2003 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students presenting for Further Mathematics Examination 2 in 2003 was 24 327, compared to 20 402 who sat in 2002. The selection of modules chosen by students in 2003 is shown in the table below.

MODULE	%
1 Number patterns and applications	51.02
2 Geometry and trigonometry	90.05
3 Graphs and relations	49.70
4 Business-related mathematics	68.60
5 Networks	40.64

The mean score for the 2003 examination indicates that it was slightly less difficult than for 2002 with the 'Data analysis' core the most challenging section. For each of the five modules, the mean scores were quite similar and indicated that modules were of comparable difficulty.

There is strong evidence that many students do not read the entire question before responding. This occurs particularly where there are more than two or three explanatory lines at the start of a question. Students should carefully note the reading and understanding required for the analysis style of questions in Examination 2; especially in the 'Data analysis' core in 2003. Many students did not obtain the correct answers for questions that were fairly straightforward *skill* questions.

Most students were able to begin each of their three selected modules but encountered difficulty in clearly understanding many of the later questions in each module. Within each module, questions were designed to become increasingly more challenging in the latter stages. In some instances students had to rely on a previously calculated result for a consequential result. The marking scheme is structured to accommodate this.

Some students still gave answers without any detailed working. Where the answer was correct, full marks were awarded. However, without working, an incorrect answer cannot be awarded any marks.

However, some students laboriously show a calculation for a mean and standard deviation by applying the formulas to a table. This is not required for Further Mathematics examinations. Students should know how to enter the data into a graphics calculator and then to use the inbuilt functions to find the sample mean standard deviation. There will be no instance in a Further Mathematics examination where a population standard deviation is required.

Rounding errors were penalised once per paper. This sometimes occurred if a student rounded off too early in a multi-step calculation. Answers written to fewer decimal places than required were not considered *rounding errors* and scored zero. Where students engaged with a question beyond the required answer, a penalty was applied if the extension was in error (e.g. an incorrect simplification).

Students should be encouraged to write a leading zero for a decimal absolute value of less than one (for example, 0.5 rather than .5). Answers often written in pencil and a faint decimal point without the leading zero can result in the student's answer not being able to be clearly read. Also, students should understand that a hyphen does not suffice in place of a required answer of zero.

Several instances were observed where students crossed out a solution without replacement. Deleted work is not assessed. Students should avoid deleting work without providing an alternative solution.

Areas of strength

Core

- finding the mean and the value of r for a table of data
- finding one quantity as a percentage of another
- calculating a predicted value using a formula.

Number patterns and application

- working with an arithmetic sequence.

Geometry and trigonometry
- using Pythagoras' theorem
- using trigonometry on a right triangle
- finding the area of one triangle using ½ bc sinA.

Graphs and relations
- plotting a straight line beginning at the origin
- completing a table of values for t^2
- plotting points on a graph
- finding the gradient of a line.

Business-related mathematics
- calculating the total of a given number of regular payments
- understanding what is meant by depreciation for one period.

Networks
- finding the duration associated with a stated path.

Areas of weakness

Core

Of those students who found the mean for the correct column of data, the sample standard deviation was often incorrect. A common error was to give the population standard deviation instead.

Plotting of a value of 11.30 on a scale with four intervals between 10 and 12 caused some difficulty. The term *trend* does not seem to be well understood and similarly students should be familiar with a range of examples of circumstances in which three-median smoothing is more suitable than three-mean smoothing.

Difficulties noted this year were:

Number patterns and applications
- recognising the initial term in a sequence for which n-1 can be applied in formulas
- finding the value of r from a description (of water usage)
- recognising the existence of a geometric sequence as opposed to an arithmetic sequence
- recognising that *total* water usage implied a *series* formula was needed
- substitution into a difference equation.

Geometry and trigonometry
- angles in a polygon
- applying Pythagoras' theorem to non-right triangles
- visualisation in 3-dimensions
- scale ratio between linear increase and volumetric increase.

Graphs and relations
- terminating a graph of a situation appropriately
- finding the value of the intercept of a line that is not shown as crossing the vertical axis.

