



# Victorian Certificate of Education 2005

## FURTHER MATHEMATICS

### Written examination 1 (Facts, skills and applications)

Monday 31 October 2005

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

### MULTIPLE-CHOICE QUESTION BOOK

#### Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
A	13	13			13
B	45	27	5	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten), one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question book of 31 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

#### Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

#### At the end of the examination

- You may keep this question book.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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Working space

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**SECTION A****Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

**Core**

*The following information relates to Questions 1 and 2.*

The table below lists the speed (in km/h) of ten cars recorded in a 60 km/h zone. Also recorded are the ages (in years) of the drivers.

Speed	Age
71.8	27
68.3	38
65.1	22
63.2	64
62.8	57
62.6	37
62.5	21
61.3	19
60.1	57
59.8	61

**Question 1**

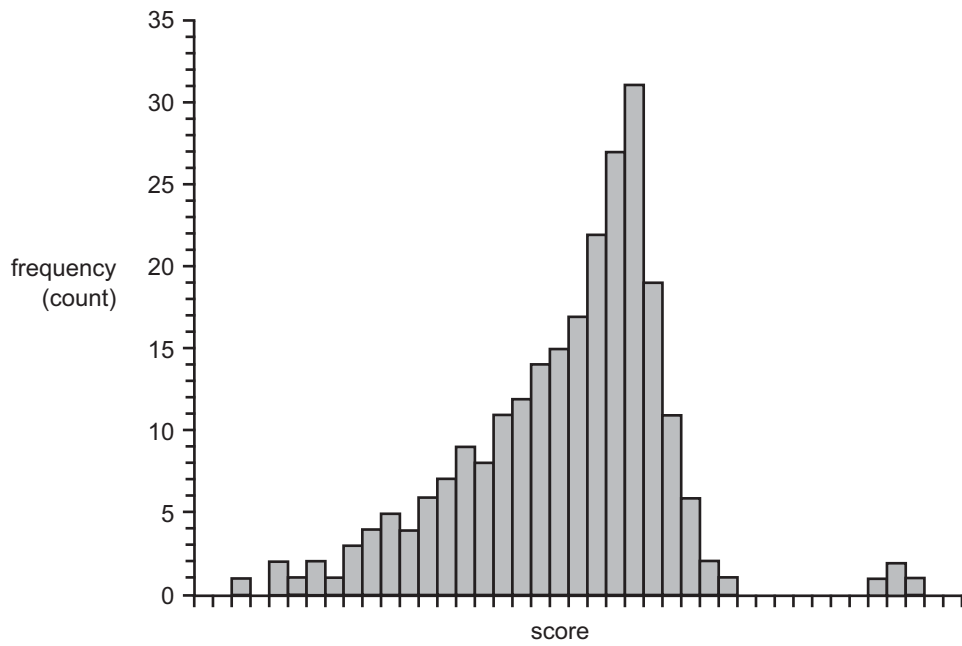
The median speed (in km/h) of the ten cars is

- A. 62.6
- B. 62.7
- C. 62.8
- D. 63.0
- E. 63.5

**Question 2**

The percentage of the drivers over the age of 25 years is

- A. 30%
- B. 40%
- C. 50%
- D. 60%
- E. 70%

**Question 3**

The histogram above is best described as

- A. negatively skewed.
- B. positively skewed.
- C. symmetric.
- D. negatively skewed with outliers.
- E. positively skewed with outliers.

*The following information relates to Questions 4 and 5.*

Text messaging use (never, sometimes, everyday) and the number of mobile phones in the household were recorded for a sample of 154 households. The results are shown in the table below.

Text messaging use	Number of mobile phones in household				Total
	0	1	2	3	
Never	34	10	3	0	47
Sometimes	0	23	12	2	37
Everyday	0	45	15	10	70
<b>Total</b>	34	78	30	12	154

**Question 4**

Of the households with two mobile phones in the sample, the percentage that never used text messaging is

- A. 0%
- B. 6%
- C. 10%
- D. 20%
- E. 30%

**Question 5**

The mean number of mobile phones in these 154 households is closest to

- A. 1.13
- B. 1.45
- C. 1.50
- D. 1.54
- E. 2.00

**Question 6**

The distribution of fuel consumption of a particular model of car is approximately bell-shaped with a mean of 8.8 km per litre and a standard deviation of 2.2 km per litre.

The percentage of this model of car that has a fuel consumption less than 6.6 km per litre is closest to

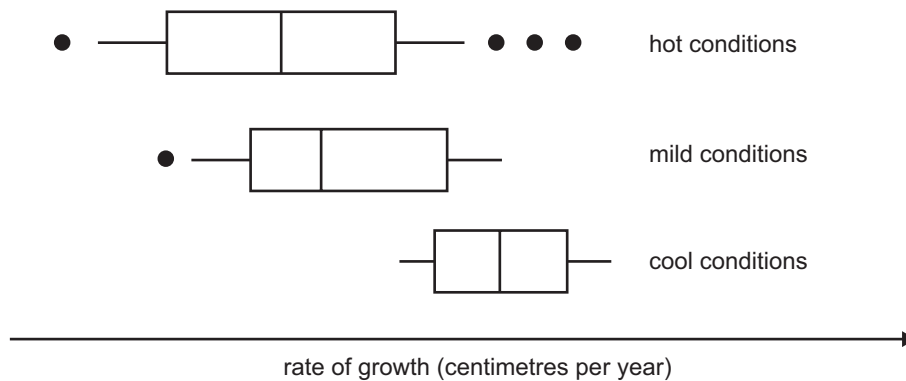
- A. 2.5%
- B. 5%
- C. 16%
- D. 32%
- E. 68%

**Question 7**

As part of an experiment, three samples of pine trees were planted. Each sample contained 50 trees.

One sample was grown under hot conditions, one sample was grown under mild conditions and one sample was grown under cool conditions.

The parallel box plots below show the rate of growth (in centimetres per year) of these three samples.



From the parallel box plots it can be concluded that, as conditions change from hot to mild to cool, the rate of growth for these trees

- A. decreases on average and becomes less variable.
- B. decreases on average and becomes more variable.
- C. does not change on average but becomes more variable.
- D. increases on average and becomes less variable.
- E. increases on average and becomes more variable.

*The following information relates to Questions 8 and 9.*

The length (in metres) and wingspan (in metres) of eight commercial aeroplanes are displayed in the table below.

