VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2007

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

FURTHER MATHEMATICS

Written examination 2

Wednesday 7 November 2007

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 30 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

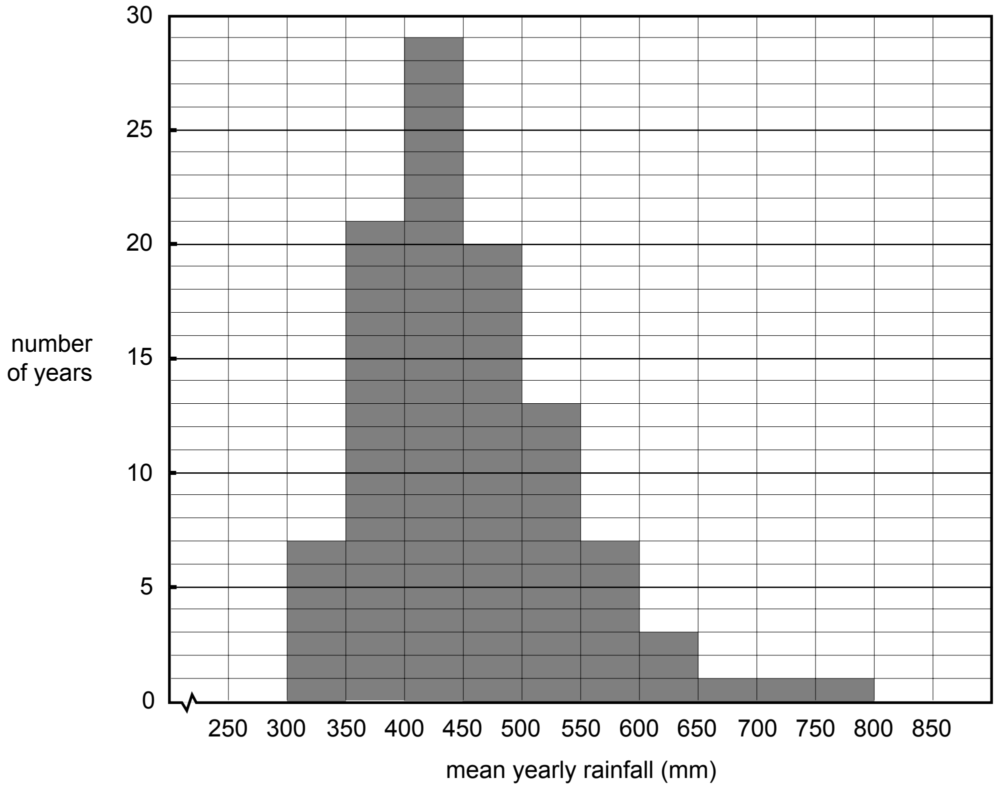
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Page

Core

Question 1

The histogram below shows the distribution of mean yearly rainfall (in mm) for Australia over 103 years.



Data source: ABS 2007

a. Describe the shape of the histogram.

1 mark

b. Use the histogram to determine

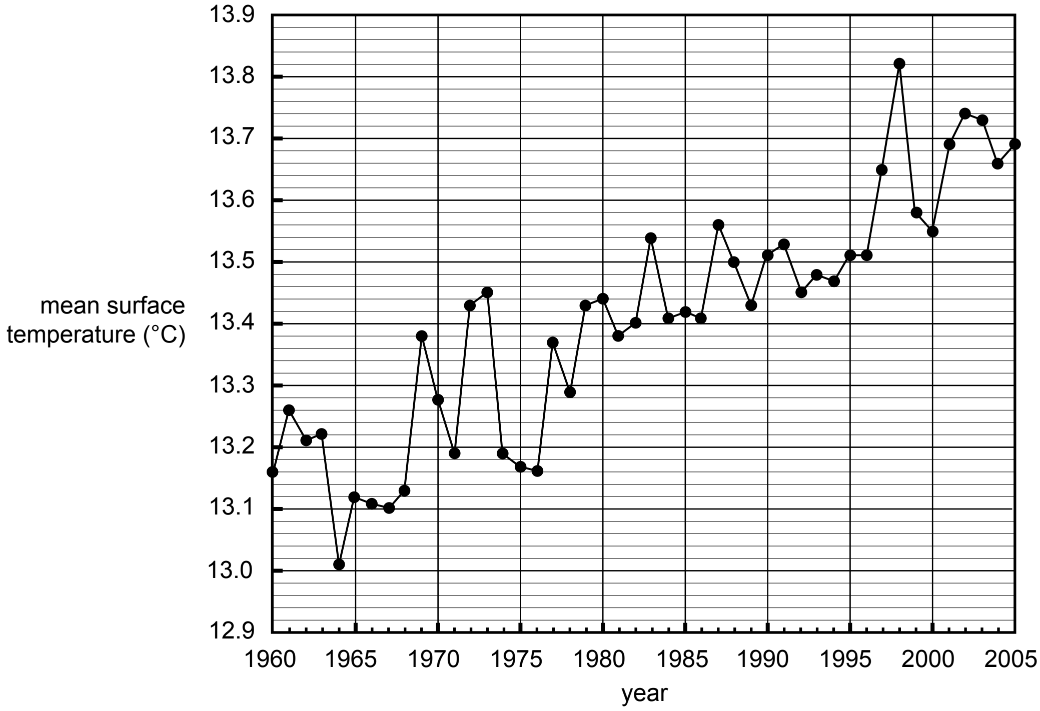
i. the number of years in which the mean yearly rainfall was 500 mm or more

ii. the percentage of years in which the mean yearly rainfall was between 500 mm and 600 mm. Write your answer correct to one decimal place.

1 + 1 = 2 marks

Question 2

The mean surface temperature (in °C) of Australia for the period 1960 to 2005 is displayed in the time series plot below.



Data source: ABS 2007

a. In what year was the lowest mean surface temperature recorded?

1 mark

The least squares method is used to fit a trend line to the time series plot.

b. The equation of this trend line is found to be

$$\textit{mean surface temperature} = -12.361 + 0.013 \times \textit{year}$$

i. Use the trend line to predict the mean surface temperature (in °C) for 2010. Write your answer correct to two decimal places.

The actual mean surface temperature in the year 2000 was 13.55°C.

ii. Determine the residual value (in °C) when the trend line is used to predict the mean surface temperature for this year. Write your answer correct to two decimal places.

iii. By how many degrees does the trend line predict Australia's mean surface temperature will rise each year? Write your answer correct to three decimal places.

