VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2005

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

FURTHER MATHEMATICS

Written examination 2
(Analysis task)

Wednesday 2 November 2005

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
1	1	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
5	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten), one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 31 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and five modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core

Question 1

Cars depreciate in value over time. Table 1 gives the average value of a car (of the same brand and model) at different ages.

Table 1.

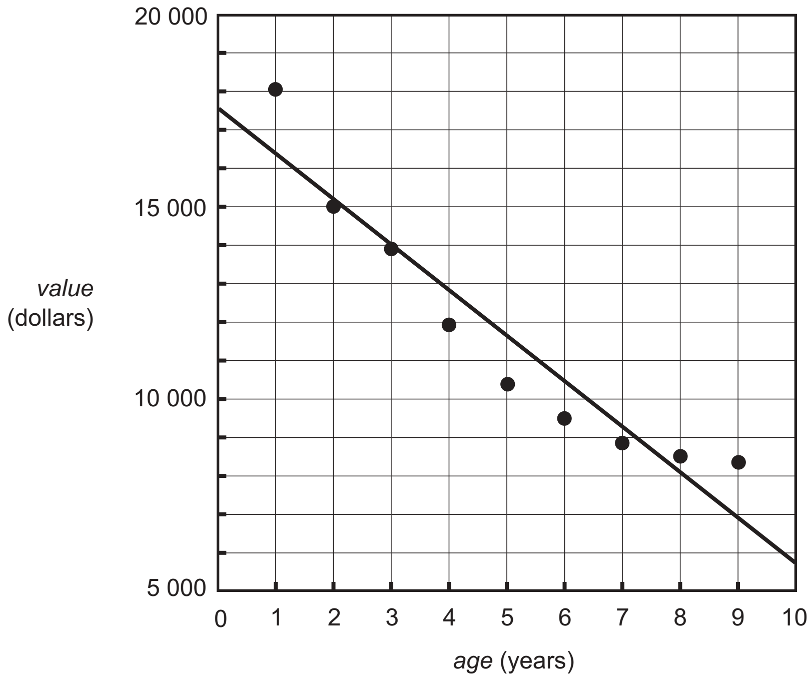
age (years)	1	2	3	4	5	6	7	8	9
value (dollars)	18 100	15 050	13 900	11 900	10 400	9 600	8 900	8 500	8 400

a. The data is to be used to build a mathematical relationship that will enable the average value of this brand and model of car to be predicted from its age.

In this situation, the **dependent** variable is [ ]

1 mark

Scatterplot 1 is constructed from the data and a least squares regression line is fitted as shown.



Scatterplot 1

b. The coefficient of determination for this data is 0.9058.

i. Find the value of the correlation coefficient correct to three decimal places.

r = ☐

ii. Write down the percentage of the variation in the value of a car that can be accounted for by the variation in its age.

percentage = ☐

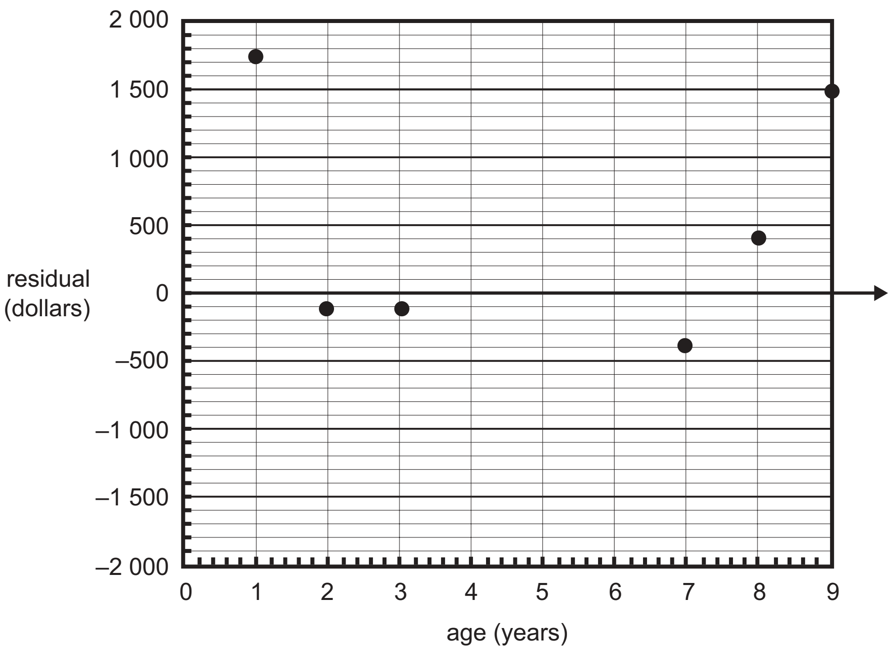
1 + 1 = 2 marks

c. Using the line shown in Scatterplot 1, or otherwise, determine the equation of the least squares regression line. Write the coefficients correct to the nearest hundred.

value = ☐ + ☐ × *age*

2 marks

Scatterplot 1 suggests that the relationship may be nonlinear. To investigate this idea, Residual plot 1 is constructed. It is incomplete.



Residual plot 1

d. Using the information in Scatterplot 1, or otherwise, complete Residual plot 1 by marking in the missing residual values for cars aged 4, 5 and 6 years.

2 marks

e. When complete, does Residual plot 1 suggest that a nonlinear relationship will provide a better fit for the data? Justify your response.

1 mark

Scatterplot 1 indicates that a logarithmic transformation of the horizontal (*age*) axis may linearise the data.

The original data has been reproduced in Table 2. An extra row has been added for the transformed variable, log(*age*). The table is incomplete.

