VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2004 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students who presented for the Further Mathematics examination 2 in 2004 was 21 173. The selection of modules chosen by the students in 2003 and 2004 is shown in the table below.

MODULE	% 2003	% 2004
1 Number patterns and applications	51.02	47.48
2 Geometry and trigonometry	90.05	89.84
3 Graphs and relations	49.70	51.94
4 Business-related mathematics	68.60	68.06
5 Networks and decision mathematics	40.64	42.68

Within each module, questions were designed to become increasingly challenging in the latter stages. In some instances, students had to rely on a previously calculated result for a consequential result. The marking scheme is structured to accommodate this.

In general, incorrect rounding off, including rounding too early in a question, remains an issue. Many students displayed poor rounding technique in Question 2eii in the Core section. It required an answer correct to one decimal place. The calculated answer was 23.947, which rounded down to 23.9. Many students apparently first rounded 23.947 to 23.95 and then gave an answer of 24.0.

In a significant number of cases, student answers suggested that the question was not carefully read. This especially occurred in Number patterns and applications (Questions 2c and 3b), Geometry and trigonometry (Questions 2a and 2c), Graphs and relations (Questions 2 and 3), Business-related mathematics (Questions 1c and 2e) and Networks and decision mathematics (Questions 1bii and 2d).

Some students still give answers without any detailed working. Where the answer was correct, full marks were awarded; however, without working, an incorrect answer or consequential answer could not be awarded any marks.

Rounding errors were penalised once per paper. Answers written to fewer decimal places than required were not considered rounding errors and scored zero. Where students engaged with a question beyond the required answer, a penalty was applied if the extension was in error (for example, an incorrect simplification).

Students should write a leading zero for a decimal absolute value of less than one (for example, 0.5 rather than .5). Answers are often written in pencil and a faint decimal point without the leading zero may result in the student's answer not being clearly read by assessors. Also, students must understand that a hyphen or blank space does not suffice in place of a required answer of zero.

Several instances were observed where the student had crossed out a solution without replacement. Deleted work is not assessed. Students must avoid deleting work without providing an alternative solution.

Areas of strengths

Core

- Finding the range of data
- Completing a two way frequency table
- Calculating percentages.

Number patterns and applications

- Calculating common ratio
- Working with an arithmetic sequence.

Geometry and trigonometry
- Working with right triangles
- Calculating the area of a non-right triangle
- Using the cosine rule to find the length of a side opposite a given angle
- Using the sine rule to find the length of one side
- Applying equilateral triangle facts.

Graphs and relations
- Substituting into linear formulas
- Drawing a straight line graph with *y*-intercept at $y \neq 0$
- Interpreting break even point on a graph.

Business-related mathematics
- Calculating the total of a given number of payments
- Understanding that the difference between repayments and initial cost represents interest paid on a loan
- Calculating depreciated value using reducing balance formula.

Networks and decision mathematics
- Finding the shortest path between two points on a simple network
- Understanding the requirements of a Hamiltonian circuit

Areas of weakness

Students should be instructed on the effective use of reading time. This will assist in minimising the number of instances where students fail to follow instructions or misread questions.

Core
- Interpreting calculator output for least squares regression line
- Interpreting boxplots
- Understanding the application of a median figure.

Number patterns and applications
- Interpreting percentage increase from a common ratio figure
- Correctly substituting into the formula for a geometric series
- Solving difference equations.

Geometry and trigonometry
- Determining bearings
- Finding the 'centre of a triangle'; frequently assumed to be halfway between the base and the apex
- Using brackets when entering an equation into the calculator. Students must be aware that, without appropriate closing brackets, an entry such as $15 \times \sin 37 / \sin 49$ will result in evaluating $15\sin\left(\frac{37}{\sin 49}\right)$ rather than $\frac{15 \times \sin 37}{\sin 49}$.

Graphs and relations
- Simplifying a profit expression where the cost is represented by a binomial expression
- Determining the selling price required to achieve a certain profit.

Business-related mathematics
- Understanding what an effective interest rate represents
- Calculating the original amount that was increased by 10% to achieve a given final amount
- Understanding that in the annuities formula A = 0 if an loan is to be fully repaid
- Understanding that repayments include an interest component and hence each repayment is not fully directed to repaying the loan.

Networks and decision mathematics

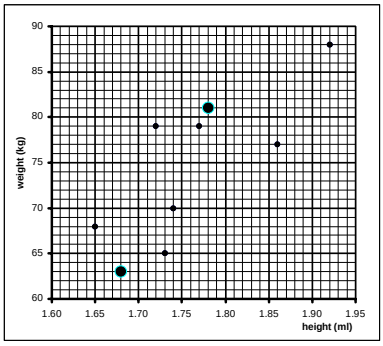
- Finding a critical path
- Performing critical path analysis
- Recognising that reducing the time to complete an activity at cost is unnecessary if it does not reduce the overall time for a project.