1 + 1 + 1 = 3 marks

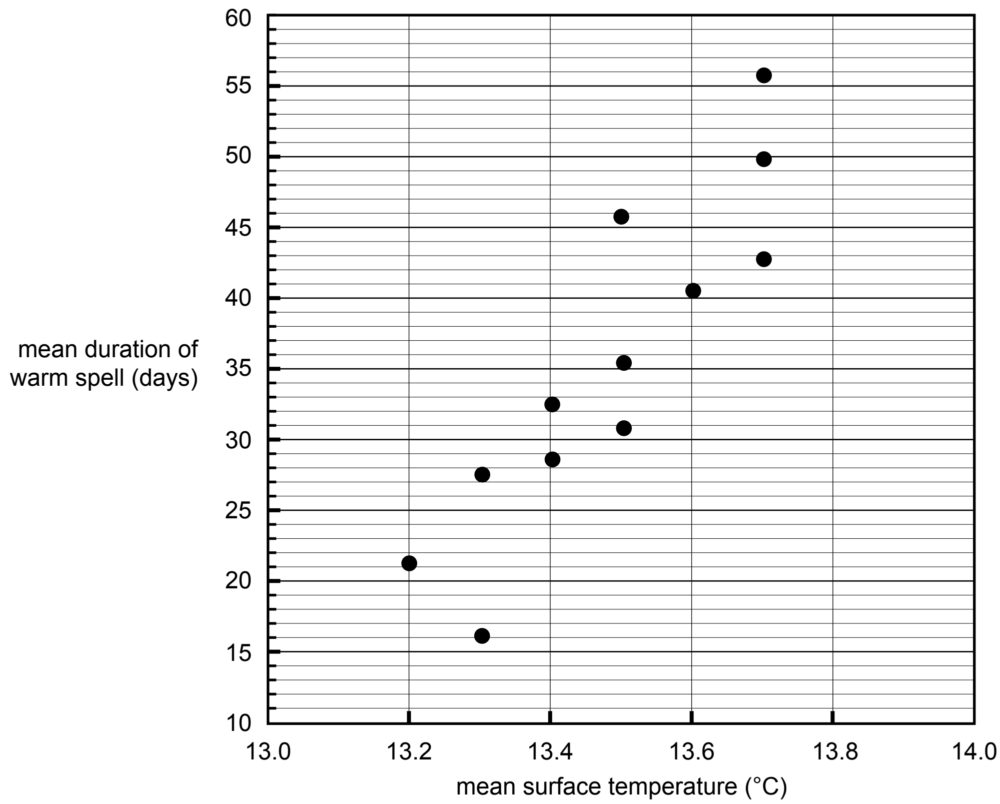
Question 3

The table below displays the mean surface temperature (in °C) and the mean duration of warm spell (in days) in Australia for 13 years selected at random from the period 1960 to 2005.

Mean surface temperature (°C)	**Mean duration of warm spell (days)**
13.2	21.4
13.3	16.3
13.3	27.6
13.4	32.6
13.4	28.7
13.5	30.9
13.5	45.9
13.5	35.5
13.6	40.6
13.7	42.8
13.7	49.9
13.7	55.8
13.8	**53.1**

This data set has been used to construct the scatterplot below. The scatterplot is incomplete.

a. Complete the scatterplot below by plotting the **bold** data values given in the table above. Mark the point with a cross (×).



Scatterplot 1

1 mark

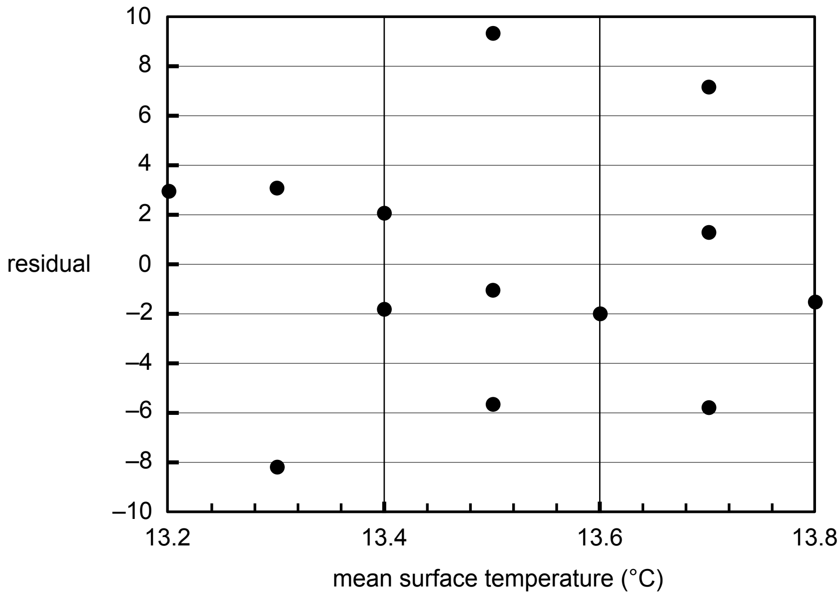
b. Mean surface temperature is the independent variable.

i. Determine the equation of the least squares regression line for this set of data. Write the equation in terms of the variables *mean duration of warm spell* and *mean surface temperature*. Write the values of the coefficients correct to one decimal place.

ii. Plot the least squares regression line on Scatterplot 1 opposite (page 6).

2 + 1 = 3 marks

The residual plot below was constructed to test the assumption of linearity for the relationship between the variables *mean duration of warm spell* and the *mean surface temperature*.



c. Explain why this residual plot supports the assumption of linearity for this relationship.

1 mark

d. Write down the percentage of variation in the mean duration of a warm spell that is explained by the variation in mean surface temperature. Write your answer correct to the nearest per cent.

1 mark

e. Describe the relationship between the mean duration of a warm spell and the mean surface temperature in terms of strength, direction and form.

