shortest path from Amity to Bevin.
Question 2	**a**	**[There are exactly two odd nodes and the race begins at one of these.]** One mark for noting there are only two odd nodes. One mark for noting that the race starts (and ends) at one of these odd nodes. Most students achieved the first mark only. Unacceptable answers included ‘there is an even number of odd nodes’ and ‘the sum of the orders of all nodes is even’ and ‘there is an even number of nodes’. Most students did not identify the second, necessary requirement for an Eulerian path.
	b	**[A combination of at least two roads chosen from S, N or P to indicate the intersection]** This was generally well answered.
	c	**[Road J must be avoided – otherwise there is no way, under the rules, of returning to the left side of Bevin to complete roads B, E, F and G]** One mark for the correct road and 1 mark for the explanation. Generally done well.
	d	**[Road S]** Generally done well.

<table>
<tr><td></td><td>e</td><td>

One mark for indicating any two correct arrows for the roads B, E and G.
Many students had the arrows reversed on roads B and G. This seemed to indicate trouble in comprehending the question as all travel on these roads was in the direction of Bevin. Many forgot to mark in the direction along road A. This, however, was regarded as a trivial omission in the context of this question and was not penalised.</td></tr>
<tr><td>Question 3</td><td>a</td><td>

One mark for drawing any tree that covered all nodes. Incorrect trees commonly included road H or excluded one node. Some included a circuit within their ‘tree’.</td></tr>
<tr><td></td><td>bi</td><td>

Most students drew this circuit but a variety of alternatives were given, some of which excluded one node.</td></tr>
<tr><td></td><td>bii</td><td>[22 km]
One mark for the correct length for an incorrect, but complete, circuit shown in 3bi.</td></tr>
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