<b>Length</b>	70.7	70.7	63.7	58.4	54.9	39.4	36.4	33.4
<b>Wingspan</b>	64.4	59.6	60.3	60.3	47.6	35.8	28.9	28.9

**Question 8**

Correct to four decimal places, the value of Pearson's product moment correlation coefficient for this data is

- A. 0.9371
- B. 0.9583
- C. 0.9681
- D. 0.9793
- E. 0.9839

**Question 9**

The equation of the least squares regression line for this data is

$$\text{wingspan} = -2.99 + 0.96 \times \text{length}$$

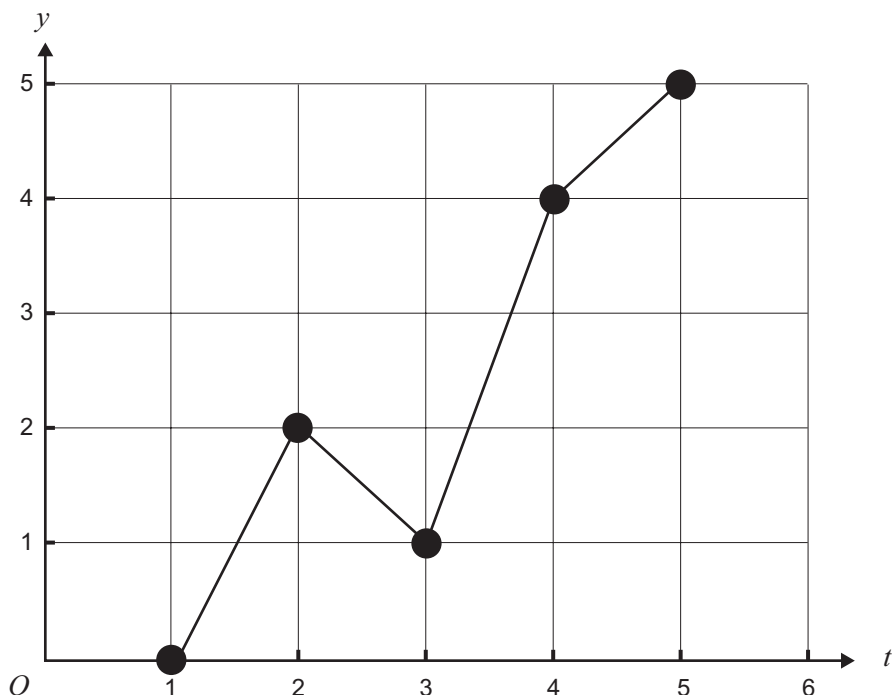
From this equation it can be concluded that, on average, for these aeroplanes, wingspan

- A. decreases by 2.03 metres with each one metre increase in length.
- B. increases by 0.96 metres with each one metre increase in length.
- C. decreases by 0.96 metres with each one metre increase in length.
- D. increases by 2.99 metres with each one metre increase in length.
- E. decreases by 2.99 metres with each one metre increase in length.



**Question 10**

The five points shown on the grid below have been taken from a time series plot that is to be smoothed using median smoothing.



The coordinates of the median of these five points are

- A. (3, 1)
- B. (3, 2)
- C. (3, 2.4)
- D. (3, 2.5)
- E. (3, 3)

**Question 11**

The quarterly seasonal indices for sales in a shop are shown in the table below.

Quarter	1	2	3	4
Seasonal index	1.3	0.9	0.7	1.1

A seasonal index of 1.1 for Quarter 4 means that sales in Quarter 4 are typically

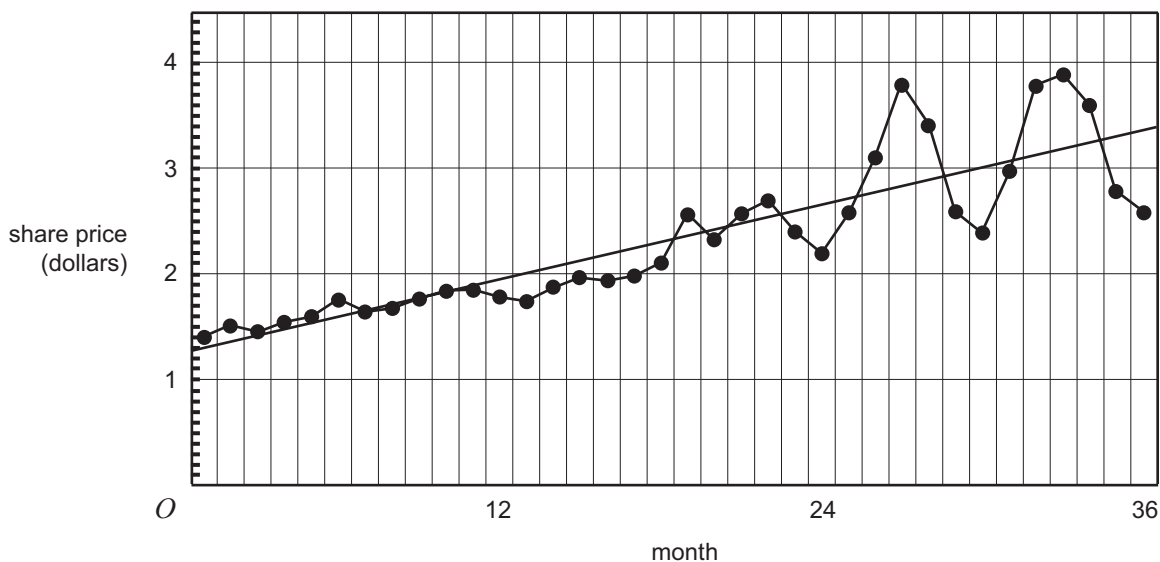
- A. 10% above the yearly average.
- B. 10% below the yearly average.
- C. 11% below the yearly average.
- D. 90% above the yearly average.
- E. 90% below the yearly average.

The following information relates to Questions 12 and 13.

The month-by-month price of a share listed on the Australian Stock Exchange is shown in the time series plot below for a 36-month period. Also shown is a least squares regression line that has been fitted to the data.

The equation of the least squares regression line is

$$\text{share price} = 1.24 + 0.06 \times \text{month}$$



**Question 12**

The least squares regression line predicts that the price of the share after 48 months will be

- A. \$1.96
- B. \$4.12
- C. \$5.04
- D. \$28.80
- E. \$62.40

**Question 13**

Which one of the following statements **best** describes the time series plot for the period shown?

- A. The share price shows no trend and no change in variability.
- B. The share price shows no trend and increases in variability.
- C. The share price shows an increasing linear trend with constant variability.
- D. The share price shows an increasing linear trend with decreasing variability.
- E. The share price shows an increasing linear trend with increasing variability.

**SECTION B****Instructions for Section B**

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

<b>Module</b>	<b>Page</b>
Module 1: Number patterns and applications	12
Module 2: Geometry and trigonometry	15
Module 3: Graphs and relations	19
Module 4: Business-related mathematics	23
Module 5: Networks and decision mathematics	26

**Module 1: Number patterns and applications**

Before answering these questions you must **shade** the Number patterns and applications box on the answer sheet for multiple-choice questions.

**Question 1**

Consider the arithmetic sequence:  $-5, 1, 7 \dots$

Another number that also appears in this sequence is

- A. 15
- B. 17
- C. 19
- D. 21
- E. 23

**Question 2**

Consider the following collection of sequences.

1.1, 1.11, 1.111, 1.1111  $\dots$

3, 6, 12, 24  $\dots$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \dots$

27, 9, 3, 1  $\dots$

The number of **geometric** sequences is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

**Question 3**

Gino paid \$1200 to hire a hall last year.