2 marks

Total 15 marks

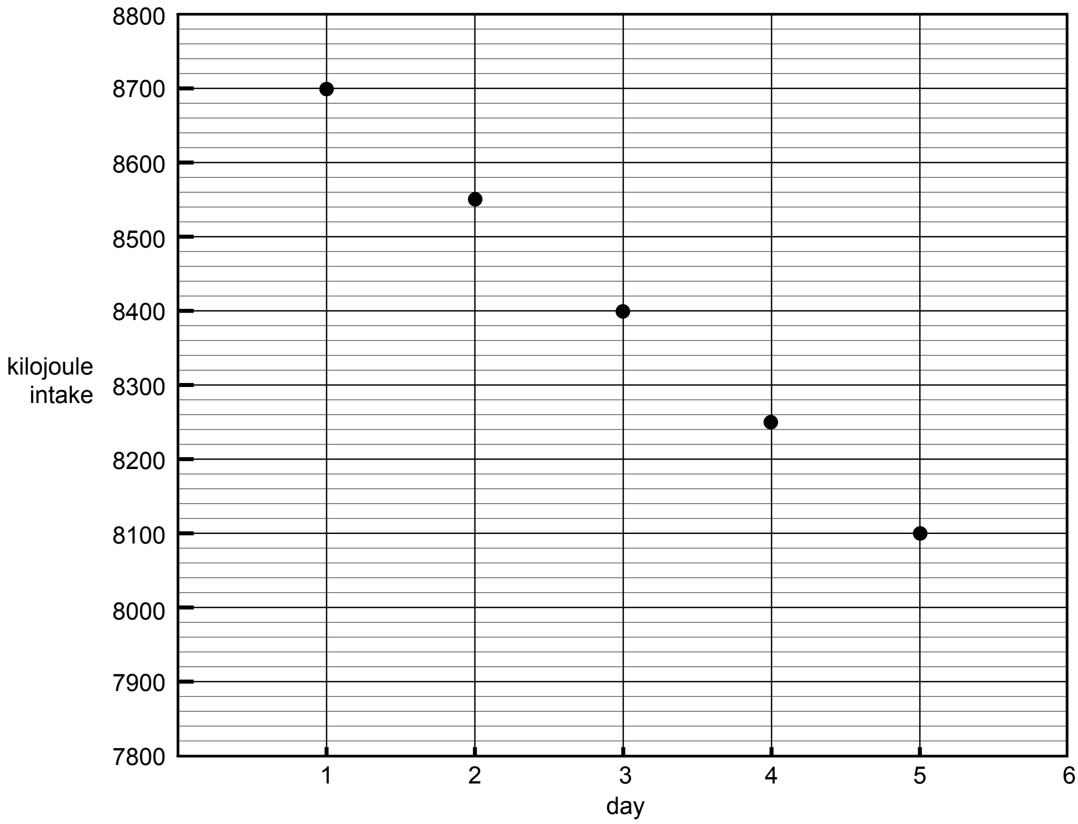
END OF CORE
TURN OVER

Module 1: Number patterns

Question 1

Maria intends to follow a healthy eating plan. She will reduce her daily kilojoule intake by a constant amount each day over a period of 14 days.

The graph below shows Maria's kilojoule intake for the first five days.



a. What was Maria's kilojoule intake on day 1?

1 mark

b. Determine Maria's kilojoule intake on day 6.

1 mark

c. Maria's kilojoule intake on the nth day is given by the equation $K_n = a - 150 \times n$. Determine the value of a.

$a =$ $\boxed{}$

1 mark

d. On which day will Maria's daily kilojoule intake be 6750?

1 mark

Module 1: Number patterns – continued

Question 2

Maria's brother, Rupert, believes he will benefit by reducing his daily kilojoule intake. His kilojoule intake over 14 days will follow a geometric sequence with a common ratio of 0.95.

On day 1, Rupert's kilojoule intake was 12 000.

a. By what percentage is Rupert's kilojoule intake reduced each day?

1 mark

b. Determine Rupert's kilojoule intake on day 3.

1 mark

c. Write an equation that gives Rupert's kilojoule intake R_n on the nth day.

1 mark

d. Find the difference between Rupert's kilojoule intake on day 9 and day 10.
Write your answer correct to the nearest kilojoule.

1 mark

e. Determine Rupert's total kilojoule intake from day 8 to day 14 inclusive.
Write your answer correct to the nearest kilojoule.

2 marks

Question 3

Maria decides to improve her fitness level by cycling each day.

The time in minutes M_n that Maria cycles on the nth day is modelled by the difference equation

$$M_{n+1} = 0.75M_n + 8 \qquad \text{where} \qquad M_2 = 20$$

a. For how many minutes will Maria cycle on day 4?

1 mark

b. Show that the time Maria cycles each day does not follow an arithmetic or a geometric sequence.

1 mark

c. For how many minutes will Maria cycle on day 1?

1 mark

Question 4

Rupert decides to include both swimming and running in his exercise plan.

On day 1, Rupert swims 100 m and runs 500 m.

Each day he will increase the distance he swims and the distance he runs.

His swimming distance will increase by 50 m each day.

His running distance will increase by 2% of the distance he ran on the previous day.

On which day will the distance Rupert swims first be greater than the distance he runs?

2 marks

Total 15 marks

END OF MODULE 1

Working space

Module 2: Geometry and trigonometry

Tessa is a student in a woodwork class.

The class will construct geometrical solids from a block of wood.

Tessa has a piece of wood in the shape of a rectangular prism.

This prism, *ABCDQRST*, shown in Figure 1, has base length 24 cm, base width 28 cm and height 32 cm.

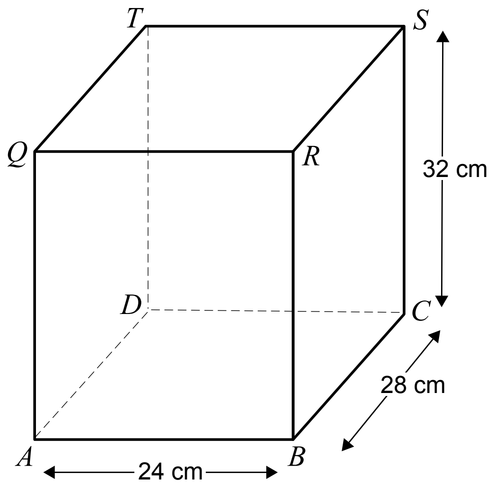


Figure 1

Question 1

On the front face of Figure 1, *ABRQ*, Tessa marks point *W* halfway between *Q* and *R* as shown in Figure 2 below. She then draws line segments *AW* and *BW* as shown.

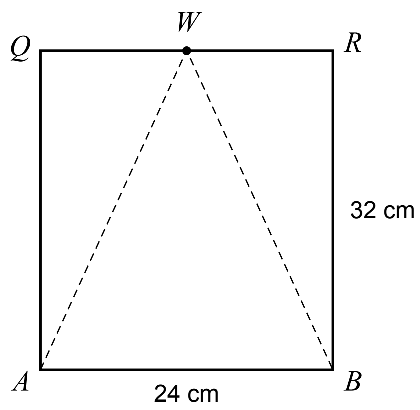


Figure 2

a. Determine the length, in cm, of *QW*.