SPECIFIC INFORMATION

Question 1

a

Marks	0	1	Average
%	8	92	0.9



As this was the first question and only one mark was allocated for two points, the mark was awarded if either point was correctly plotted.

b-c, 2a

Marks	0	1	2	3	4	Average
%	4	16	23	17	40	2.7

b

-60.8, 76.8

Some students reversed these numbers and indicated some confusion in interpreting their calculator output. One of the two marks was then awarded as long as both numbers were correct and the negative sign preceded 60.8.

Some gave integer answers only.

c

weight, height

Some reversed these and gained no marks.

In a few cases, students referred to the *dependent variable* or *independent variable*. This was not accepted unless one of the terms was specifically, and correctly, written as *weight* or *height*.

Question 2

a

23.1

Generally well done although several forgot to square *height*.

b-c

Marks	0	1	2	3	Average
%	2	3	12	83	2.8

b
10.8

Most students wrote this correct number as the preferred answer.

Several wrote '20.6 – 31.4' which was accepted as the student did at least have some concept of *range*.

c

6	3
11	18

One mark per correct column

Generally done well although percentages were sometimes given as an answer. If these were correct figures, one of the two marks was awarded.

d-e

Marks	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Average**
%	5	9	13	18	23	19	8	5	3.6

d
Yes. 35.3% of men compared with 14.3% of women are overweight.

Required the comparison of two relevant, comparable and quoted percentages; one for males and the other for females. Students could not just quote raw numbers. No marks were given for 'yes' without any justification. No marks were given for comparing fractions that did not at least have common denominators.

Incorrect answers appeared to result from misinterpretation of the information. Some students ignored the data and expressed their own observations of weight distribution according to gender. Other incorrect answers used fractions such as 6/11 or 3/18, or their percentage equivalent.

ei
Interquartile range and median

Most students gave *median* as one answer. The most common incorrect term was *range*.

eii
23.9

Generally well done, but the most common error came from incorrect rounding techniques. Many students appeared to find the correct number of 23.947 which they rounded off to 23.95, with two decimal places. As this number is not **correct to one decimal place** students were not awarded the mark for the question. Some did further round 23.95 up to 24.0.

eiii
The median gives the better indication of the typical *BMI* as it is not affected by extreme values as is the mean.

This question was not done well as students did not generally appreciate the effect of extreme values on the mean. Nominating the median without some appropriate justification was not sufficient to gain a mark. The most common incorrect response suggested that the *mean* gave the better indication as 'the *mean* takes all data values into account while the *median* is only the middle number'. Another common incorrect interpretation was 'the *mean* gives the average but the *median* is only the midpoint'.

Most students who correctly named the *median* then inappropriately referred to it not being affected so much by *outliers*. This answer scored only one mark as there was no information in the boxplots to suggest the presence of any *outliers*, although *extreme values* were evident.

A reference to the median being more appropriate to skewed distributions such as here was acceptable.

Module 1 – Number patterns and applications

Question 1

a-e

Marks	0	1	2	3	4	5	6	Average
%	12	7	12	18	19	15	16	3.3

a

$\frac{2420}{2200} = 1.1$

Most students were able to show some appropriate division or multiplication that showed the common ratio was 1.1.

b

10%

Many students got this wrong. A common incorrect answer was 110%.

c

2928

Some students did not round this answer to a whole number of heating systems.

d

31 874

A number of students did not round this answer to a whole number of heating systems. Several tabulated the sequence and added the terms rather than use the appropriate series formula. Some students found only the number of heating systems produced in the tenth year.

e

$b = 1.1$, $c = 0$

Generally, this question was poorly answered, suggesting that many students continue to struggle with difference equations. Most were unable to apply the concept of the geometric sequence and common ratio to the difference equation. Many students left the box for c blank. This was not accepted even though $c = 0$. A common incorrect answer was $c = 2000$.

Question 2

a-c

Marks	0	1	2	3	4	5	Average
%	8	9	12	21	19	31	3.3

a

$3740

Students should write a dollar sign before a number that signifies an amount of money. Many did not do this, but lost no mark if their number was correct.

b

16 outlets

The most common incorrect answer was 11 outlets, which did not allow for the additional five outlets that came with the $3500 basic heating system.

c

$4780

Many students failed to find the *cost* of the required system. Common answers of 21 outlets were found by ratio and gained one of the two marks. The second mark was applicable to the *cost* of this system and this had to take into account that a basic system plus 16 additional outlets was needed.