Gino hired the same hall this year.

The ratio of the amount paid to hire the hall last year to the amount paid to hire the hall this year was 3:5.

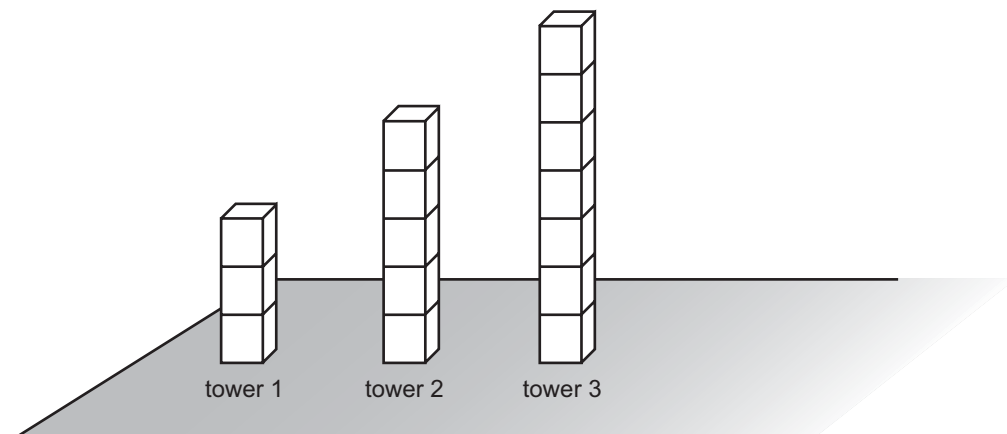
The amount Gino paid to hire the hall this year was

- A. \$450
- B. \$720
- C. \$750
- D. \$2000
- E. \$3200

**Question 4**

A child has a box of 50 blocks.

She lays them out in block towers, starting with tower 1, following the pattern as shown below.



If she continues to follow the same pattern, the maximum number of block towers that she can make is

- A. 4
- B. 5
- C. 6
- D. 7
- E. 24

**Question 5**

The first term of an infinite geometric sequence is 80.

Each successive term is 75% of the value of the previous term.

The sum of the infinite sequence is

- A. 80
- B. 106
- C. 140
- D. 240
- E. 320

**Question 6**

The first term in a sequence is 3.

The second term is obtained by multiplying the value of the first term by 5, then subtracting 2.

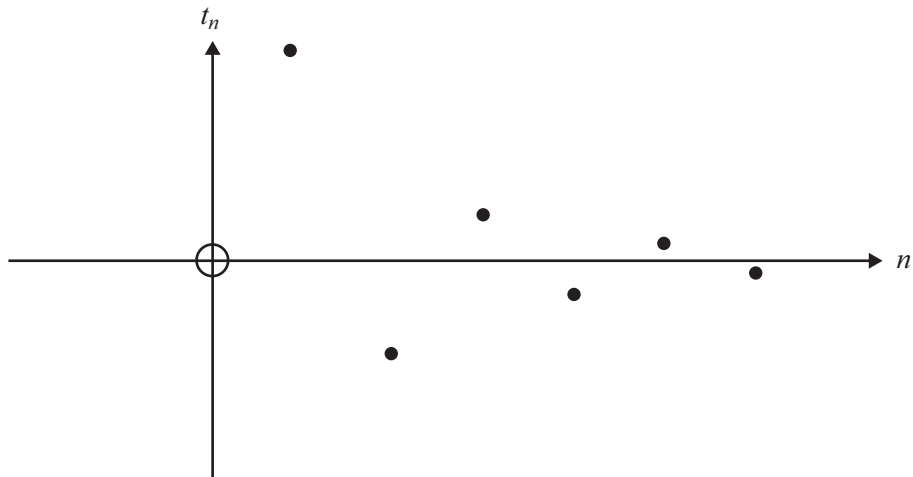
The third term is obtained by multiplying the value of the second term by 5, then subtracting 2.

This pattern continues.

The difference equation which generates this sequence is

- A.  $t_n = 5t_{n+1} - 2$       where       $t_1 = 3$
- B.  $t_{n+1} = 5t_n - 2$       where       $t_1 = 3$
- C.  $-5t_{n+1} = t_n - 2$       where       $t_1 = 3$
- D.  $5t_{n+1} = t_n - 2$       where       $t_1 = 3$
- E.  $t_{n+1} = -5t_n - 2$       where       $t_1 = 3$

**Question 7**



The graph above shows the first six terms of a sequence.

This sequence is **best** described as

- A. geometric with  $-1 < r < 0$
- B. geometric with  $0 < r < 1$
- C. geometric with  $r > 1$
- D. arithmetic with  $d < 0$
- E. arithmetic with  $0 < d < 1$

**Question 8**

A sequence is generated from the difference equation

$$t_{n+1} = at_n + b \quad \text{where} \quad t_1 = 3$$

The sequence is arithmetic if

- A.  $a = -1, b = 0$
- B.  $a = -1, b = 1$
- C.  $a = 1, b = 2$
- D.  $a = 2, b = 0$
- E.  $a = 2, b = -1$

**Question 9**

The rule for a difference equation is

$$P_n = 2P_{n-1} - 200$$

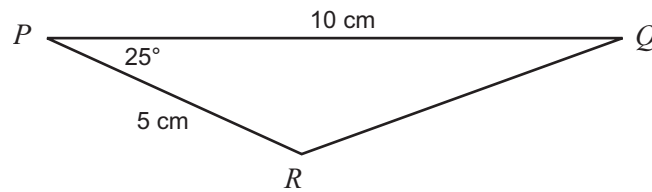
If  $P_6 = 1000$ , then  $P_4$  is equal to

- A. 100
- B. 400
- C. 600
- D. 1800
- E. 3400

## Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

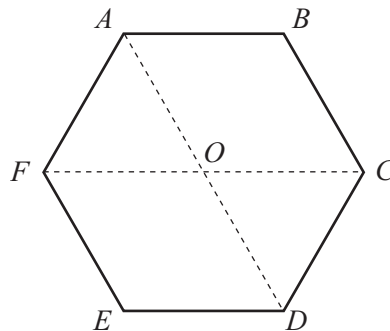
### Question 1



The area of the triangle  $PQR$  is closest to

- A.  $3.3\text{ cm}^2$
- B.  $6.3\text{ cm}^2$
- C.  $10.6\text{ cm}^2$
- D.  $12.5\text{ cm}^2$
- E.  $22.7\text{ cm}^2$

### Question 2

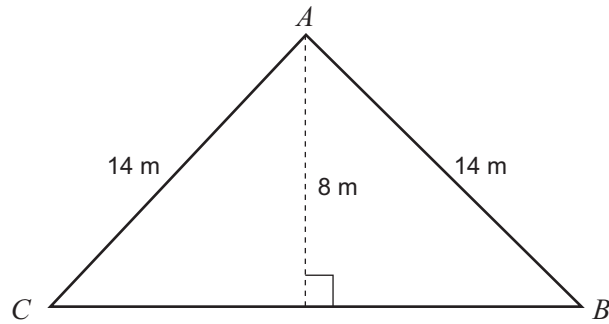


$ABCDEF$  is a regular hexagon.