---

1 mark

b. Calculate the angle *WAQ*. Write your answer in degrees, correct to one decimal place.

1 mark

c. Calculate the angle *AWB* correct to one decimal place.

1 mark

d. What fraction of the area of the rectangle *ABRQ* does the area of the triangle *AWB* represent?

1 mark

Question 2

Tessa carves a triangular prism from her block of wood.

Using point *V*, halfway between *T* and *S* on the back face, *DCST*, of Figure 1, she constructs the triangular prism shown in Figure 3.

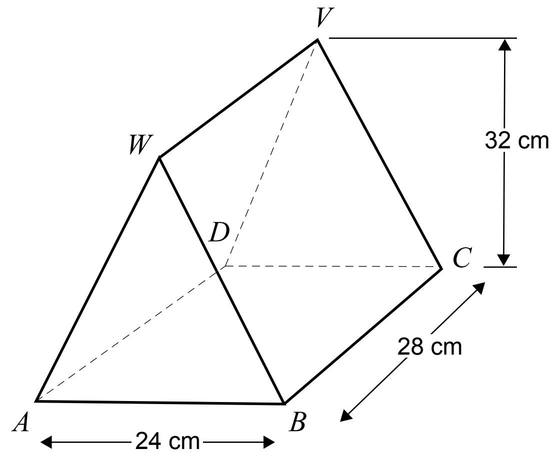


Figure 3

a. Show that, correct to the nearest centimetre, length *AW* is 34 cm.

1 mark

b. Using length *AW* as 34 cm, find the total surface area, in cm^2, of the triangular prism *ABCDWV* in Figure 3.

2 marks

Question 3

Tessa's next task is to carve the right rectangular pyramid *ABCDY* shown in Figure 4 below.

She marks a new point, *Y*, halfway between points *W* and *V* in Figure 3. She uses point *Y* to construct this pyramid.

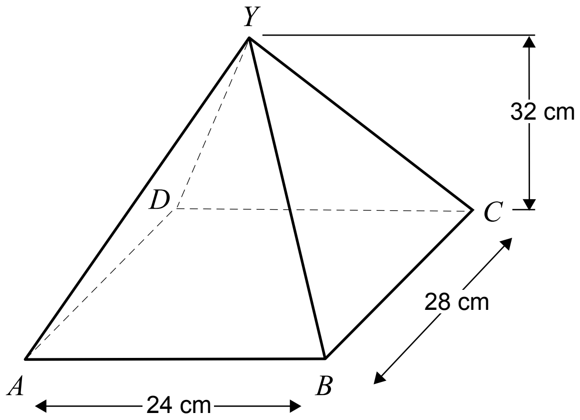


Figure 4

a. Calculate the volume, in cm^3, of the pyramid *ABCDY* in Figure 4.

1 mark

b. Show that, correct to the nearest cm, length *AY* is 37 cm.

2 marks

c. Using *AY* as 37 cm, demonstrate the use of **Heron's formula** to calculate the area, in cm^2, of the triangular face *YAB*.

2 marks

Question 4

Tessa's final task involves removing the top 24 cm of the height of her pyramid (Figure 4).

The shape remaining is shown in Figure 5 below. The top surface, *JKLM*, is parallel to the base, *ABCD*.

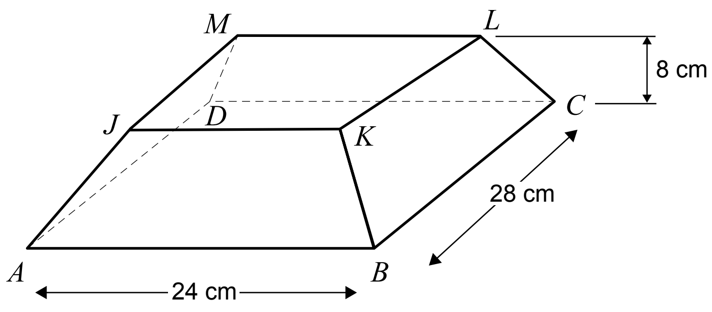


Figure 5

a. What fraction of the height of the pyramid in Figure 4 has Tessa removed to produce Figure 5?

1 mark

b. What fraction of the volume of the pyramid in Figure 4 remains in Figure 5?