Table 2.

age (years)	1	2	3	4	5	6	7	8	9
log(*age*)	0	0.30	0.48	0.60	0.70	0.78	0.85	0.90	
value (dollars)	18 100	15 050	13 900	11 900	10 400	9 600	8 900	8 500	8 400

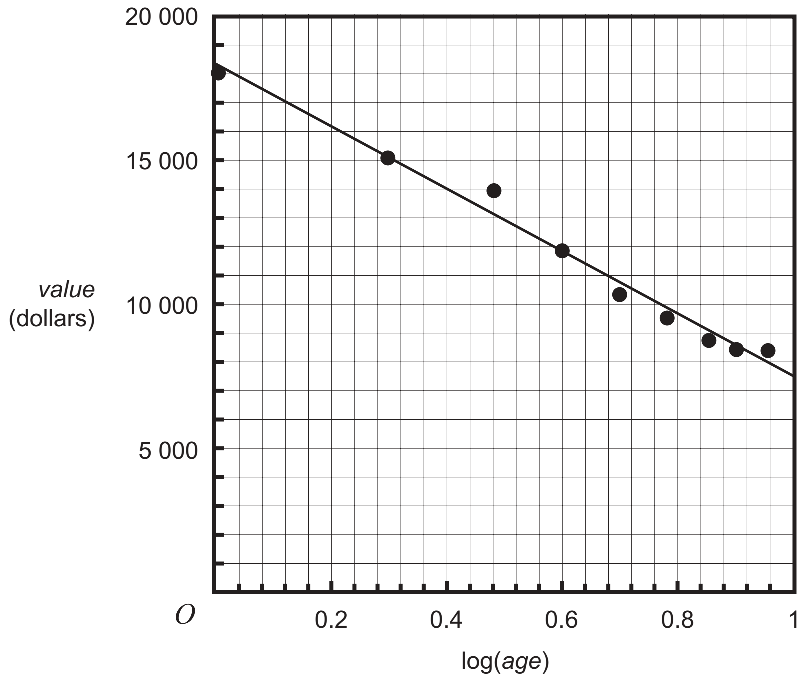
f. Complete the table above.

Write your answer correct to two decimal places.

1 mark

g. In Scatterplot 2 below, *value* is plotted against log(*age*).

A least squares regression line fitted to the transformed data is also drawn.



Scatterplot 2

Use the information in Scatterplot 2 to describe the relationship between *value* and log(*age*) in terms of **direction, form** and **strength**.

3 marks

h. The equation of this least squares regression line is

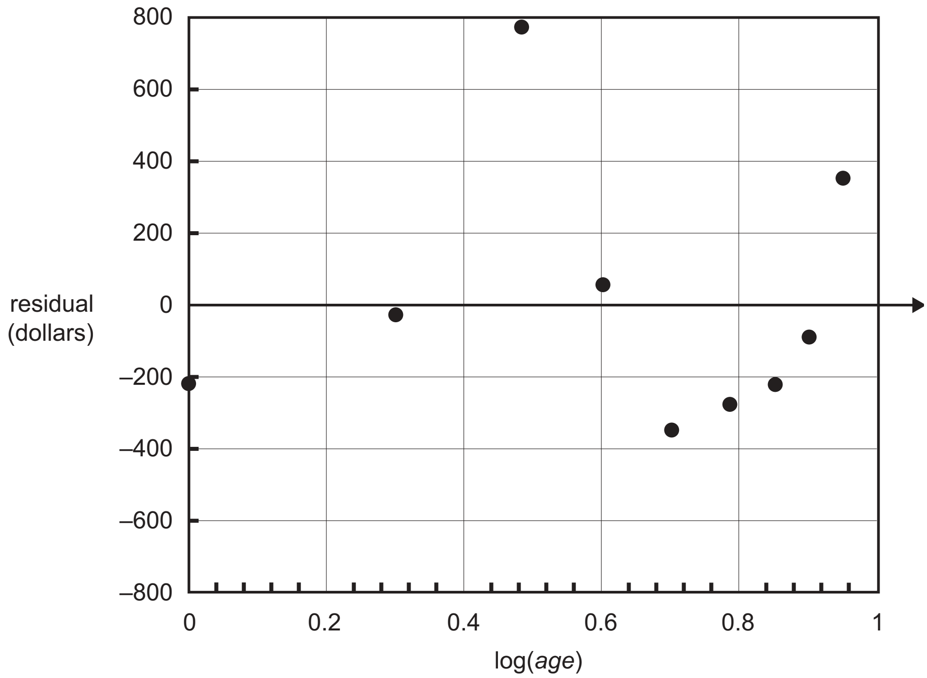
$$value = 18\,300 - 10\,800 \times \log(age)$$

Use this equation to predict the value of a car that is three years old.

Write your answer correct to the nearest hundred dollars.

1 mark

The residual plot for this relationship is shown in Residual plot 2.



Residual plot 2

i. Residual plot 2 suggests that the log(*age*) transformation has been successful in linearising the data. What feature of this residual plot shows that it has been successful?

1 mark

j. A transformation applied to the *value* axis can also linearise the original data displayed in Scatterplot 1. Suggest a suitable transformation and **explain** why it will work.

1 mark

Total 15 marks

END OF CORE

Working space

Module 1: Number patterns and applications

Question 1

Noel is an enthusiastic runner. He decides to follow a training program over several weeks.

The distance (in kilometres) that Noel runs each day forms an arithmetic sequence.

The distances for the first three days of this training program are shown in Table 1.

Table 1.

Day	1	2	3
Distance (km)	4.2	4.5	4.8

a. What distance (in kilometres) will Noel run on Day 5?

1 mark

b. On which day will Noel first run more than 7 km?

1 mark

c. Determine the total distance that Noel runs from Day 3 to Day 12 inclusive.
Write your answer in kilometres, correct to one decimal place.

2 marks

d. An expression for the nth term of this sequence can be written as $t_n = 0.3n + b$
Determine the value of b.

$b =$ ☐

1 mark

Question 2

Ray enjoys walking. The distance (in kilometres) that Ray walks each day forms a sequence.
The distances for the first three days of his walking program are shown in Table 2.

Table 2.

Day	1	2	3
Distance (km)	3	3.5	4.05

a. Show that this sequence is neither arithmetic nor geometric.

2 marks

b. A difference equation that generates the terms of this sequence is

$$w_{n+1} = 1.1w_n + c \qquad \text{where} \qquad w_1 = 3$$

What is the value of c?

$c =$ ☐

1 mark

c. What distance will Ray walk on Day 5?
Write your answer in kilometres, correct to two decimal places.

1 mark

Question 3

Catherine likes both running and walking. She commences a two week training program.

The distance (in kilometres) that Catherine travels each day of the training program is always 5% greater than that of the previous day.

The distance travelled in the first and third days of her training program are shown in Table 3.

Table 3.

Day	1	2	3
Distance (km)	10		11.025

a. Complete the table above.

1 mark

b. Calculate the total distance that Catherine travels in 14 days.

Write your answer correct to the nearest kilometre.

1 mark

c. Catherine's training each day consists of both running and walking.

i. On Day 1, Catherine travelled 10 km.

The ratio of her running distance to her walking distance was 3:2.

Determine the distance that Catherine walked on Day 1.

ii. On Day 3, Catherine ran the first 25% of the total distance then walked the remaining distance.

Determine the ratio of her running distance to her walking distance on Day 3.

1 + 1 = 2 marks

d. Catherine will begin a new training program at the start of next month.
On the first day of this new program she will travel 5 km.
Each day thereafter she will travel 90% of the previous distance plus an additional 1.2 km.

i. Write a difference equation that **specifies** the distance Catherine will travel on the nth day.

Catherine follows this training program for many months.

ii. Explain why she will never travel more than 12 km in any one day.

1 + 1 = 2 marks

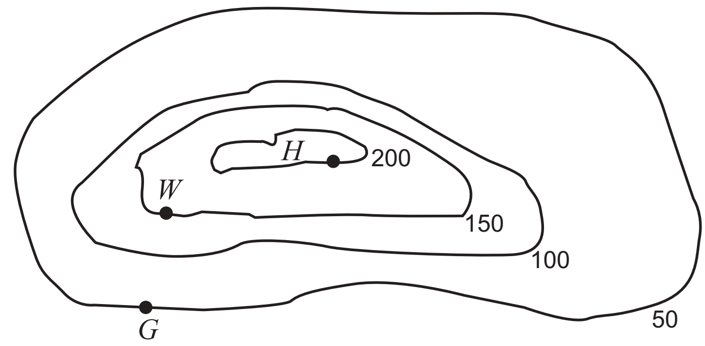
Total 15 marks

END OF MODULE 1
TURN OVER

Module 2: Geometry and trigonometry

Question 1

The contour map below shows a region in country Victoria. It has contours drawn at intervals of 50 metres. There is a camping ground at *G*, a hut at *H* and a water source at *W*.



a. What is the difference in height (in metres) between hut *H* and the water source *W*?

1 mark

b. Camping ground *G* obtains its water from *W*.

The horizontal distance between *G* and *W* is 500 m.

Find the shortest length of water pipe that can connect *G* with *W*.

Write your answer correct to the nearest metre.

1 mark

c. The average slope between camping ground *G* and hut *H* is 0.12.

Find the horizontal distance (in metres) between *G* and *H*.

1 mark

d. The scale used on this contour map is 1:40 000.

Determine the length of a line (in centimetres) on the contour map that represents a distance of 2 km.

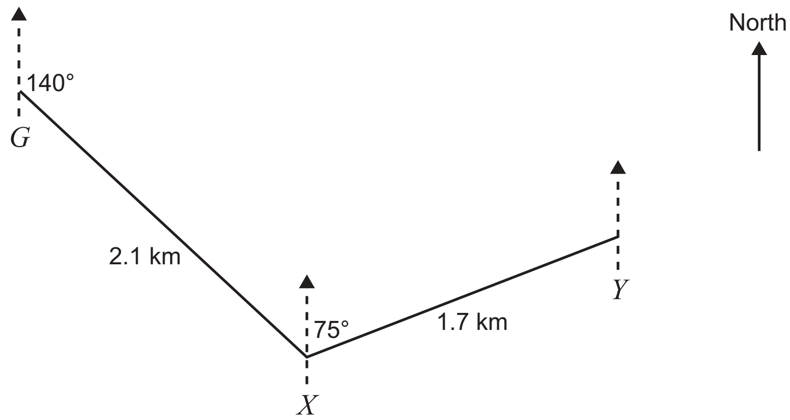
1 mark

Question 2

The land near the camping ground is flat and suitable for orienteering.

Checkpoint *X* is situated 2.1 km from camping ground *G* on a bearing of 140°.

Checkpoint *Y* is situated 1.7 km from checkpoint *X* on a bearing of 075°.



a. **i.** How far south of *G* is checkpoint *X*?

Write your answer in kilometres, correct to one decimal place.

ii. How far south of *G* is checkpoint *Y*?

Write your answer in kilometres, correct to one decimal place.

1 + 2 = 3 marks

b. Determine the size of angle *GXY*.

1 mark

c. Calculate the distance *GY*.

Write your answer in kilometres, correct to one decimal place.

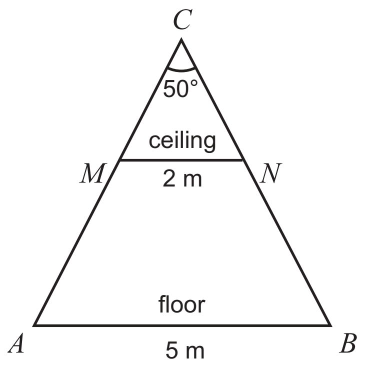
1 mark

d. Determine the bearing of checkpoint *Y* from camping ground *G*.
Write your answer correct to the nearest degree.

2 marks

Question 3

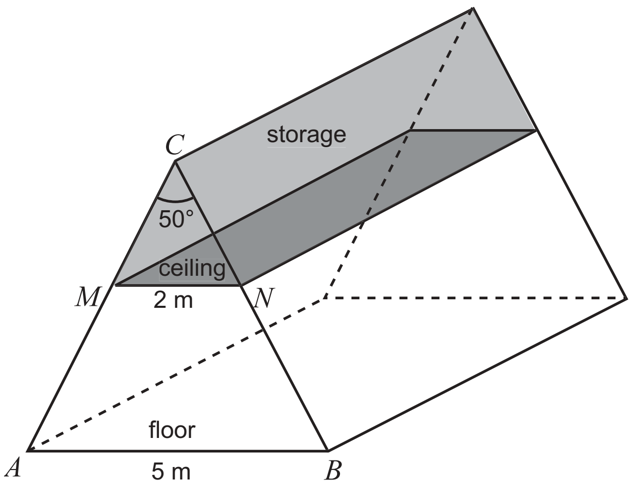
The cross-section, *ABC*, of an A-frame hut near the camping ground is an isosceles triangle as shown below.
Angle *ACB* is 50°.
AB is 5 metres in length and is the floor line.
MN is 2 metres in length and is the ceiling line inside the hut.
AB and *MN* are parallel.



a. Calculate the vertical height of the ceiling, *MN*, above the floor, *AB*.
Write your answer in metres, correct to one decimal place.

2 marks

b. The hut has the shape of a triangular prism.
The space inside the hut above the ceiling is used for storage.



The total space inside the hut (including storage) is V m^3.
What fraction of V is used for storage?

2 marks

Total 15 marks

Module 3: Graphs and relations

Question 1

Pepi manages two street stands in the city. One is in Xenon Street and the other is in Yarra Street. These street stands sell newspapers and magazines.

On average, per hour, each stand sells

Xenon Street: 20 newspapers and 2 magazines
Yarra Street: 15 newspapers and 4 magazines.

In total, at least 900 newspapers and at least 160 magazines must be sold each week.

Let x be the number of hours per week the Xenon Street stand operates
y be the number of hours per week the Yarra Street stand operates.

This information can be expressed as Inequalities 1 to 4.

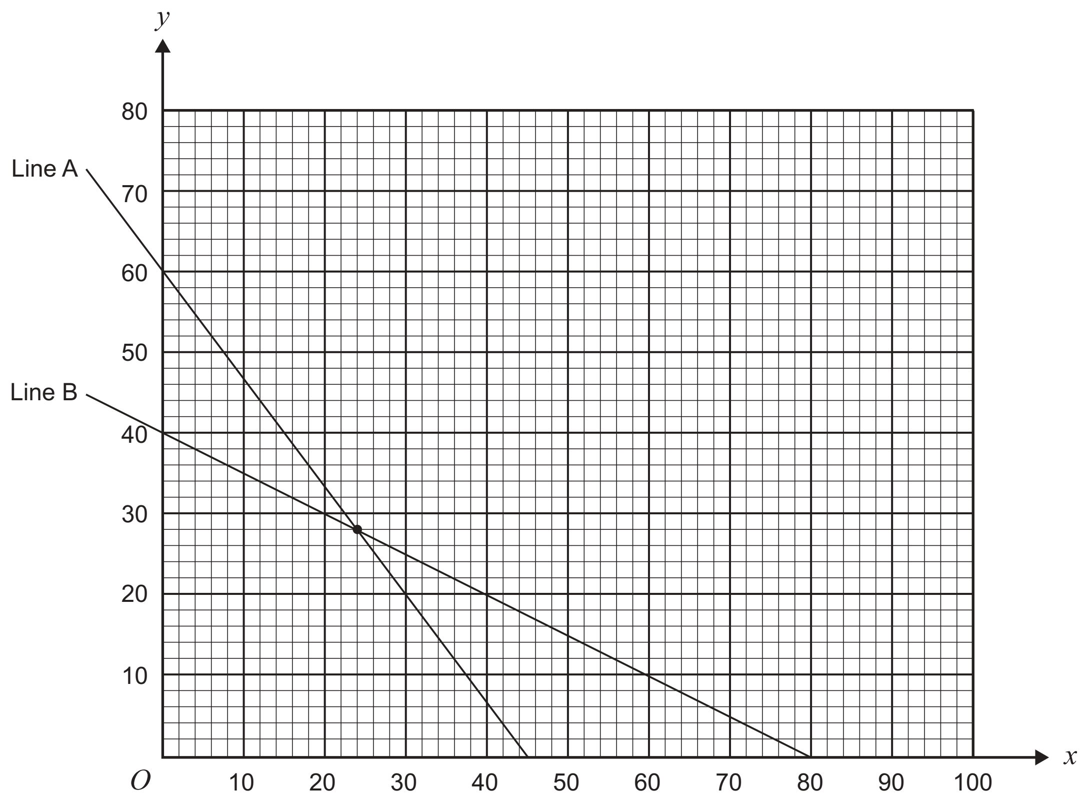
Inequality 1: $20x + 15y \geq 900$
Inequality 2: $2x + 4y \geq 160$
Inequality 3: $x \geq 0$
Inequality 4: $y \geq 0$

a. Which line (Line A or Line B) in Graph 1 below forms the boundary of the region defined by Inequality 1: $20x + 15y \geq 900$?

1 mark

b. Write down the coordinates of the point of intersection of Line A and Line B in Graph 1.

1 mark



Graph 1

c. Pepi is allowed to operate the Xenon Street stand for up to 64 hours per week and the Yarra Street stand for up to 52 hours per week.

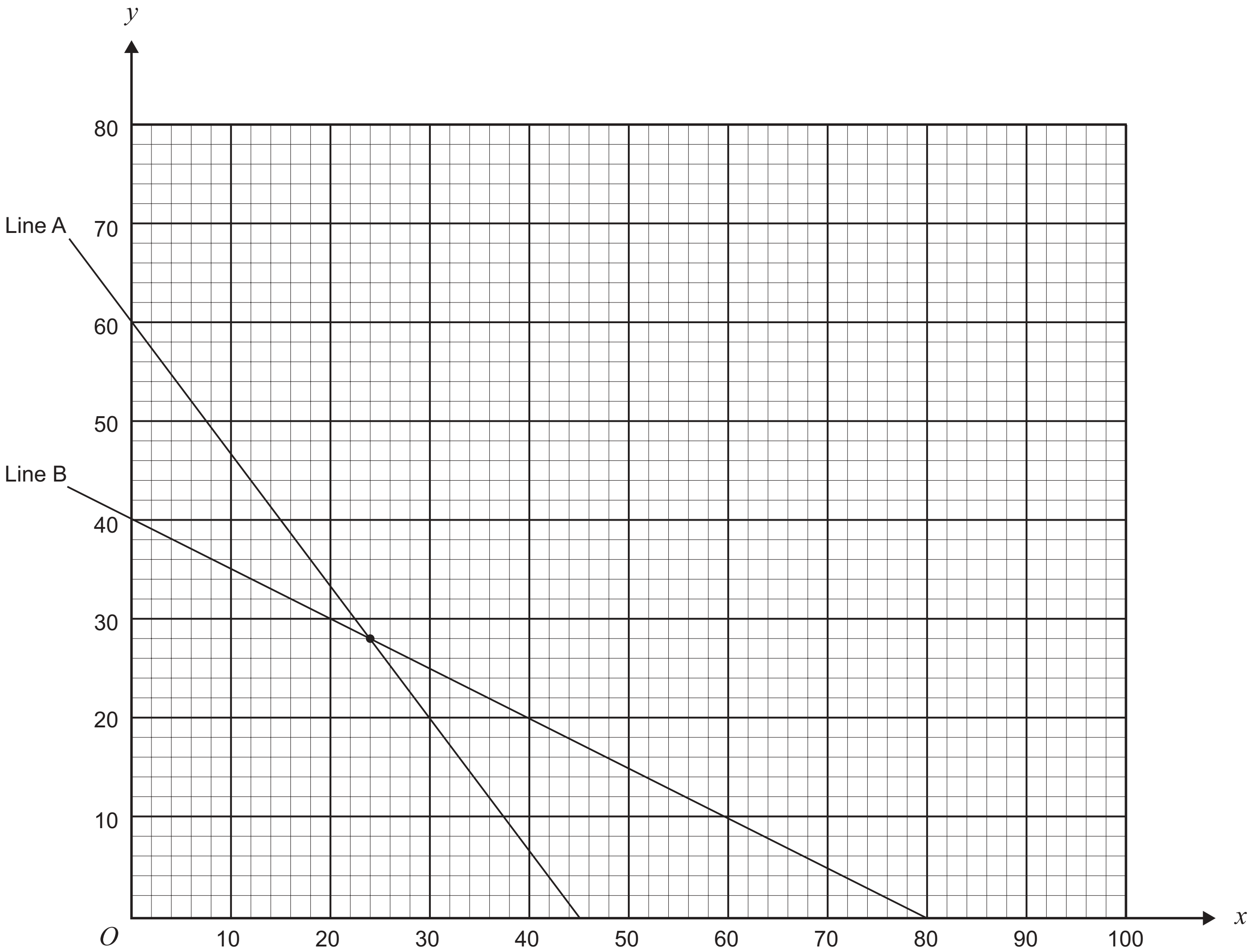
Write two corresponding inequalities.

Inequality 5:

Inequality 6:

1 mark

d. Using Inequalities 1 to 6, construct and shade the feasible region for operating the two street stands for one week on Graph 2 below.



Graph 2

3 marks

e. The cost, per hour, of operating each stand is

Xenon Street: $100

Yarra Street: $70

Let C be the total cost of operating the two street stands for one week.

Write an equation for C in terms of x and y.

1 mark

f. Under the conditions described, what is the minimum total cost of operating the street stands for one week?

2 marks

Pepi has decided to sell books at each stand in addition to the newspapers and magazines that are being sold.

On average, per hour, each stand will be able to sell

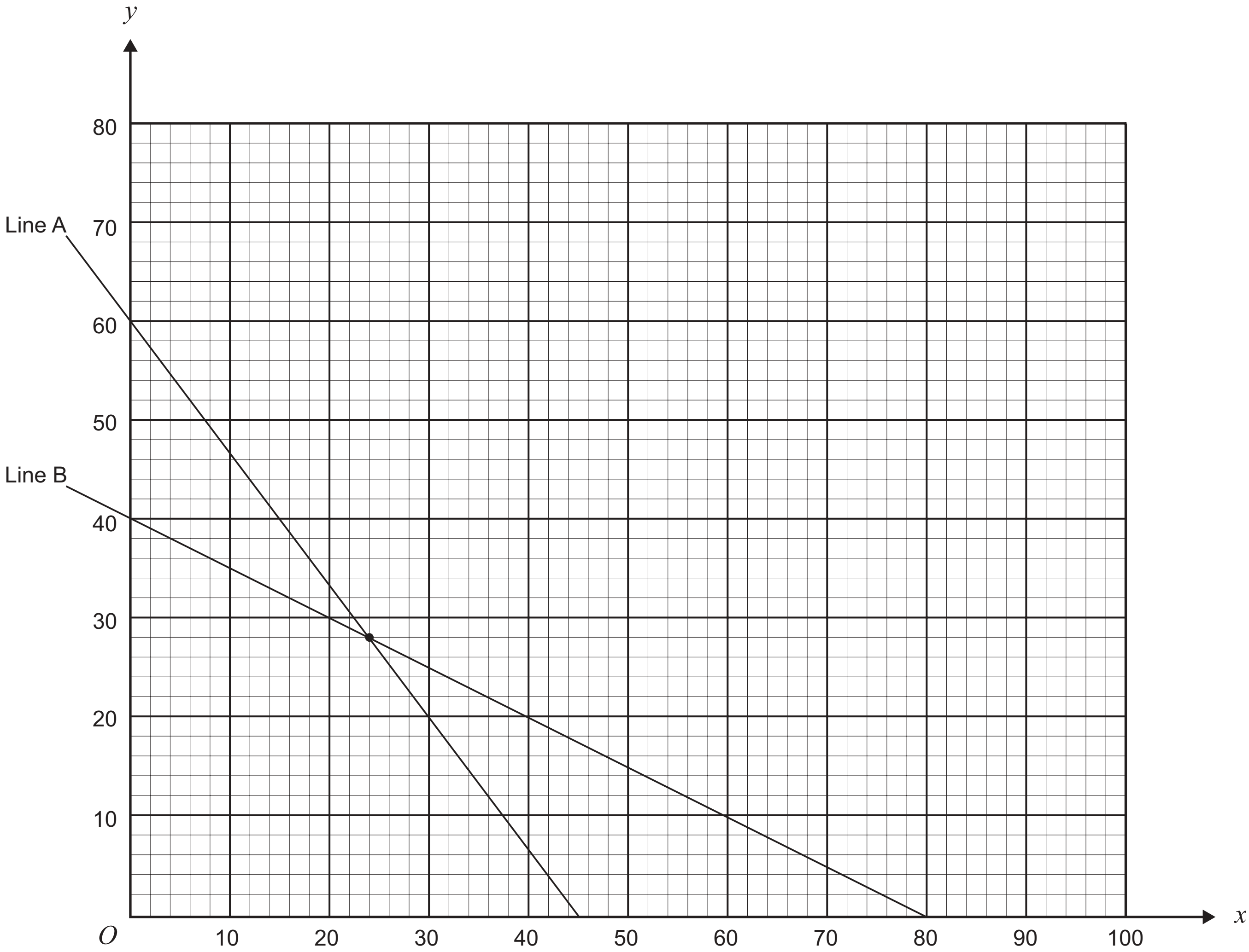
Xenon Street: 2 books
Yarra Street: 3 books.

In total, at least 150 books must be sold each week.

g. Write an inequality in terms of x and y for the total number of books that must be sold each week.

Inequality 7:

1 mark



Graph 3

h. Draw the line that forms the boundary of the region defined by Inequality 7 on Graph 3.

1 mark

i. In one particular week Pepi decided to operate the Xenon Street stand for 25 hours and the Yarra Street stand for 32 hours.

Will he be able to sell the required number of books if he operates the stands for these hours?

Justify your answer.

1 mark

j. Pepi is now allowed to operate each street stand for as many hours as he wants each week. He sells newspapers, magazines and books at each stand and meets all sales requirements for these items.

The cost, per hour, of operating each stand increases to

Xenon Street: $120
Yarra Street: $90

i. Determine the new minimum total cost of operating the street stands for one week.

ii. Find all solutions for the number of hours per week that each stand can operate so that the minimum total operating cost is achieved.

1 + 2 = 3 marks

Total 15 marks

END OF MODULE 3

Working space

Module 4: Business-related mathematics

Question 1

Stan bought a \$4 000 computer under a hire-purchase agreement. He paid \$500 deposit and will repay the balance in equal monthly instalments over two years. A flat rate of interest is charged. The total amount Stan will pay for the computer (including the deposit) is \$4 560.

a. Determine the total amount of interest Stan pays.

1 mark

b. Show that the flat rate of interest for this agreement is 8% per annum.

1 mark

c. Determine the **effective** rate of interest per annum.

Write your answer correct to one decimal place.

1 mark

d. Explain why an effective interest rate differs from a flat interest rate.

1 mark

Question 2

For taxation purposes, Stan will depreciate his $4 000 computer over five years. At the end of five years the book value of his computer will be $1 000.

a. If Stan uses **flat rate depreciation**, determine the annual depreciation rate.

2 marks

b. If Stan uses **reducing balance depreciation**, determine the annual depreciation rate.
Write your answer correct to one decimal place.

2 marks

Question 3

Stan's friend Lena has some money that she wishes to invest for a period of five years. She is considering three investment options.

a. Investment Option A

$10 000 is deposited into an account with an interest rate of 4.8% per annum compounding monthly for five years.

i. To determine the value of this investment at the end of five years, Lena correctly applies the compound interest formula $A = PR^n$.

Write down the values of P, R and n that Lena uses.

$P =$ ☐ $R =$ ☐ $n =$ ☐

ii. Calculate the value of Investment Option A at the end of five years.

Write your answer correct to the nearest cent.

2 + 1 = 3 marks

b. Investment Option B

$4 000 is deposited into an account with an interest rate of 4.8% per annum compounding monthly. At the end of each month, for a period of five years, a further $100 is deposited after interest has been paid.

Determine the value of Investment Option B at the end of five years (immediately after the $100 has been deposited). Write your answer correct to the nearest cent.

1 mark

c. Investment Option C

Investment Option B is followed for two years. After this, the amount deposited at the end of each month changes. With the new monthly deposit, Investment Option C is worth $13 000 at the end of the five years.

i. Find the new amount deposited at the end of each month for the remaining three years.

Write your answer correct to the nearest cent.

ii. Determine the total amount of interest earned by Investment Option C over the **five-year** period.

Write your answer correct to the nearest cent.

1 + 2 = 3 marks

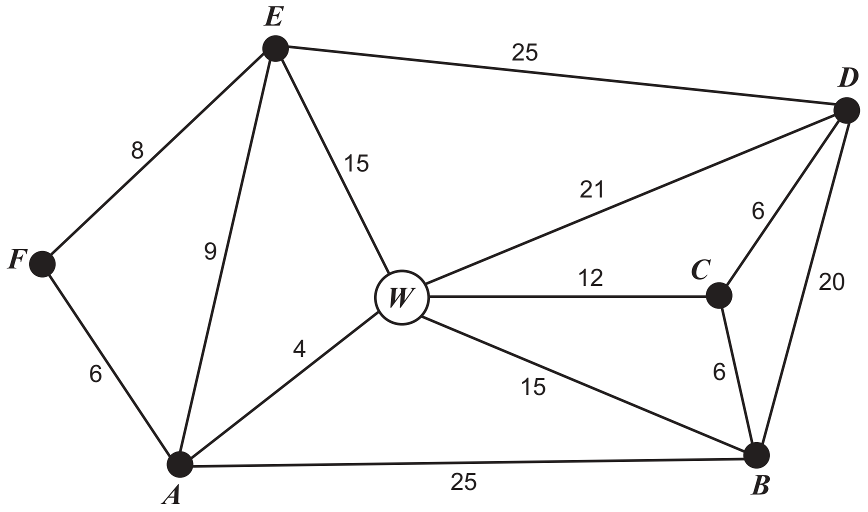
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

The network diagram below shows the location of a warehouse, *W*. This warehouse supplies equipment to six factories *A*, *B*, *C*, *D*, *E* and *F*. The numbers on the edges indicate the shortest distance (in kilometres) to drive along each of the connecting roads.



a. The degree of vertex *W* is ☐

1 mark

b. A delivery van is at factory *B*. It must first make a delivery to factory *D* and then drive to the warehouse, *W*.
Determine the minimum distance travelled on this journey.

____

____

1 mark

c. A salesman plans to leave factory E, first visit the warehouse, W, and then visit every other factory. He is to visit each location only once. He will not return to factory E.

i. Write down the mathematical term used to describe the route that he plans to take.

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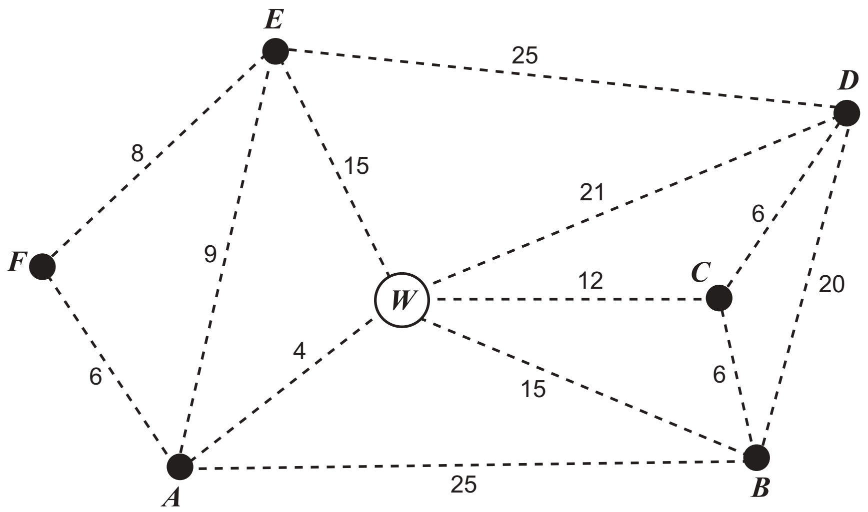
ii. On the network diagram below, mark in a complete route for his planned journey.



1 + 1 = 2 marks

d. The company plans to build an office along one of the roads in the network.

The manager wishes to drive along a route through the network which follows an Euler **circuit**. She will start at the warehouse, W.

i. Explain why the journey that the manager plans to take is **not** possible for this network.

___

___

___

ii. A journey that follows an Euler **path**, starting at the warehouse, W, is possible for this network.

At which vertex will this Euler path end?

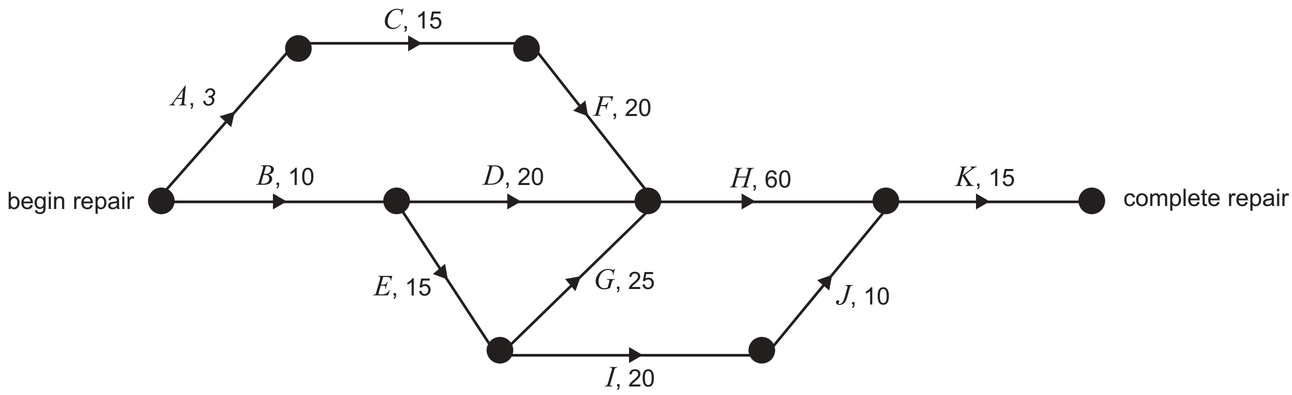
___

1 + 1 = 2 marks

Question 2

One of the delivery vans has to be repaired.

This repair usually involves every activity shown in the network diagram below. The duration of each activity, in minutes, is also shown.



The incomplete table below shows this same information and includes predecessor activities and the earliest starting times (EST).

Activity	Predecessor(s)	Duration of activity (minutes)	Earliest starting time EST (minutes)
A	–	3	0
B	–	10	0
C	A	15	3
D	B	20	10
E	B	15	10
F	C	20	18
G	E	25	25
H		60	
I	E	20	25
J	I	10	45
K	H, J	15	110

a. Complete the two shaded cells for activity *H* in the table above.

2 marks

b. All activities are required for this repair. What is the minimum time needed to complete this repair?

___

1 mark

c. During the repair to this delivery van it is found that activity *F* will take longer than the usual 20 minutes.

i. Explain why the duration of activity *F* can be increased without delaying this repair.

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ii. What is the maximum duration of activity *F* that will not delay completion of this repair?

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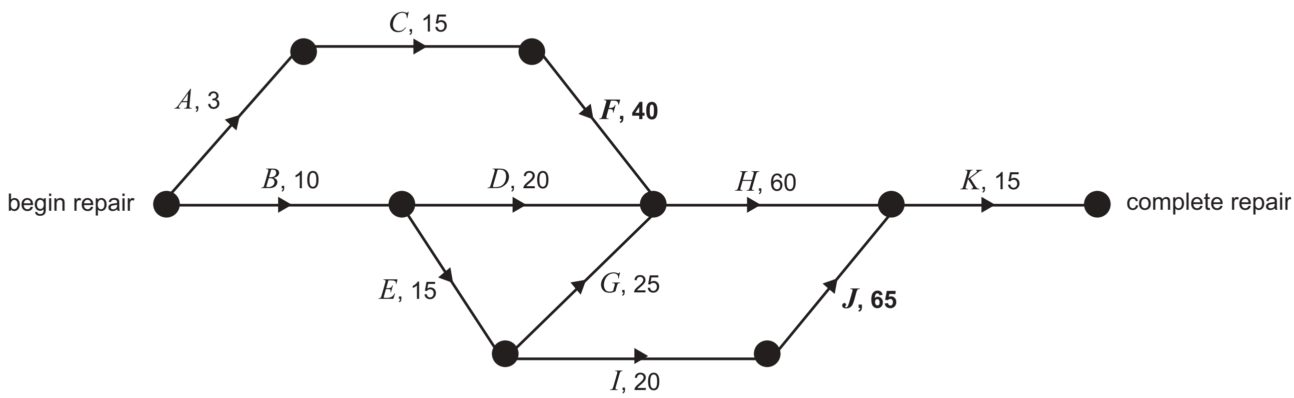
1 + 1 = 2 marks

d. Unfortunately further complications arise with this repair. Activity *F* will now take 40 minutes and activity *J* will now take 65 minutes. The network diagram below shows the increased duration of these activities.



The mechanic also finds that activity *J* now cannot start until activity *G* has been completed.

i. **Alter the network diagram above** to allow for this requirement.

ii. What is the latest starting time (LST) for activity *H* that will not further delay completion of this repair?

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iii. From beginning to completion, what is the minimum time needed to repair this delivery van?

Several activities for this repair can be delayed without increasing the minimum completion time.

iv. Which of these activities can be delayed for the longest time?

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1 + 1 + 1 + 1 = 4 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$, *where* $R = 1 + \dfrac{r}{100}$

Geometry and trigonometry

area of a triangle: $\dfrac{1}{2} bc \sin A$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Graphs and relations

Straight line graphs

gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y - y_1 = m(x - x_1)$ gradient-point form

$y = mx + c$ gradient-intercept form

$\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ two-point form

Number patterns and applications

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, |r| < 1$

linear difference equations: $t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1}-1)}{a-1}, a \neq 1$

$$= a^n t_0 + b\frac{(a^n - 1)}{a-1}$$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

END OF FORMULA SHEET