The size of obtuse angle  $FOD$  is

- A.  $60^\circ$
- B.  $90^\circ$
- C.  $120^\circ$
- D.  $135^\circ$
- E.  $165^\circ$

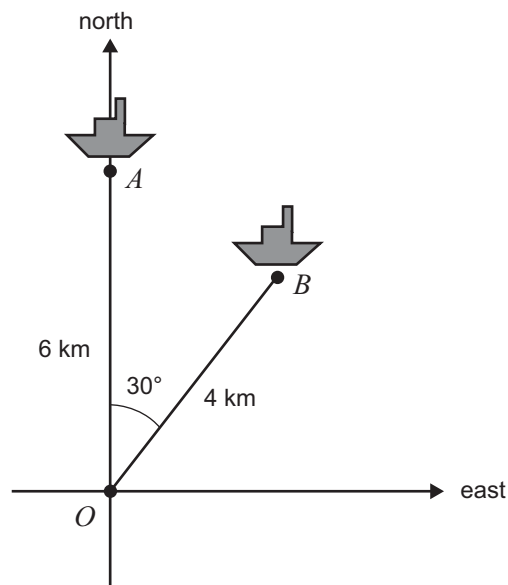
**Question 3**



In triangle  $ABC$ , the length  $CB$  is closest to

- A. 11.5 m
- B. 12.0 m
- C. 16.1 m
- D. 19.8 m
- E. 23.0 m

**Question 4**



Two ships are observed from point  $O$ . At a particular time their positions  $A$  and  $B$  are as shown. The distance between the ships at this time is

- A. 3.0 km
- B. 3.2 km
- C. 4.5 km
- D. 9.7 km
- E. 10.4 km

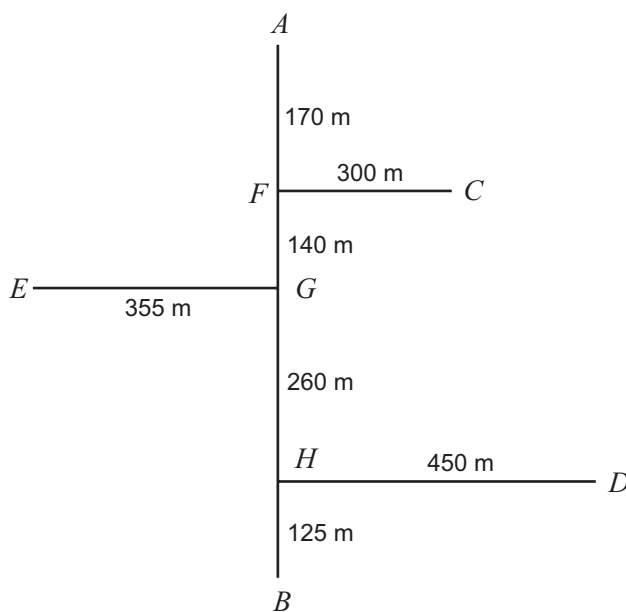


**Question 5**

The bearing of an aeroplane,  $X$ , from a control tower,  $T$ , is  $055^\circ$ . Another aeroplane,  $Y$ , is due east of control tower  $T$ . The bearing of aeroplane  $X$  from aeroplane  $Y$  is  $302^\circ$ .

The size of the angle  $TXY$  is

- A.  $32^\circ$
- B.  $35^\circ$
- C.  $55^\circ$
- D.  $58^\circ$
- E.  $113^\circ$

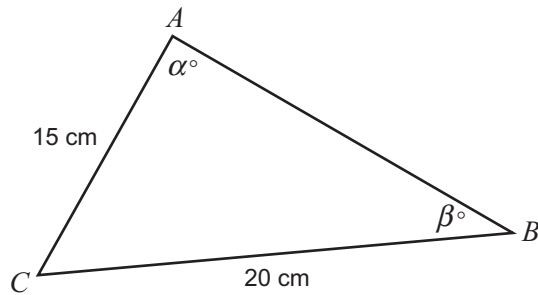
**Question 6**

The diagram above shows a traverse survey.

Correct to the nearest metre, the length  $CD$  is

- A. 427 m
- B. 493 m
- C. 500 m
- D. 550 m
- E. 602 m

**Question 7**



In triangle  $ABC$ ,  $\sin \alpha^\circ = 0.8$   
 $\sin \beta^\circ$  is equal to

- A. 0.5
- B. 0.6
- C. 0.75
- D. 0.8
- E. 0.9375

**Question 8**

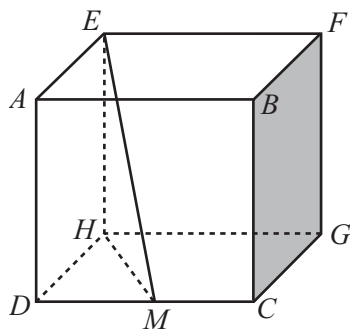
A factory floor is rectangular in shape with an area of  $1440 \text{ m}^2$ . One of the linear dimensions of the original floor is 36 m.

It is to be enlarged to a similar shape with an area of  $2250 \text{ m}^2$ .

When the area of the floor is enlarged, the corresponding linear dimension will be

- A. 40 m
- B. 45 m
- C. 50 m
- D. 56.25 m
- E. 62.5 m

**Question 9**



The cube above has sides four metres long.  
 $M$  is the midpoint of  $DC$ .

The angle  $EMH$  is closest to

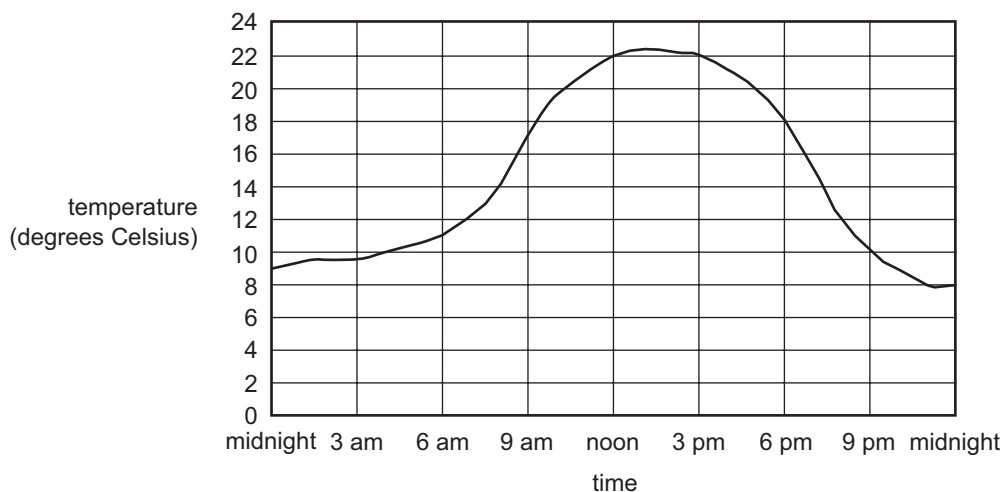
- A.  $41.8^\circ$
- B.  $48.2^\circ$
- C.  $49.1^\circ$
- D.  $54.7^\circ$
- E.  $70.5^\circ$

### Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

#### Question 1

The graph below shows the temperature (in degrees Celsius) over a 24-hour period.