2 marks

Total 15 marks

END OF MODULE 2
TURN OVER

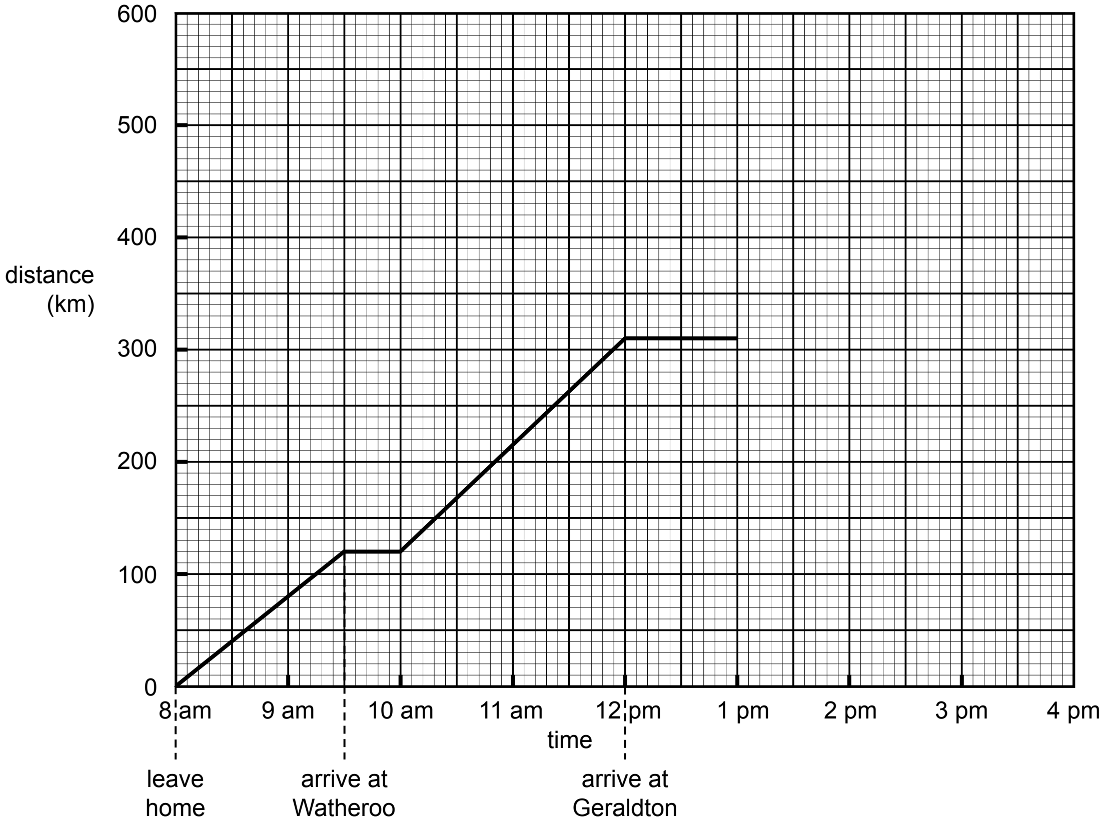
Module 3: Graphs and relations

Question 1

The Goldsmith family are going on a driving holiday in Western Australia.

On the first day, they leave home at 8 am and drive to Watheroo then Geraldton.

The distance–time graph below shows their journey to Geraldton.



At 9.30 am the Goldsmiths arrive at Watheroo.

They stop for a period of time.

a. For how many minutes did they stop at Watheroo?

1 mark

After leaving Watheroo, the Goldsmiths continue their journey and arrive in Geraldton at 12 pm.

b. What distance (in kilometres) do they travel between Watheroo and Geraldton?

1 mark

c. Calculate the Goldsmiths' average speed (in km/h) when travelling between Watheroo and Geraldton.

1 mark

The Goldsmiths leave Geraldton at 1 pm and drive to Hamelin. They travel at a constant speed of 80 km/h for three hours. They do not make any stops.

d. On the graph above, draw a line segment representing their journey from Geraldton to Hamelin.

1 mark

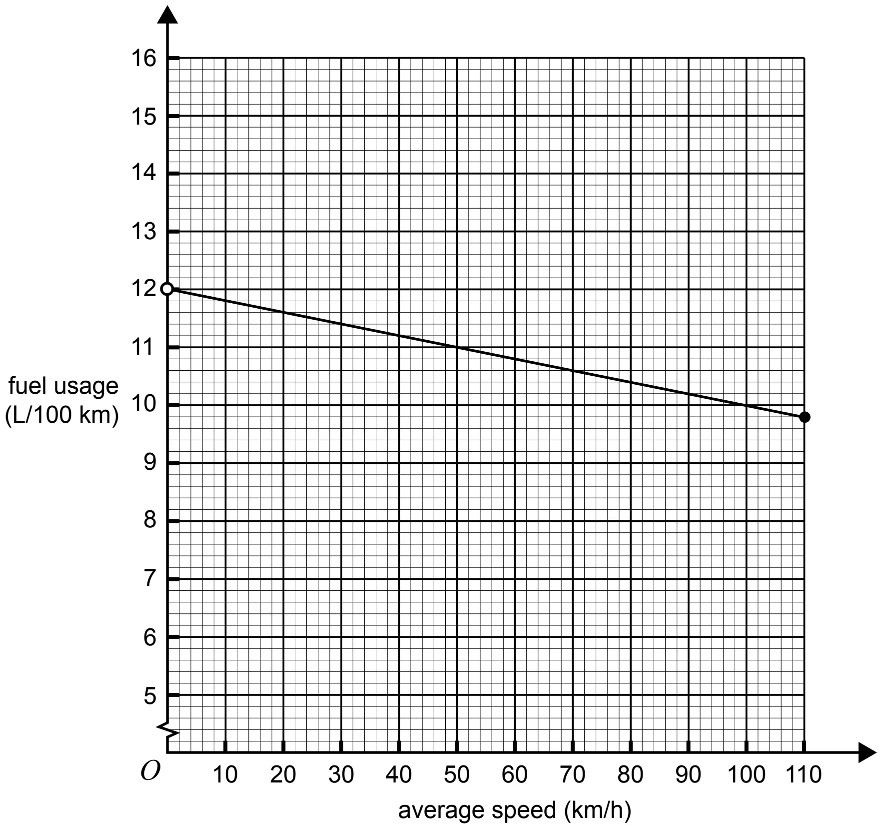
Question 2

The Goldsmiths' car can use either petrol or gas.

The following equation models the fuel usage of petrol, P, in litres per 100 km (L/100 km) when the car is travelling at an average speed of s km/h.

$$P = 12 - 0.02s$$

The line $P = 12 - 0.02s$ is drawn on the graph below for average speeds up to 110 km/h.



a. Determine how many litres of petrol the car will use to travel 100 km at an average speed of 60 km/h. Write your answer correct to one decimal place.

1 mark

The following equation models the fuel usage of gas, G, in litres per 100 km (L/100 km) when the car is travelling at an average speed of s km/h.

$$G = 15 - 0.06s$$

b. On the axes above, draw the line $G = 15 - 0.06s$ for average speeds up to 110 km/h.

1 mark

c. Determine the average speeds for which fuel usage of gas will be less than fuel usage of petrol.

1 mark

The Goldsmiths' car travels at an average speed of 85 km/h. It is using gas.
Gas costs 80 cents per litre.

d. Determine the cost of the gas used to travel 100 km.
Write your answer in dollars and cents.

2 marks

Question 3

Gas is generally cheaper than petrol.
The car must run on petrol for some of the driving time.

Let x be the number of hours driving using gas
y be the number of hours driving using petrol

Inequalities 1 to 5 below represent the constraints on driving a car over a 24-hour period.
Explanations are given for Inequalities 3 and 4.

Inequality 1: $x \geq 0$

Inequality 2: $y \geq 0$

Inequality 3: $y \leq \frac{1}{2}x$ The number of hours driving using petrol must not exceed half the number of hours driving using gas.