Business-related mathematics
- reducing the purchase price by the deposit to determine the amount of A
- understanding to what the value of A in the annuities formula refers
- converting 3.0 cents to $0.03.

Networks
- calculating the value of a cut where a path goes from the *sink* toward the *source*
- the value of a cut if a path is <u>drawn</u> as going from right to left on the page
- requirement for a 'valid' cut that applies to *maximum flow*
- the significance of a *critical path*
- the meaning of *Earliest Starting Time (EST).*

SPECIFIC INFORMATION

Question 1

a–c

Marks	0	1	2	3	4	5	Average
%	8	13	18	17	19	25	**3.00**

a

mean = $ 6.51, standard deviation = $ 3.31 for one mark each.

Most students had no difficulty with finding the correct mean. Some incorrectly used the data for Number of calls instead of Cost. A common incorrect value given for the mean was $6.50. The standard deviation was very poorly done. Many students seemed to have little idea of what was required with some quite unlikely answers given despite having the mean correct. Some students incorrectly quoted the population standard deviation rather than that for the sample. A standard deviation of $3.30 was considered a round-off error.

b

75 %

ci

0.19

cii

0.8054

d–h

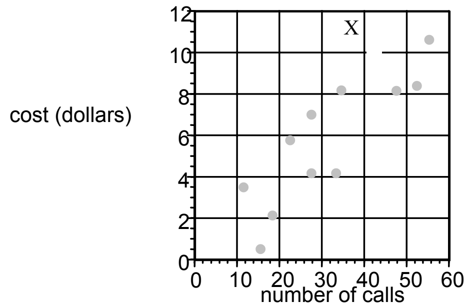
Marks	0	1	2	3	4	5	6	Average
%	14	20	22	18	12	9	5	**2.43**

d

linear, accept 'straight'

Unacceptable answers included 'positive', 'strong' or 'constant'.

e



The point had to be located in the upper right quarter of the correct square. Many placed the point at the halfway mark, either vertically or horizontally (this was not accepted) or left this question.

f

Cost, accept 'dollars'

A small percentage incorrectly quoted 'number of calls'.

gi

19 cents

This number had to be the coefficient of Question 1ci converted to cents for a consequential mark. An answer of 0.19 was not accepted. This answer often did not match the coefficient stated in Question 1ci.

gii

65 %

Many did not seem to relate this to the value of r in Question 1cii.

h

1.38 dollars, (accepted -1.38)

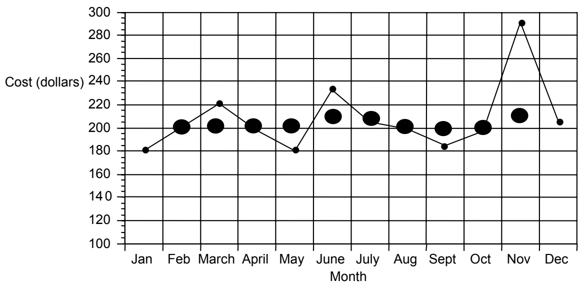
Many simply quoted the predicted cost of $7.12 without then calculating a residual value. A mark was given here for calculating an appropriate residual value using their equation.

Question 2

a–c

Marks	0	1	2	3	4	Average
%	56	13	10	12	9	**1.06**

a



Many seemed to have confused 3-median smoothing with either 3-mean smoothing or with the 3-median regression line (which is not smoothing). Neither of these was acceptable. The absence of plots for January and for December was essential for full marks. At least one mark was lost if more than two points were in error.

b

No. The smoothed time series is basically horizontal and shows no trend. An explanation was needed to support the negation of the supervisor's belief. There was confusion about 'trend' with numerous responses referring to 'an up and down trend' or 'a flat trend that goes up in November' or 'the trend goes up and down'. Students who incorrectly smoothed the data generally missed this mark as they usually indicated an increasing trend, which is not supported by the original data.

c

An outlier exists in November. A single outlier can have an undue influence in calculating a mean since its actual *value* is important. A median calculation simply regards the *position* of an outlier as a point 'at the edge', regardless of its value. Students performed poorly on the question. Some asserted that a time series or seasonal data should always be smoothed by the 3-median method.