The period of greatest temperature increase was

- A. 6 am–9 am
- B. 9 am–noon
- C. noon–3 pm
- D. 3 pm–6 pm
- E. 6 pm–9 pm

#### Question 2

Two lines have equations  $y = -5$  and  $y = -x + 5$  respectively.

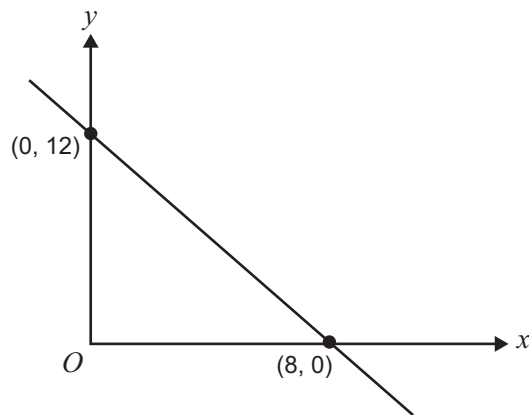
The point that lies on both of these lines is

- A.  $(-10, 5)$
- B.  $(-5, 5)$
- C.  $(0, -5)$
- D.  $(5, -5)$
- E.  $(10, -5)$

#### Question 3

Which one of the following statements is **not** true?

- A. The line with equation  $7x - 4y = 0$  passes through the point  $(4, 7)$ .
- B. The point  $(3, 5)$  lies in the region defined by  $7x - 4y \geq 0$ .
- C. The line with equation  $3x + 5y = 0$  has a positive gradient.
- D. The lines  $7x - 4y = 0$  and  $3x + 5y = 0$  meet at the origin.
- E. For the line with the equation  $7x - 4y = 0$ ,  $y$  increases as  $x$  increases.

**Question 4**

If the line above has equation  $3x + 2y = 4k$ , then the value of  $k$  must be

- A. 2
- B. 3
- C. 6
- D. 8
- E. 12

**Question 5**

An electrician charges a fixed call-out fee of \$50 and then charges \$65 per hour for each hour worked.

For  $n$  hours worked, the total charge in dollars is

- A. 115
- B.  $n + 115$
- C.  $50n + 65$
- D.  $65n + 50$
- E.  $115n$

**Question 6**

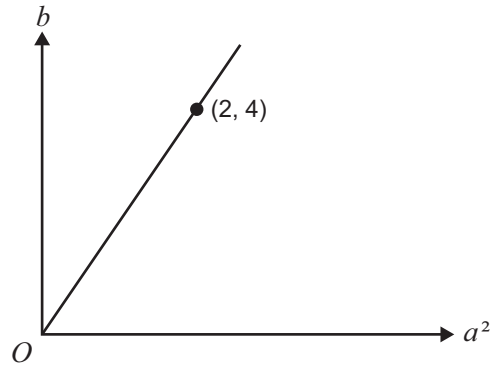
One afternoon at the beach Mr Smith bought four ice creams and three drinks for his family at a cost of \$21.40.

Mrs Brown bought five of the same ice creams and two of the same drinks for \$20.80.

Based on these prices, the cost of one drink is

- A. \$2.80
- B. \$2.90
- C. \$3.00
- D. \$3.30
- E. \$3.40

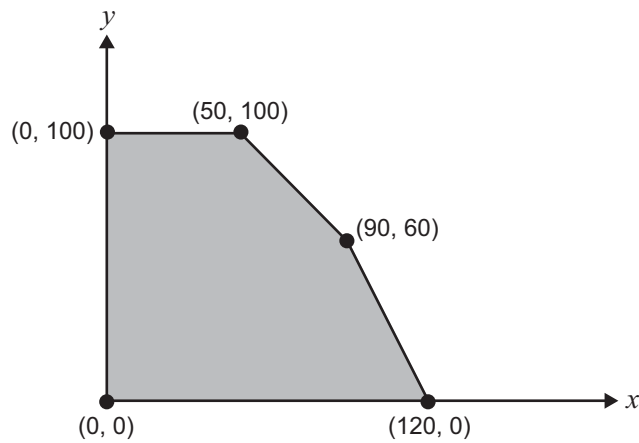
**Question 7**



The relationship between the variables  $a$  and  $b$  as shown in the graph above is

- A.  $b = a^2$
- B.  $b = 2a^2$
- C.  $b = 2a$
- D.  $a = 2b$
- E.  $a^2 = 2b$

**Question 8**



For the shaded region above (with boundaries included), the value of the objective function  $P = 4x - 3y$  is a **maximum** at the point

- A.  $(0, 0)$
- B.  $(0, 100)$
- C.  $(50, 100)$
- D.  $(90, 60)$
- E.  $(120, 0)$

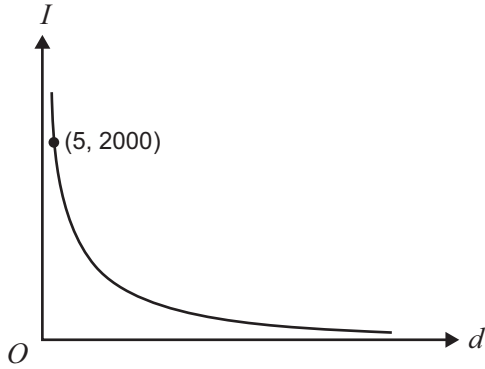
**Question 9**

The relationship between the intensity of sound, and the distance from the source of the sound, is  $I = \frac{k}{d^2}$ , where  $I$  is the intensity of sound measured in watts per square metre ( $\text{W}/\text{m}^2$ ),  $d$  is the distance in metres from the source, and  $k$  is a constant.

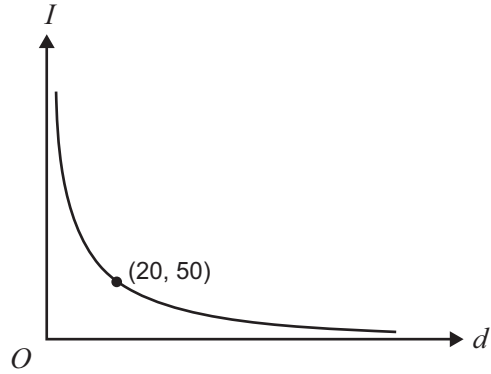
The intensity of sound is  $20 \text{ W}/\text{m}^2$  at a distance of 50 m from the source.