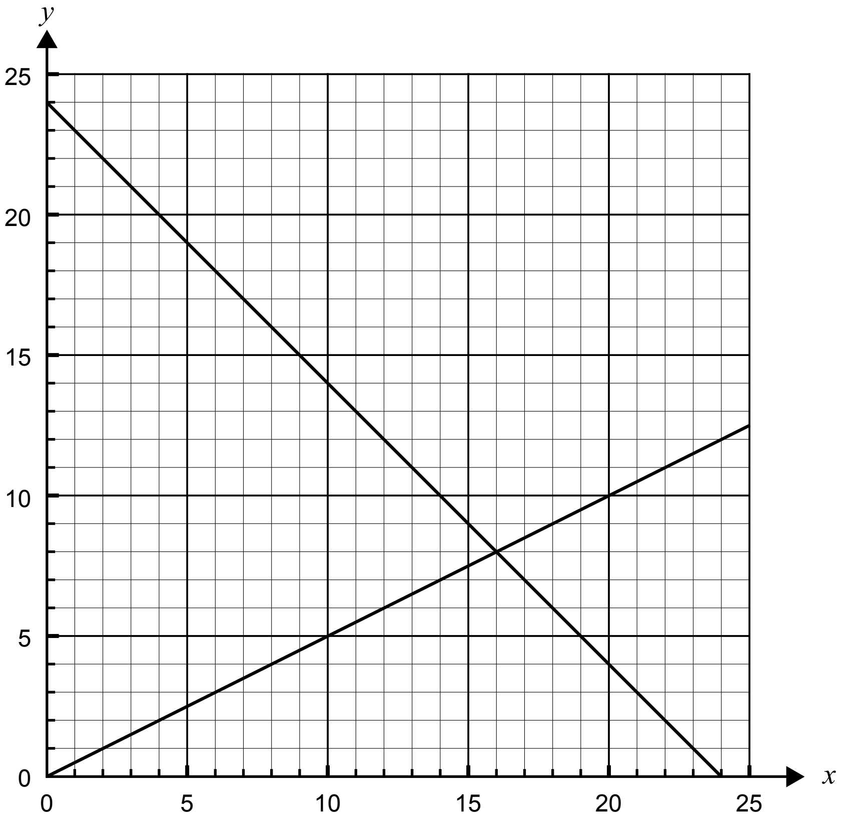
Inequality 4: $y \geq \frac{1}{3}x$ The number of hours driving using petrol must be at least one third the number of hours driving using gas.

Inequality 5: $x + y \leq 24$

a. Explain the meaning of Inequality 5 in terms of the context of this problem.

1 mark

The lines $x + y = 24$ and $y = \frac{1}{2}x$ are drawn on the graph below.



b. On the graph above

i. draw the line $y = \frac{1}{3}x$

ii. clearly shade the feasible region represented by Inequalities 1 to 5.

1 + 1 = 2 marks

On a particular day, the Goldsmiths plan to drive for 15 hours. They will use gas for 10 of these hours.

c. Will the Goldsmiths comply with all constraints? Justify your answer.

1 mark

On another day, the Goldsmiths plan to drive for 24 hours.

Their car carries enough fuel to drive for 20 hours using gas and 7 hours using petrol.

d. Determine the **maximum** and **minimum** number of hours they can drive using **gas** while satisfying all constraints.

Maximum = ☐ hours

Minimum = ☐ hours

2 marks

Total 15 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1

Khan wants to buy some office furniture that is valued at $7000.

a. **i.** A store requires 25% deposit. Calculate the deposit.

The balance is to be paid in 24 equal monthly instalments. No interest is charged.

ii. Determine the amount of each instalment. Write your answer in dollars and cents.

1 + 1 = 2 marks

b. Another store offers the same $7000 office furniture for $500 deposit and 36 monthly instalments of $220.

i. Determine the total amount paid for the furniture at this store.

ii. Calculate the **annual flat rate** of interest charged by this store.

Write your answer as a percentage correct to one decimal place.

1 + 2 = 3 marks

A third store has the office furniture marked at $7000 but will give 15% discount if payment is made in cash at the time of sale.

c. Calculate the cash price paid for the furniture after the discount is applied.

1 mark

Question 2

Khan decides to extend his home office and borrows \$30 000 for building costs. Interest is charged on the loan at a rate of 9% per annum compounding monthly.

Assume Khan will pay only the interest on the loan at the end of each month.

a. Calculate the amount of interest he will pay each month.

1 mark

Suppose the interest rate remains at 9% per annum compounding monthly and Khan pays \$400 each month for five years.

b. Determine the amount of the loan that is outstanding at the end of five years.

Write your answer correct to the nearest dollar.

1 mark

Khan decides to repay the \$30 000 loan fully in equal monthly instalments over five years.

The interest rate is 9% per annum compounding monthly.

c. Determine the amount of each monthly instalment. Write your answer correct to the nearest cent.

1 mark

Question 3

Khan paid $900 for a fax machine.

This price includes 10% GST (goods and services tax).

a. Determine the price of the fax machine **before** GST was added. Write your answer correct to the nearest cent.

1 mark

b. Khan will depreciate his $900 fax machine for taxation purposes.

He considers two methods of depreciation.

Flat rate depreciation

Under flat rate depreciation the fax machine will be valued at $300 after five years.

i. Calculate the annual depreciation in dollars.

Unit cost depreciation

Suppose Khan sends 250 faxes a year. The $900 fax machine is depreciated by 46 cents for each fax it sends.

ii. Determine the value of the fax machine after five years.

1 + 1 = 2 marks

Question 4

The books in Khan's office are valued at $10 000.

a. Calculate the value of these books after five years if they are depreciated by 12% per annum using the reducing balance method. Write your answer correct to the nearest dollar.

1 mark

Khan believes his books should be valued at $4000 after five years.

b. Determine the annual reducing balance depreciation rate that will produce this value. Write your answer as a percentage correct to one decimal place.

2 marks

Total 15 marks

END OF MODULE 4

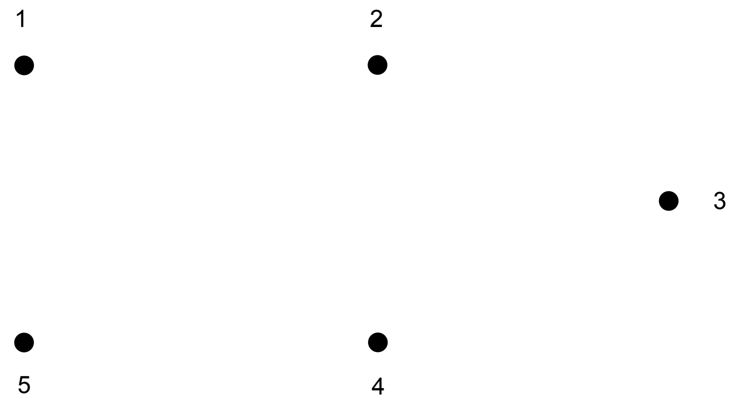
Module 5: Networks and decision mathematics

Question 1

A new housing estate is being developed.