Question 3
a-b

Marks	0	1	2	3	4	Average
%	67	7	10	5	12	0.9

a
1850

Generally very poorly answered, again indicating concerns with difference equations. Most students did not transpose. One mark was available for a correct attempt at substituting $S_3 = 2224$ into a reasonable attempt at transposition.

b
92.1%

Very few students got this answer correct. Many used only one or both figures from the third year rather than the totals of the first three years. The most common incorrect answer among those who attempted this question was
$\frac{S_3}{M_3} = \frac{2224}{2420} = 91.9\%$

Module 2 – Geometry & trigonometry

A significant number of students wrote only the answer to questions in this module, suggesting that they may have used calculator programs to generate their answers. While correct answers to the correct number of decimal places were awarded full marks, teachers are advised to warn their students that if their answer is incorrect they will have no access to method or consequential marks. This especially impacts on students who write their answers with the incorrect digit in the required decimal place or beyond, or who round to fewer places than required.

Several students evidently operated in radian mode throughout. They were awarded some marks within the module if their working out was evident. It is the student's responsibility to know how to operate their calculator.

There continues to be evidence that some students do not appreciate the formulas in this module and use them inappropriately; for example, using the cosine rule to find an area.

Question 1
a-d

Marks	0	1	2	3	4	Average
%	8	11	15	25	41	2.8

a
70°

The most common incorrect answer was 20°. Some used the length *BC* to find the angle *ABC*. Premature rounding off then produced an incorrect answer.

b
10.6 m

Most were able to apply Pythagoras' theorem here. Some used $\cos 70^\circ = \frac{x}{3.6}$ to find *BC* = 10.5 which was allowed if the working was shown. Some used the sine rule to find *BC*.

c
10.2 m

Generally well done but several students forgot to find the square root at the end of the cosine rule application.

Many students rounded off too early before finding the square root, resulting in incorrect answers of 10.25 or 10.3.

d
8.6 m², accept 8.8 m² if Heron's formula was used with rounded data

A common expression of the answer was Area = 8.6^2 which was accepted here since the required calculation had obviously been done. Some used Heron's formula which gave an answer of 8.77 m^2.

Question 2
a-c

Marks	0	1	2	3	4	5	Average
%	24	13	10	20	10	23	2.5

a
282° T

Very poorly answered. Many students continue to struggle with bearings. The most common incorrect answer was 102°, which was awarded one of the two marks as it is the back bearing of the one required.

A significant number of students were unable to find an appropriate angle that could usefully be applied to find the bearing.

b
6.9 km

Generally well done

c
18.6 km

One of the two marks was awarded for a correct calculation for the length of *UV*. Many incorrectly quoted the length of the shortest of the three sides of the triangular course.

Question 3
a-b

Marks	0	1	2	3	Average
%	17	42	15	26	1.5

a
60°

Incorrect answers of 90° and 45° were seen.

b
0.289 m

Generally very poorly answered. This question required a simple inverse tangent calculation using 30° from the bisection of angle *YXZ*. One of the two marks was awarded if the student recognised the correct 30° angle. Most commonly, students calculated the height of triangle *XYZ* and then said point *O* was halfway up this height. Incorrect use of the calculator was evident in some cases where $0.5 \times \tan 30^o$ was entered as $\tan 30^o \times 0.5$ which resulted in finding $\tan 15^o \approx 0.268$.

c

Marks	0	1	2	3	Average
%	75	19	2	4	0.4

0.214 m

Very poorly answered, as the ratio was commonly misinterpreted. Many students left this question blank.

Most students incorrectly stated that the circle area was half the triangle area rather than one third. For these students, a mark was available for working that attempted to solve $\pi r^2 = \frac{1}{2} area\ of\ the\ triangle$ for r.

Module 3 – Graphs & relations

Question 1

a-c

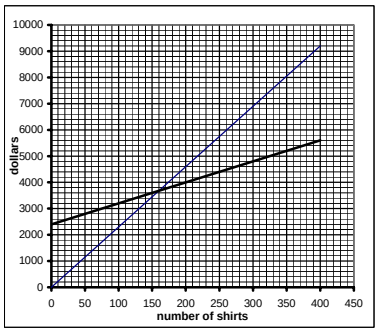
Marks	0	1	2	3	4	Average
%	6	6	21	11	56	3.1

a
$5600

b
75 units

Generally well answered.

c



Some students appeared to miss this question.

d-f, 2

Marks	0	1	2	3	4	5	6	Average
%	17	8	13	13	14	10	25	3.3

d
160

This could have been calculated or read off the graph.

e
$P = 15x - 2400$

The most common error was in simplifying $P = R - C$ where brackets were not applied to the cost function. This gave the incorrect answer of $P = 15x + 2400$. This was awarded one of the two available marks.

The relationship of $P = R - C$ does not seem to be well understood by a number of students.

f
$2775

This mark was awarded for a correct substitution in the student's equation for Question 1e.

Question 2
$31.20

Several students wrote this as $31.2 which was awarded the mark, although teachers should address such notation in class.

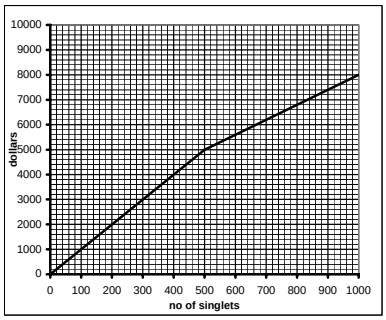
Question 3
a-b

Marks	0	1	2	3	Average
%	40	28	12	20	1.1

a
$5720

Many students did not understand how to interpret this formula. Many used both formulas and quoted both answers, while others decided that the way to get only one answer was to subtract or add the two results.

b



Many students sketched only one line. Others typically sketched two complete, intersecting lines and gained one mark if these were correct.

The second mark came from correctly terminating each section of the graph at (500, 5000).

c

Marks	0	1	2	Average
%	79	4	17	0.4

750

Poorly answered. The solution to the equation $2000 = R_s - C_s$ was needed, with the appropriate revenue formula applied. If $R_s = 10x$ was used, x = 583 should have been discarded as inappropriate to $R_s = 10x$. The most common error was to write $2000 = 4x + 1500$, giving an answer of 125.

Module 4 – Business-related mathematics

A significant number of students wrote only the answer to questions in this module. This suggests that they may be using calculator programs to generate their answers. While correct answers to the correct number of decimal places were awarded full marks, teachers are advised to warn their students that if their answer is incorrect they will have no access to method or consequential marks if only answers are provided. This especially impacts on students who write their answers with the incorrect digit in the required decimal place or beyond or who round to fewer places than required.

Teachers should be aware that a significant number of students used inappropriate formulas, thus indicating a very poor grasp of a very practical topic.

Question 1

a-c

Marks	0	1	2	3	4	5	Average
%	6	8	42	30	11	2	2.4

ai

$720

aii

$70

bi

21.5%

Quite poorly answered. Many students used $T = 6$ rather than $T = \frac{6}{12}$ or $T = 0.5$. Several found $\frac{70}{650} \approx 10.8\%$.

bii

The effective rate is greater than the simple interest rate as *SI* does not take into account reductions in the balance owing as repayments are made.

Students demonstrated poor understanding of the concept of effective interest rate. A common incorrect explanation was that 'the simple interest rate in this example was for only six months while the effective rate is for a whole year'.

Many quoted the formula $Effective\ \ rate \approx 2 \times Flat\ rate$ or $Effective\ \ rate = \frac{2n}{n+1} \times Flat\ \ rate$

c

$590.91, accept $590.90

Very poorly answered. Most students incorrectly calculated $650 - 10\% \times 650 = \$585$.

d

Marks	0	1	2	3	Average
%	46	22	13	19	1.1

Flat rate gives a greater depreciation by $28.41

Most students calculated the *book value* with reducing balance depreciation of $680 \times (0.85)^5 = \$288.41$. Many found the flat depreciation amount of $12\% \times 650 \times 5 = \$390$, but then misinterpreted this as being equivalent to the book value.

A large number of students quoted and misused a formula written as $BV_T = P - dT$. The most common error by those who used this formula was to substitute $d = 0.12$ or $d = 12$. Such a formulaic approach should be undertaken with caution; students would be better served by gaining a greater appreciation of the concepts.

Several answers ignored the percentages altogether and suggested that 'flat rate will give the greater depreciation as it reduces by the same value while the reducing balance depreciation gets smaller every time'.

Question 2

a-e

Marks	0	1	2	3	4	5	6	7	Average
%	29	21	11	7	8	8	10	6	2.4

a

$A = 0$, $n = 48$, $P = 12\,000$

The first mark was awarded for any **two** answers correct. The second was given for all three correct.

A common error was A = 12 000, or many students simply left it blank. Other incorrect values included $A = 48$ and $n = 4$.

b
$290.15

Several examples of $\frac{12000}{48} = \$250$ were found.

Several students substituted into the annuities formula rather than use the financial function on their calculator. Teachers should explain the quantities, terms and signs of the annuities formula, but students need not use the formula in a calculation. They can be expected to explain, for example, 'why is a fractional term subtracted?' or 'what does *A* stand for?' Teachers should instruct students on how to use the TVM solver on their calculators. Students should show a listing of their input, including negative signs as appropriate.

c
$1927

A consequential mark was available for a correct calculation of (48 × their reasonable answer Q2b) – $12 000.

d
$1311

Very poorly answered. A common incorrect answer was $1740. The required calculation used $n = 6$, $I = 7.5$, $PV = 12\,000$, $PMT = 290.15$ to find $FV = 10\,688.78$ as the amount owing after six monthly payments. This was then subtracted off the initial loan of $12 000.

Many students reduced the loan by the value of six repayments without consideration of the interest component in each of those repayments.

ei
$n = 42$, $P = 10\,689$

A mark was available here if $n = 42$ and the value for *P* given related to their answer in question 2d.

eii
$293

Many students did not use the new interest rate of 8%. A mark was available here if their answer matched $I = 8$ and their numbers from question 2ei.

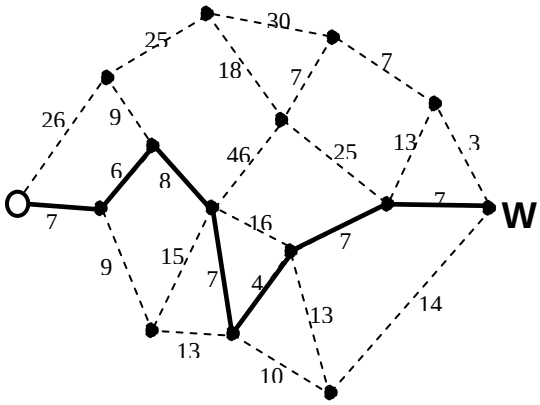
Module 5 – Networks

Question 1

a-b

Marks	0	1	2	3	4	5	Average
%	2	5	11	10	24	49	4.0

ai

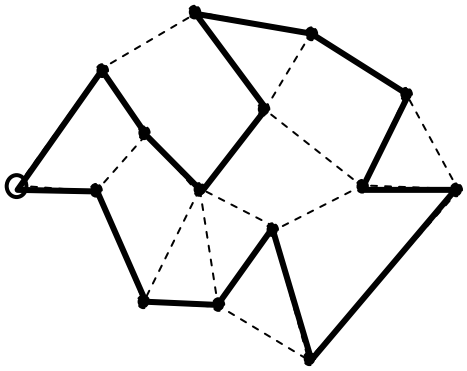


aii
46 km

A mark was available if the total matched the path in question 1ai.

bi
Hamiltonian circuit

bii



Incorrect answers included paths that were not circuits back to *Q*, that were not Hamiltonian or where points were missed. One mark out of two was available for a circuit with one missed point or one retraced edge only.

Question 2
a-b

Marks	0	1	2	3	4	5	Average
%	26	23	17	14	10	10	1.9

a
$A \rightarrow 1$, $D \rightarrow 4$, $F \rightarrow 10$, $K \rightarrow 12$

One mark per answer.

Very poorly answered, with many blanks. Generally, $D \rightarrow 4$ was the most common correct value found by subtracting time for *B* from the EST of *H*. $A \rightarrow 0$ was a common incorrect value, although *A* has a 'float time' of one hour.

The value $F \rightarrow 10$ comes from a 'float time' found by subtracting the sum of the times for *F* and *I* from the sum of the times for *E*, *G* and *J*. The value $K \rightarrow 12$ comes from subtracting the starting time for *K* from the quoted *least time required for the project* (30 hours).

b
B-C-E-G-J-K

A significant number of students incorrectly found their critical path started with *A*.

c-d

Marks	0	1	2	3	4	5	Average
%	36	30	11	6	5	12	1.5

c
15 hours

Again, poorly answered, indicating limited understanding of critical paths.

di
B & *F*

Many students paid to reduce activities that would not affect the critical path. Activities *A* and *E* should not have been reduced. Some suggested that *C* and/or *I* should be reduced, but these options were not available. Clearly activity *T* could not be reduced to below zero.

One mark was awarded for reducing *B* without reducing *A* as well. One mark was awarded for reducing *F* without reducing *E* as well. An earned mark was subtracted for each inclusion of activities *C*, *T* or *I*.

dii
3 hours

Poorly answered as a result of errors in question 2di.

diii
$200

Poorly answered as a result of errors in question 2di.