A graph showing the relationship between intensity of sound and distance from the source is

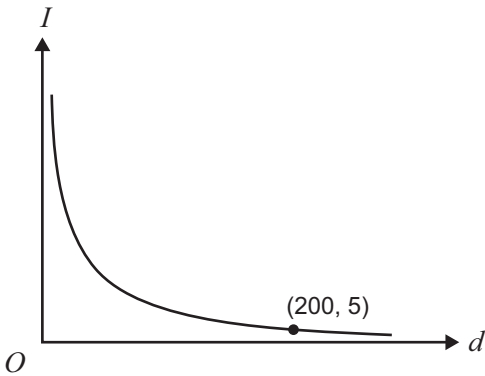
**A.**



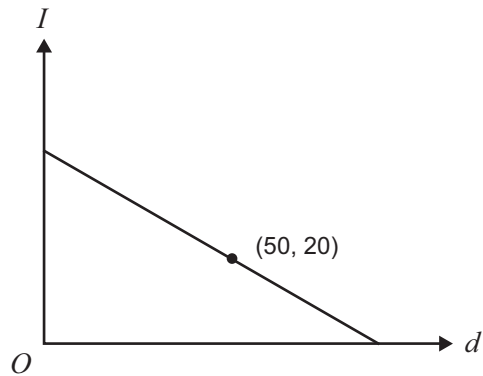
**B.**



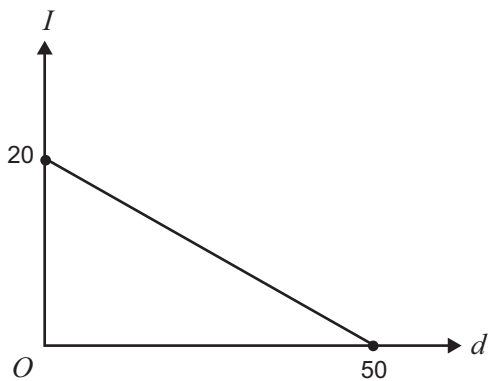
**C.**



**D.**



**E.**



## Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions.

### Question 1

An investment of \$1200 for one year returns \$140.38 interest.

The simple interest rate per annum for this investment is closest to

- A. 0.88%
- B. 3.21%
- C. 4.38%
- D. 11.70%
- E. 15.99%

### Question 2

The price of a book is \$22. It includes a Goods and Services Tax (GST) of 10%.

The price before this Goods and Services Tax (GST) is added is

- A. \$19.80
- B. \$20.00
- C. \$24.00
- D. \$24.20
- E. \$24.40

### Question 3

Tamara's bank statement for September has been damaged by spilt ink as shown below.

Date	Transaction details	Withdrawals (\$)	Deposits (\$)	Balance (\$)
01 Sept	Opening balance			2143.50
01 Sept	Interest		2.45	
13 Sept	Payment – Telstra	616.40		
30 Sept	Pay/Salary			
<b>Totals at the end of month</b>		<b>616.40</b>		<b>1971.75</b>

Tamara's Pay/Salary was deposited on 30 September.

What is the value of this deposit?

- A. \$441.95
- B. \$442.20
- C. \$444.65
- D. \$785.70
- E. \$788.15

**Question 4**

A machine that makes boxes costs \$45 000.

Its value depreciates by five cents for every box it makes.

Each year it makes 120 000 boxes.

The depreciated value of this machine at the end of **two** years is

- A. \$33 000
- B. \$38 000
- C. \$39 000
- D. \$45 000
- E. \$115 000

**Question 5**

An investor borrows \$200 000 for five years to buy an apartment.

The interest rate is 8.5% per annum compounding monthly.

It is an interest only loan, that is, at the end of five years, the investor will still owe \$200 000.

He is required to make monthly repayments.

Correct to the nearest cent, his monthly repayment will be

- A. \$666.67
- B. \$1416.67
- C. \$1757.67
- D. \$4103.31
- E. \$6789.95

**Question 6**

Tim invests \$3000 in a term deposit account that adds 6.5% interest annually, calculated on the account balance at the end of each year.

The interest paid in the fourth year is

- A. \$195.00
- B. \$221.16
- C. \$235.55
- D. \$3623.85
- E. \$3859.40

**Question 7**

Gregor invests \$10 000 and earns interest at a rate of 6% per annum compounding quarterly.

Every quarter, after interest has been added, he withdraws \$500.

At the end of four years, after interest has been added and he has made the \$500 withdrawal, the value of the remaining investment will be closest to

- A. \$3 720
- B. \$4 220
- C. \$5 440
- D. \$21 660
- E. \$22 160



**Question 8**

On Monday, a clock has a certain price.

In a crazy week of sales the price goes up and down as follows.

Tuesday – the price of the clock is increased by 10%.

Wednesday – Tuesday's price is reduced by 10%.

Thursday – Wednesday's price is increased by 20%.

Friday – Thursday's price is reduced by 20%.

When compared with Monday's price, the price on Friday is closest to

- A. the same as Monday's price.
- B. 4% lower than Monday's price.
- C. 4% higher than Monday's price.
- D. 5% lower than Monday's price.
- E. 5% higher than Monday's price.

**Question 9**

Sally planned to repay a loan fully with six equal monthly repayments of \$800.

Interest was calculated monthly on the reducing balance.

Sally missed the third payment, but made a **double** payment of \$1600 in the fourth month.

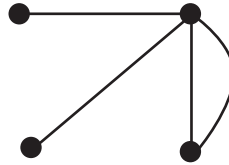
Which of the following statements is true?

- A. The same amount of interest is paid each month.
- B. The amount owing after three months is the same as the amount owed after two months.
- C. The amount owing after three months is less than the amount owed after two months.
- D. To fully repay the loan, Sally will pay less than \$4800.
- E. To fully repay the loan, Sally will pay more than \$4800.

**Module 5: Networks and decision mathematics**

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions.

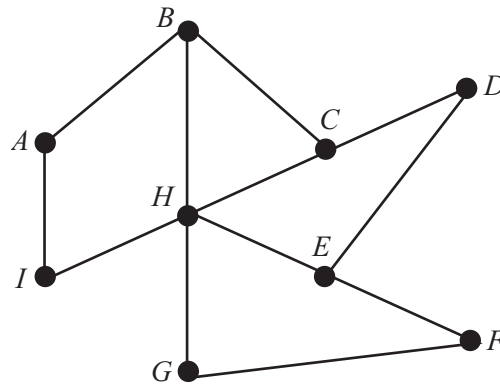
**Question 1**



The sum of the degrees of all the vertices in this network is

- A. 6
- B. 7
- C. 8
- D. 9
- E. 10

**Question 2**

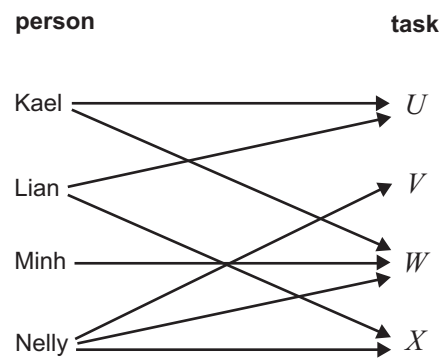


For the network above, an Euler **path** can be found

- A. without altering the network.
- B. by adding an edge that joins *A* to *H*.
- C. by adding an edge that joins *C* to *F*.
- D. by removing the edge that joins *B* to *C*.
- E. by removing the edge that joins *D* to *E*.