There are five houses under construction in one location.

These houses are numbered as points 1 to 5 below.



The builders require the five houses to be connected by electrical cables to enable the workers to have a supply of power on each site.

a. What is the **minimum** number of edges needed to connect the five houses?

---

1 mark

b. On the diagram above, **draw** a connected graph with this number of edges.

1 mark

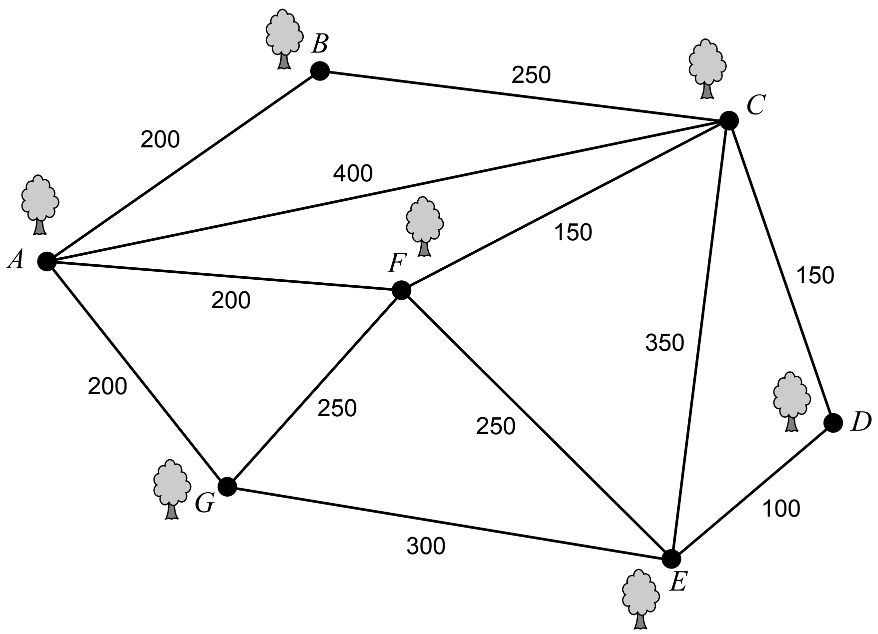
Question 2

The estate has large open parklands that contain seven large trees.

The trees are denoted as vertices A to G on the network diagram below.

Walking paths link the trees as shown.

The numbers on the edges represent the lengths of the paths in metres.



a. Determine the sum of the degrees of the vertices of this network.

1 mark

b. One day Jamie decides to go for a walk that will take him along each of the paths between the trees.
He wishes to walk the minimum possible distance.

i. State a vertex at which Jamie could begin his walk.

ii. Determine the total distance, in metres, that Jamie will walk.

1 + 1 = 2 marks

Michelle is currently at F.

She wishes to follow a route that can be described as the **shortest** Hamiltonian circuit.

c. Write down a route that Michelle can take.

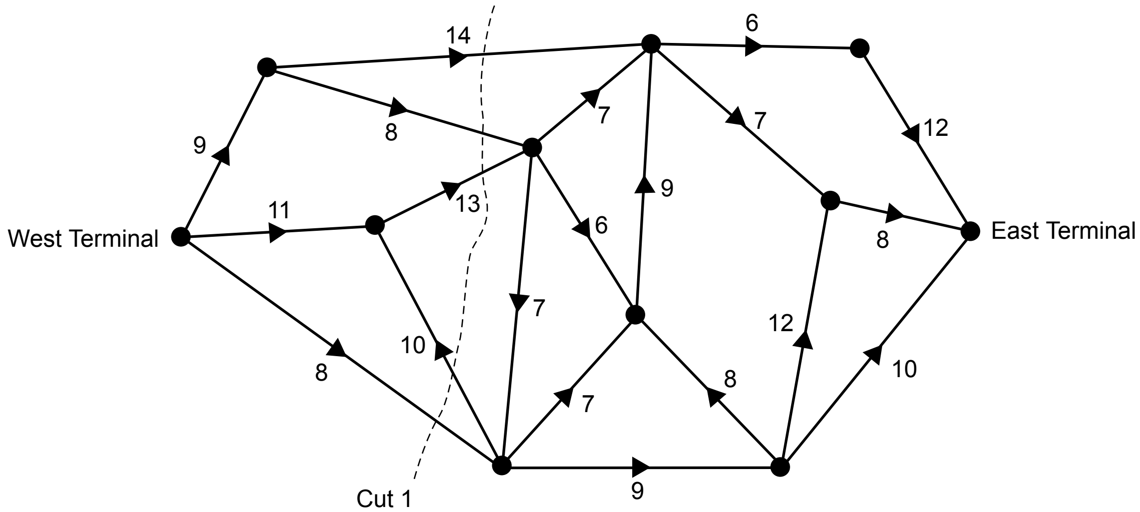
1 mark

Question 3

As an attraction for young children, a miniature railway runs throughout the new housing estate.

The trains travel through stations that are represented by nodes on the directed network diagram below.

The number of seats available for children, between each pair of stations, is indicated beside the corresponding edge.



Cut 1, through the network, is shown in the diagram above.

a. Determine the capacity of Cut 1.

---

1 mark

b. Determine the maximum number of seats available for children for a journey that begins at the West Terminal and ends at the East Terminal.

---

1 mark

On one particular train, 10 children set out from the West Terminal.

No new passengers board the train on the journey to the East Terminal.

c. Determine the maximum number of children who can arrive at the East Terminal on this train.