Module 1 – Number patterns and applications

Question 1

a–d

Marks	0	1	2	3	4	5	Average
%	2	5	8	18	23	44	**3.88**

a

10 800

Some students did not consider the tank already contained 10 000 litres to begin with, and others incorrectly *reduced* the water in the tank by 800 litres.

b

14 000

$t_5 = 10\,000 + 5\times800$ or $10\,800 + (5-1)\times800$

One mark was given where a reduction of 800 litres had been made from the initial 10 000 litres in Q1a and their answer here reflected five such reductions.

c

a = 10 000, b = 800 (worth one mark each).

Most got this although some had them incorrectly reversed. A value $b = -800$ was accepted for one mark if this reduction occurred in Question 1a.

d

25 minutes

Most students got this answer; even if their answer for Question 1c was incorrect.

Question 2

a–c

Marks	0	1	2	3	4	Average
%	26	25	21	9	19	**1.69**

a

28 500 litres

Most students got this answer. Several incorrect examples were seen of attempting to reduce 30 000 by 5% by calculating $\frac{5}{30000}\times\frac{100}{1}$

b
24 435 litres
$t_4 = 30\ 000 \times (0.95)^4$ or $28\ 5000 \times (0.95)^{(4-1)}$

Many had little difficulty, although some regarded this as an arithmetic sequence. The answer was quite often completed by tabulation and a subsequent correct result was acceptable. Some incorrect applications of the formula included $t5 = 30\ 000 \times (0.95)^{(4-1)}$

c
Week 22

One mark was given for a correct tabulation attempt or for writing an appropriate exponential equation. Many tabulated $30\ 000 \times (0.95)^n$ to find their answer. Some may have used the Tables function on their graphics calculator and simply wrote an answer. Very few students tried to set up the equation $10\ 000 = 30\ 000 \times (0.95)^n$. Some solved this using logarithms.

Question 3

a–c

Marks	0	1	2	3	4	Average
%	47	16	12	10	15	**1.29**

a
1.1

Common incorrect answers were 0.1, 1:10, 600 and 10.

b
36 631 litres

One mark was given for setting up a relevant series equation, and another mark for an appropriate answer for a series using the student's appropriate, incorrect value of *r* from Question 3a. However, a number of students tried to calculate this using $S_5 = \frac{6000\left(1.1^{(5-1)}\right)}{1.1-1}$.

Many incorrectly tried to use the geometric sequence equation.

c
7881 litres

Commonly shown as a tabulation but was poorly done by many students.

d

Marks	0	1	2	Average
%	80	2	18	**0.38**

Week 4

Poorly done by many students, especially where they did not understand Question 3c. One mark was given for an appropriate side-by-side tabulation for the water usage for the two families.

Module 2 – Geometry and trigonometry

Question 1

1a–b and 2a

Marks	0	1	2	3	Average
%	12	17	28	43	**2.01**

a
8.50 metres

An answer of 8.49 was very common and was treated as a round-off error.

b
15 degrees

Most students got this although a number calculated the other angle of 75° and some used the Sine rule.

Question 2

a
360º ÷ 8 equal angles at the centre = 45°

Generally not well done. Some calculated the sum of angles at the perimeter, the single angle of 135°. Hence the single angle of 67.5° at the base of each isosceles triangles and then the 45°. Several circular arguments were seen that referred to the angles in one triangle. Some stated, without proof, there were two triangles in 90° and then said that one triangle must be 45° at the centre.

b–c

Marks	0	1	2	3	4	Average
%	25	9	15	20	31	**2.24**

b

15 square metres

Area = 8 X $\frac{1}{2} \times 2.3 \times 2.3 \times \sin(45^\circ)$

Generally quite well started. Many gained 1 mark for finding the area of only one triangle. Some rounded off too early and caused a round-off error.

c

1.76 metres

$GH = \sqrt{(2.3)^2 + (2.3)^2 - 2(2.3)(2.3)\cos 45^\circ}$ = 1.76 or $GH = 2 \times 2.3 \times \sin(22.5^\circ) = 1.76$

Many did this question quite well although some thought *OGH* was a right triangle.

One mark was available for an appropriate substitution into a valid formula. Many students forgot to take the square root at the end of applying the cosine rule.

d–e

Marks	0	1	2	3	4	Average
%	25	25	14	13	23	**1.84**

di

3.25 metres

$GK = \sqrt{(1.76)^2 + (1.76)^2 - 2(1.76)(1.76)\cos 135^\circ}$ or $GK = \frac{2.3}{\sin 45^\circ}$ or $GK = \sqrt{2.3^2 + 2.3^2}$

The use of 2.3 in a right triangle calculation is applicable as *GK* is the hypotenuse of a triangle with 90° at the centre of the octagon. Some used the cosine rule but with different values for *b* and *c*. Marks were available for correct use of an incorrect answer at Question 2c.

dii

5.55 metres

Most students got this mark. One mark was available here for correct use of an incorrect answer at Question 2di.

e

5.05 metres

This required solution of $\frac{PR}{\sin 105^\circ} = \frac{3}{\sin 35^\circ}$

f

Marks	0	1	2	Average
%	69	14	17	**0.47**

481 cm (accept 4.81 metres)

A method mark was available for an appropriate attempt at calculating the height of *R* above *Q*. A common error was to disregard the three-dimensional nature of this situation. This commonly led to an incorrect calculation of angle *RQY* by considering point *Q* as the centre of a 360° two-dimensional rotation. Others treated *QRZY* as a square and used 45° for angle *QYR*.

Question 3

a–b

Marks	0	1	2	Average
%	53	34	13	**0.59**

a

0.26 cubic metres

Most used the correct formula from the formula sheet provided with the paper. Some used the diameter measure as the radius. Other used inappropriate formulas. A response of 0.26^3 was given in several cases rather than 0.26 m^{3}. Teachers are urged to ensure students can deal effectively with measurement units in such problems.

b
1:8

Many incorrectly wrote the linear ratio of 1:2. Most calculated the volume of the second cone and quoted a ratio of 0.26:2.11 (this was not accepted).

Students are expected to know the relationship between a scale linear change and corresponding scalings for the change in area and volume.

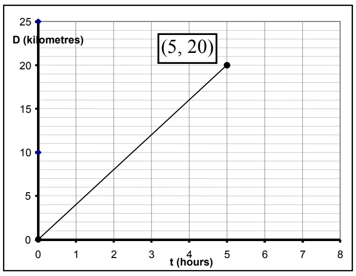
Module 3 – Graphs and relations

Question 1

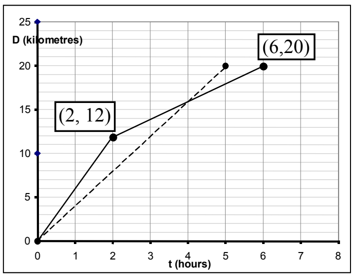
a–c

Marks	0	1	2	3	4	5	Average
%	11	19	5	11	25	29	**3.07**

a
6 km
b



Most students got both marks although many did not terminate their line at (5, 20). In this question, the terminal was disregarded. Some did not join points with a line (necessary for a time series) and lost 1 mark. One mark was given for a reasonable line with positive gradient from (0, 0).
c



Most students were able to draw a two-segment graph which had to end at (6, 20). Many stopped at t = 5, below the point for Malinda. Others went beyond (6, 20). The correct terminal point was required for full marks and the points had to be joined. A method mark was available for a reasonable two-segment graph from (0, 0).
d–e

Marks	0	1	2	3	4	Average
%	33	29	17	12	9	**1.35**

d
4 hours
e
a = 6, b = 2, h = 8, d = 6

One mark was awarded for any two correct, a second mark for the third and then a third mark for the final correct value. Most got the value for a but fewer got the value for b which had a variety of answers offered. The value of h caused most problems with h = 12 a common error. Similarly, d = 5 was a common error and usually reflected terminating their second portion of the graph for D_c at (5, 18)

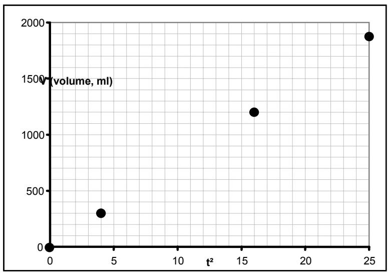
Question 2

a–b

Marks	0	1	2	3	Average
%	12	5	30	53	**2.23**

ai
4, 16, 25

Most students got this although some found values for 2*t* rather than t^2.
aii



A line was not necessary as the question asked only for the points. One mark was available for correctly plotting incorrect values from the table in Question 2ai.
b
$k = 75$
c–d

Marks	0	1	2	3	Average
%	49	19	18	14	**0.96**

c
675 millilitres

Many got this but others did not square the value of *t* in the formula. One mark was available for correctly using an incorrect *k* from Question 2b.
di
3.65 hours

A common error was to solve for t^2 and forget to take the square root. One mark was available for correctly using an incorrect *k* from Question 2b. Some students converted 3.65 hours into 3 hours 65 minutes.
dii
0.7 kilometres

Many were unable to calculate this distance.

Module 4 – Business-related mathematics

Question 1

a–b

Marks	0	1	2	3	4	Average
%	12	28	30	22	8	**1.85**

a
10.1 %

A common incorrect answer was 10%.
bi
$ 3500

Most students got this although a number forgot to include the deposit of $200 and incorrectly offered $3300 instead.
bii
13.8 %

Most calculated the interest based upon a loan of $3100 rather than $2900. One mark was available for a clear calculation of the interest amount. A common incorrect answer was 12.9% by using $3100 as the loan amount. Several students tried to use inappropriate formulas they likely included in their four pages of A4 notes. In many cases it was clear they did not understand how to use them properly.
c–d

Marks	0	1	2	3	4	5	Average
%	25	23	14	11	12	15	**2.05**

ci
$n = 24$, $P = 3100$, $A = 0$

One mark was awarded for any two correct. The third correct value scored the second mark. The correct values of n and P were hypothetically used but $n = 2$ or $n = 12$ were common errors. The value for A was poorly answered. Many students did not understand the value needed for A if the loan is to be paid out entirely as they had $A > 0$.

cii

$ 141.630 (accept $ 142.61 or $ 142.62)

The alternate answers were accepted to allow for early rounding if a student had performed a tabulation.

ciii

$ 3399

Most gained the mark in this question. One mark was available for correct use of a wrong answer from Question 1cii.

d

Discount King is cheapest by $ 101

Most gained the mark for this question. One mark was available for correct, and clear, interpretation of incorrect results from Question 1bi. and Question 1ciii.

Question 2

a–c

Marks	0	1	2	3	4	5	6	Average
%	31	9	23	9	14	4	10	**2.19**

a

$ 1904

A number of students incorrectly treated this as simple interest and reduced $3100 by five lots of (15% of $3100). One mark was available for some demonstration of a single reduction by 15%.

b

$ 1750

The main source of error was converting 3 cents to $0.03 or ignoring the three-year period of depreciation. One mark was available for correct calculation of one year's depreciation.

c

2.7 cents, accept $0.027

This required solution of $1904 = 3100 – (15\,000 \times 3 \times x)$ was generally poorly done. One mark was available for setting up an appropriate equation. Many tried substitution to get a figure close to the result from Question 2a. One mark was available for correctly using incorrect answer from Question 2a.

Module 5 – Networks and decision mathematics

Question 1

a–c

Marks	0	1	2	3	4	5	Average
%	8	11	29	27	17	8	**2.56**

a

Cut A = 14, Cut B = 23, Cut C = 12

Cut A – generally OK.
Cut B – poorly done with a common incorrect answer of 15. This ignored the value of the edge marked with an 8 and which is directed toward Bowen despite the direction apparently to the left as drawn on the diagram.
Cut C – poorly done with common incorrect figure of 20 or 4. In this cut, the edge marked with an 8 is directed toward Arlie and must be ignored, not subtracted nor added.

In general text books draw flow networks that have the *source* at the left and the *sink* at the right of the page with all edges flowing left to right. This can mislead students who take the direction of arrows in the diagram to represent the actual direction of flow. Teachers are encouraged to give students practise working with isomorphic images of graphs shown in textbook diagram forms.

b

Cut E does not entirely isolate Bowen from Arlie

A cut that is to be relevant in a maximum flow problem must totally cut all access between the *source* (start, or Arlie in this case) and the *Sink* (end, or Bowen in this case). In this example, there is still access from Arlie to Bowen if we were to exclude all the edges cut by E. Many incorrect answers referred to the direction of the edge marked with a 7 and suggested that this was heading in the wrong direction; or certainly not toward Bowen.

c
12 seats

One mark was available for correctly using smallest, incorrect answer from either Question 1a or from another labelled cut on the diagram. The most common error was to add up all the seats available along what may have regarded as the ‘shortest path’. Another common error was *maximum flow* = *maximum cut*.

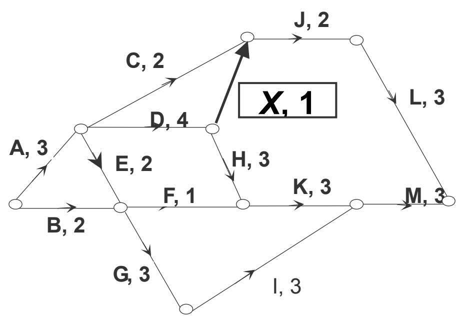
Question 2
a–d

Marks	0	1	2	3	4	5	6	7	Average
%	3	9	18	20	19	18	10	3	**3.51**

a
Immediate predecessor for *E* is *A*

EST for *I* is 8 and EST for *M* is 13 (one mark for each answer).

The predecessor was generally found by most students. The EST figures were generally poorly found; many did not fully understand the table or the application.
b



D is a predecessor of *X* and so *X* must start at the end of *D*.

Also, *X*, (EST of 7) is a predecessor for *J* (EST of 8). Therefore the duration of *X* must be 1 and must terminate at the start of *J*.

This question was very poorly done and many left this question unanswered. A common incorrect diagram for *X* began at the end of *D* but then did not connect with any other point on the network but simply finished at a disconnected point. Others were able to find the correct location for *X* but assigned it the value of 7.
ci
16 hours

Most got this as a simple addition of durations.
cii
The critical path gives the minimum time for completing the whole project.

Most were unable to clearly explain the significance of the critical path in terms of the overall project completion time. Common unacceptable errors were: ‘It gives the maximum time for the completion of the project’ and ‘Nothing else can be done until this path is finished’.
d
7 hours

The entire project can be completed in 16 hours as found in Q2c(i). Therefore any path involving *B* can be up to 16 hours long.

Since *B* is not on the original critical path, we can increase its duration until it is on a new, and second, critical path. This new path becomes *B* – *G* – *I* – *M* which was originally 11 hours long.

Therefore *B,*2 can be increased *by* 5 hours to a total of 7 hours as a maximum value without increasing the whole project beyond the established 16 hours.

Common and incorrect answers included 3 and 5.

Question 3
a–c

Marks	0	1	2	3	Average
%	28	22	21	29	**1.51**

a
6 hours

Many simply looked at $B - F$ as providing the EST for K and gave an answer of 3. This indicates confusion with a 'shortest path' problem.
b
$A - C - J - L$

Generally not well done as many students did not explore all possible paths. Several students included M in their critical path by writing $A - C - J - L - M$. Other common incorrect answers were $A - E - F - K - M$ and $B - F - K - M$
c
8 hours

Many did not get this as they had not found the correct critical path at Question 3b.

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