**Question 3**

The bipartite graph below represents the tasks that four people are able to undertake.



The matrix representation for this task allocation is

**A.**

$$\begin{array}{l}
 \text{Kael} \\
 \text{Lian} \\
 \text{Minh} \\
 \text{Nelly}
 \end{array}
 \begin{array}{cccc}
 U & V & W & X \\
 \left[ \begin{array}{cccc}
 1 & 1 & 0 & 0 \\
 1 & 0 & 0 & 1 \\
 0 & 0 & 1 & 0 \\
 0 & 1 & 1 & 1
 \end{array} \right]
 \end{array}$$

**B.**

$$\begin{array}{l}
 \text{Kael} \\
 \text{Lian} \\
 \text{Minh} \\
 \text{Nelly}
 \end{array}
 \begin{array}{cccc}
 U & V & W & X \\
 \left[ \begin{array}{cccc}
 1 & 0 & 1 & 0 \\
 1 & 0 & 0 & 1 \\
 0 & 0 & 1 & 0 \\
 0 & 1 & 1 & 1
 \end{array} \right]
 \end{array}$$

**C.**

$$\begin{array}{l}
 \text{Kael} \\
 \text{Lian} \\
 \text{Minh} \\
 \text{Nelly}
 \end{array}
 \begin{array}{cccc}
 U & V & W & X \\
 \left[ \begin{array}{cccc}
 1 & 0 & 1 & 0 \\
 1 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 1 & 1 & 1
 \end{array} \right]
 \end{array}$$

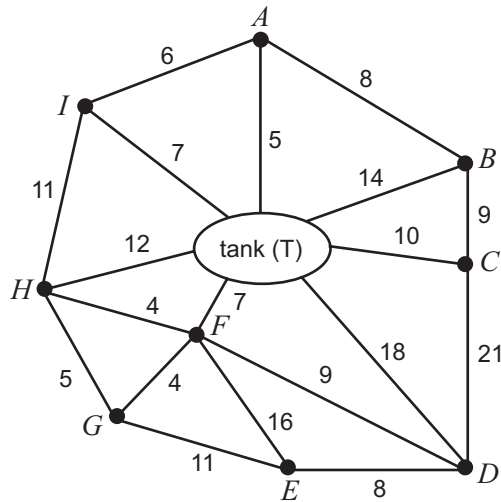
**D.**

$$\begin{array}{l}
 \text{Kael} \\
 \text{Lian} \\
 \text{Minh} \\
 \text{Nelly}
 \end{array}
 \begin{array}{cccc}
 U & V & W & X \\
 \left[ \begin{array}{cccc}
 1 & 0 & 1 & 0 \\
 1 & 0 & 0 & 1 \\
 0 & 0 & 1 & 1 \\
 0 & 1 & 1 & 1
 \end{array} \right]
 \end{array}$$

**E.**

$$\begin{array}{l}
 \text{Kael} \\
 \text{Lian} \\
 \text{Minh} \\
 \text{Nelly}
 \end{array}
 \begin{array}{cccc}
 U & V & W & X \\
 \left[ \begin{array}{cccc}
 1 & 0 & 1 & 0 \\
 1 & 1 & 0 & 1 \\
 0 & 0 & 1 & 0 \\
 0 & 1 & 1 & 1
 \end{array} \right]
 \end{array}$$

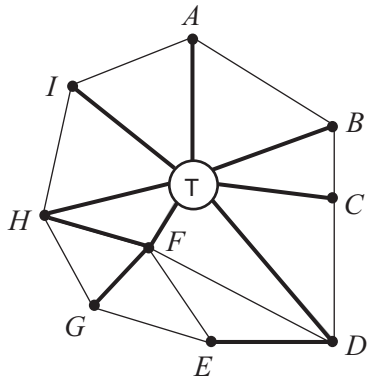
**Question 4**



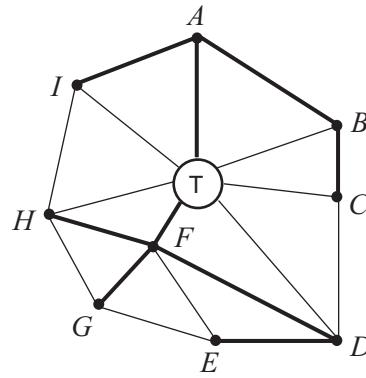
The network diagram above shows distances (in metres) between different points in a garden,  $A, B, C, D, E, F, G, H$  and  $I$ . Also shown are the distance from a tank (T) and these points. Pipes will carry water from the tank to each of these nine points. In the network diagrams below, the water pipes are indicated in bold.

Which of these network diagrams shows the minimum length of water pipe required to connect the tank (T) to each of the nine points?

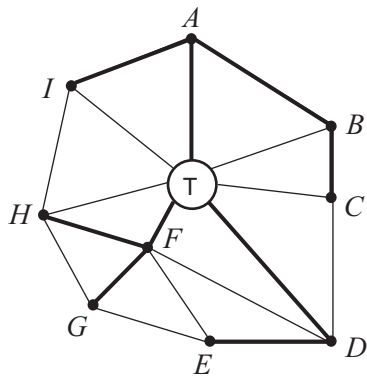
**A.**



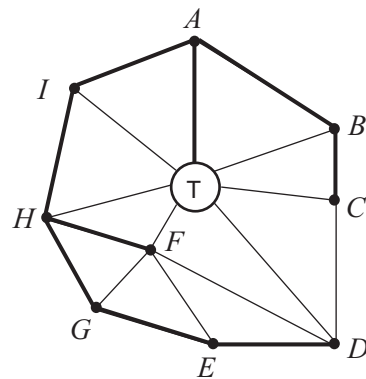
**B.**



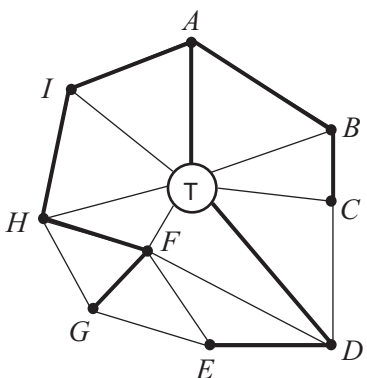
**C.**



**D.**



**E.**



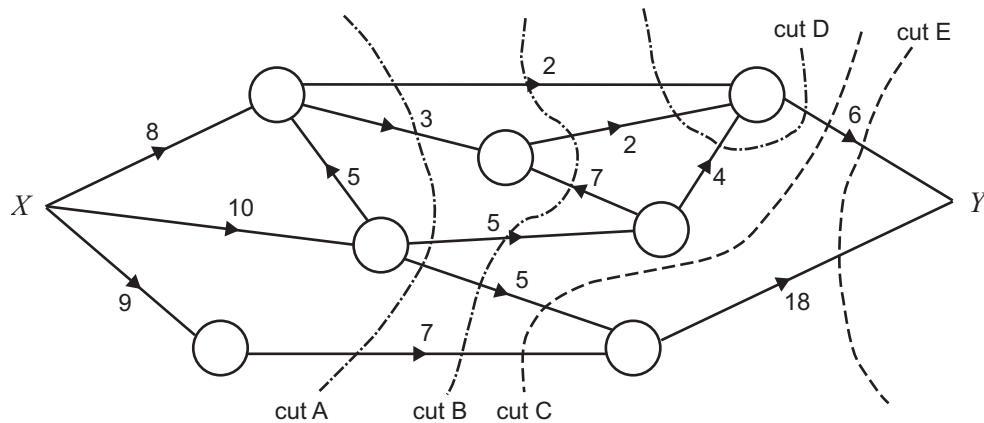
**Question 5**

A connected planar graph has 10 vertices and 15 edges.

A number of edges are removed to leave a connected graph with 10 vertices and 3 faces.

The number of edges that were removed is

- A. 4
- B. 5
- C. 6
- D. 7
- E. 8

**Question 6**

On the directed graph above, the values on the edges give the maximum flow between nodes in the direction of the arrows.

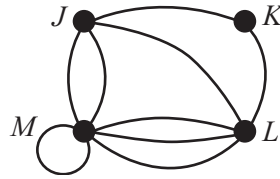
Five cuts have been made on the diagram.

Which cut allows you to find the maximum flow from point  $X$  to point  $Y$ ?

- A. cut A
- B. cut B
- C. cut C
- D. cut D
- E. cut E

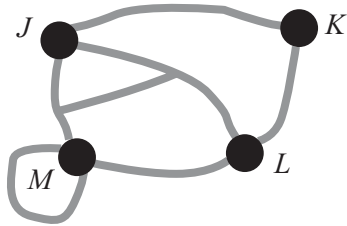
**Question 7**

The diagram below represents a network of road connections between the four towns, *J*, *K*, *L* and *M*.

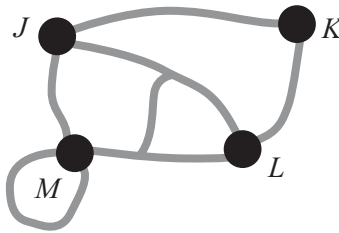


The map that corresponds to the roads that are represented by the network diagram could be

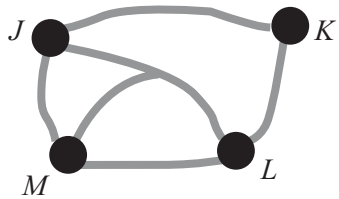
**A.**



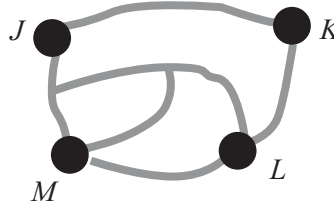
**B.**



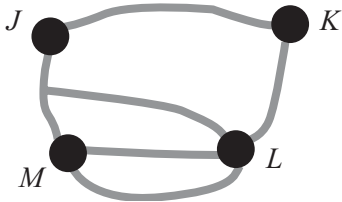
**C.**

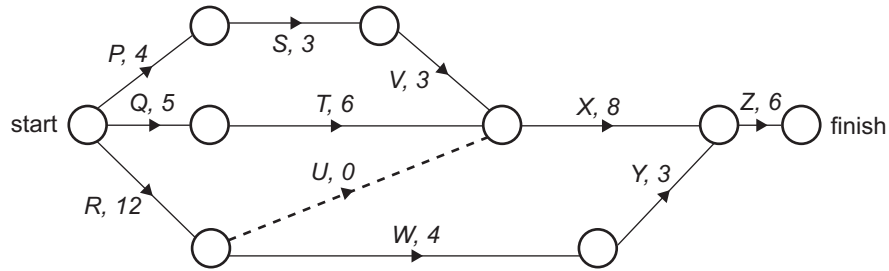


**D.**



**E.**



**Question 8**

The activities and their completion times (in hours) that are needed to complete a project are shown in the network above.

For this project, the minimum time to complete the whole project would be increased if one of the activities was delayed.

This activity is

- A. *P*
- B. *Q*
- C. *R*
- D. *T*
- E. *W*

**Question 9**

The matrix below shows the roads between five towns, *Q*, *R*, *S*, *T*, *U* and the distances in kilometres between the towns. A dash (–) indicates that there is no direct connection between two particular towns.

	<i>Q</i>	<i>R</i>	<i>S</i>	<i>T</i>	<i>U</i>
<i>Q</i>	0	8	15	–	4
<i>R</i>	8	0	5	–	–
<i>S</i>	15	5	0	10	7
<i>T</i>	–	–	10	0	–
<i>U</i>	4	–	7	–	0

The shortest route, in kilometres, between *Q* and *T* is

- A. 10
- B. 21
- C. 23
- D. 25
- E. 27

# **FURTHER MATHEMATICS**

## **Written examinations 1 and 2**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.



## Further Mathematics Formulas

### Business-related mathematics

simple interest:  $I = \frac{PrT}{100}$

compound interest:  $A = PR^n$  where  $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest  $\approx \frac{2n}{n+1} \times \text{flat rate}$

annuities:  $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ , where  $R = 1 + \frac{r}{100}$

### Geometry and trigonometry

area of a triangle:  $\frac{1}{2}bc \sin A$

area of a circle:  $\pi r^2$

volume of a sphere:  $\frac{4}{3}\pi r^3$

volume of a cone:  $\frac{1}{3}\pi r^2 h$

Pythagoras' theorem:  $c^2 = a^2 + b^2$

sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule:  $c^2 = a^2 + b^2 - 2ab \cos C$

### Graphs and relations

#### Straight line graphs

gradient:  $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation:  $y - y_1 = m(x - x_1)$  gradient-point form

$y = mx + c$  gradient-intercept form

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$  two-point form

## Number patterns and applications

arithmetic series: 
$$a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

geometric series: 
$$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

infinite geometric series: 
$$a + ar + ar^2 + ar^3 + \dots = \frac{a}{1-r}, |r| < 1$$

linear difference equations: 
$$t_n = at_{n-1} + b = a^{n-1}t_1 + b \frac{(a^{n-1} - 1)}{a - 1}, a \neq 1$$

$$= a^n t_0 + b \frac{(a^n - 1)}{a - 1}$$

## Networks and decision mathematics

Euler's formula: 
$$v + f = e + 2$$

## Statistics

seasonal index: 
$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$