---

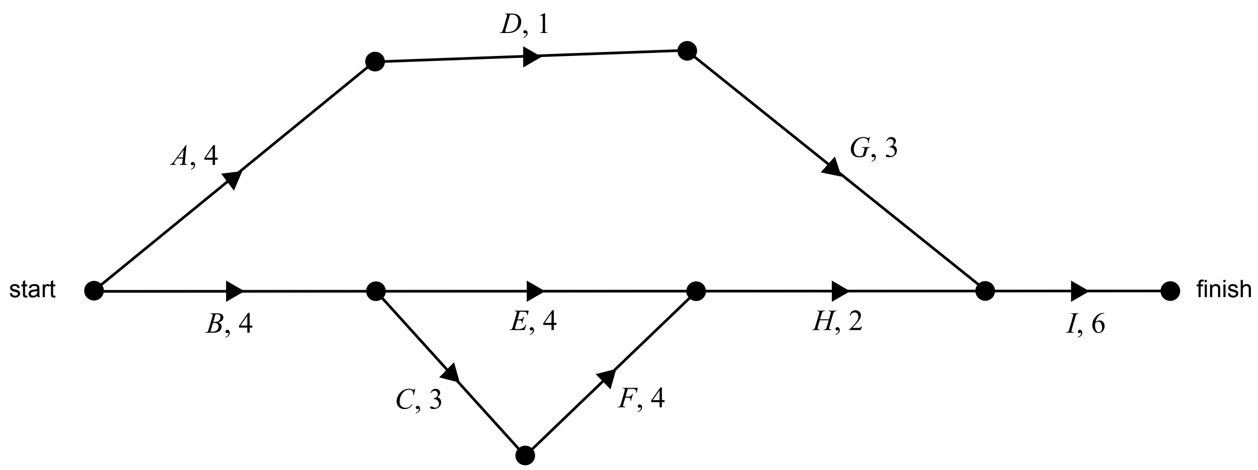
1 mark

Question 4

A community centre is to be built on the new housing estate.

Nine activities have been identified for this building project.

The directed network below shows the activities and their completion times in weeks.



a. Determine the minimum time, in weeks, to complete this project.

1 mark

b. Determine the slack time, in weeks, for activity *D*.

2 marks

The builders of the community centre are able to speed up the project.

Some of the activities can be reduced in time at an additional cost.

The activities that can be reduced in time are *A*, *C*, *E*, *F* and *G*.

c. Which of these activities, if reduced in time individually, would **not** result in an earlier completion of the project?

1 mark

The owner of the estate is prepared to pay the additional cost to achieve early completion.

The cost of reducing the time of each activity is $5000 per week.

The maximum reduction in time for each one of the five activities, *A*, *C*, *E*, *F*, *G*, is 2 weeks.

d. Determine the minimum time, in weeks, for the project to be completed now that certain activities can be reduced in time.

1 mark

e. Determine the minimum additional cost of completing the project in this reduced time.

1 mark

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

The table below displays the energy content and amounts of fat, carbohydrate and protein contained in a serve of four foods: bread, margarine, peanut butter and honey.

Food	Energy content (kilojoules/serve)	Fat (grams/serve)	Carbohydrate (grams/serve)	Protein (grams/serve)
Bread	531	1.2	20.1	4.2
Margarine	41	6.7	0.4	0.6
Peanut butter	534	10.7	3.5	4.6
Honey	212	0	12.5	0.1

a. Write down a 2×3 matrix that displays the fat, carbohydrate and protein content (in columns) of bread and margarine.

1 mark

b. A and B are two matrices defined as follows.

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 531 \\ 41 \\ 534 \\ 212 \end{bmatrix}$$

i. Evaluate the matrix product AB.

ii. Determine the order of matrix product BA.

Matrix A displays the number of servings of the four foods: bread, margarine, peanut butter and honey, needed to make a peanut butter and honey sandwich.

Matrix B displays the energy content per serving of the four foods: bread, margarine, peanut butter and honey.

iii. Explain the information that the matrix product AB provides.

1 + 1 + 1 = 3 marks

c. The number of serves of bread (b), margarine (m), peanut butter (p) and honey (h) that contain, in total, 53 grams of fat, 101.5 grams of carbohydrate, 28.5 grams of protein and 3568 kilojoules of energy can be determined by solving the matrix equation

$$\begin{bmatrix} 1.2 & 6.7 & 10.7 & 0 \\ 20.1 & 0.4 & 3.5 & 12.5 \\ 4.2 & 0.6 & 4.6 & 0.1 \\ 531 & 41 & 534 & 212 \end{bmatrix}\begin{bmatrix} b \\ m \\ p \\ h \end{bmatrix} = \begin{bmatrix} 53 \\ 101.5 \\ 28.5 \\ 3568 \end{bmatrix}$$

Solve the matrix equation to find the values b, m, p and h.

---

2 marks

Question 2

To study the life-and-death cycle of an insect population, a number of insect eggs (E), juvenile insects (J) and adult insects (A) are placed in a closed environment.

The initial state of this population can be described by the column matrix

$$S_0 = \begin{array}{c} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} E \\ J \\ A \\ D \end{array}$$

A row has been included in the state matrix to allow for insects and eggs that die (D).

a. What is the total number of insects in the population (including eggs) at the beginning of the study?

---

1 mark

In this population

- eggs may die, or they may live and grow into juveniles
- juveniles may die, or they may live and grow into adults
- adults will live a period of time but they will eventually die.

In this population, the adult insects have been sterilised so that no new eggs are produced. In these circumstances, the life-and-death cycle of the insects can be modelled by the transition matrix

$$\begin{array}{c} \text{this week} \\ \begin{array}{cccc} E & J & A & D \end{array} \end{array}$$

$$T = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}\begin{array}{l} E \\ J \\ A \\ D \end{array} \quad \text{next week}$$

b. What proportion of **eggs turn into juveniles** each week?

---

1 mark

c. **i.** Evaluate the matrix product $S_1 = T\,S_0$

$$S_1 = T\,S_0 = \begin{array}{c|c} \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix} & \begin{array}{c} E \\ J \\ A \\ D \end{array} \end{array}$$

ii. Write down the number of live **juveniles** in the population after one week.

iii. Determine the number of live **juveniles** in the population after four weeks. Write your answer correct to the nearest whole number.

iv. After a number of weeks there will be no live eggs (less than one) left in the population.
When does this first occur?

v. Write down the exact steady-state matrix for this population.

$$S_{\text{steady state}} = \begin{array}{c|c} \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix} & \begin{array}{c} E \\ J \\ A \\ D \end{array} \end{array}$$

1 + 1 + 1 + 1 + 1 = 5 marks

d. If the study is repeated with unsterilised adult insects, eggs will be laid and potentially grow into adults. Assuming 30% of adults lay eggs each week, the population matrix after one week, S_1, is now given by

$$S_1 = T\,S_0 + B\,S_0$$

where $B = \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $S_0 = \begin{array}{c} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} E \\ J \\ A \\ D \end{array}$

i. Determine S_1

$$S_1 = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}\begin{array}{l} E \\ J \\ A \\ D \end{array}$$

This pattern continues. The population matrix after n weeks, S_n, is given by

$$S_n = T\,S_{n-1} + B\,S_{n-1}$$

ii. Determine the number of live eggs in this insect population after two weeks.

1 + 1 = 2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

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Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET