VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2004

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

										Letter
Figures										
Words										

FURTHER MATHEMATICS

Written examination 2 (Analysis task)

Wednesday 3 November 2004

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
5	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 25 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

This examination consists of a core and five modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core

Question 1

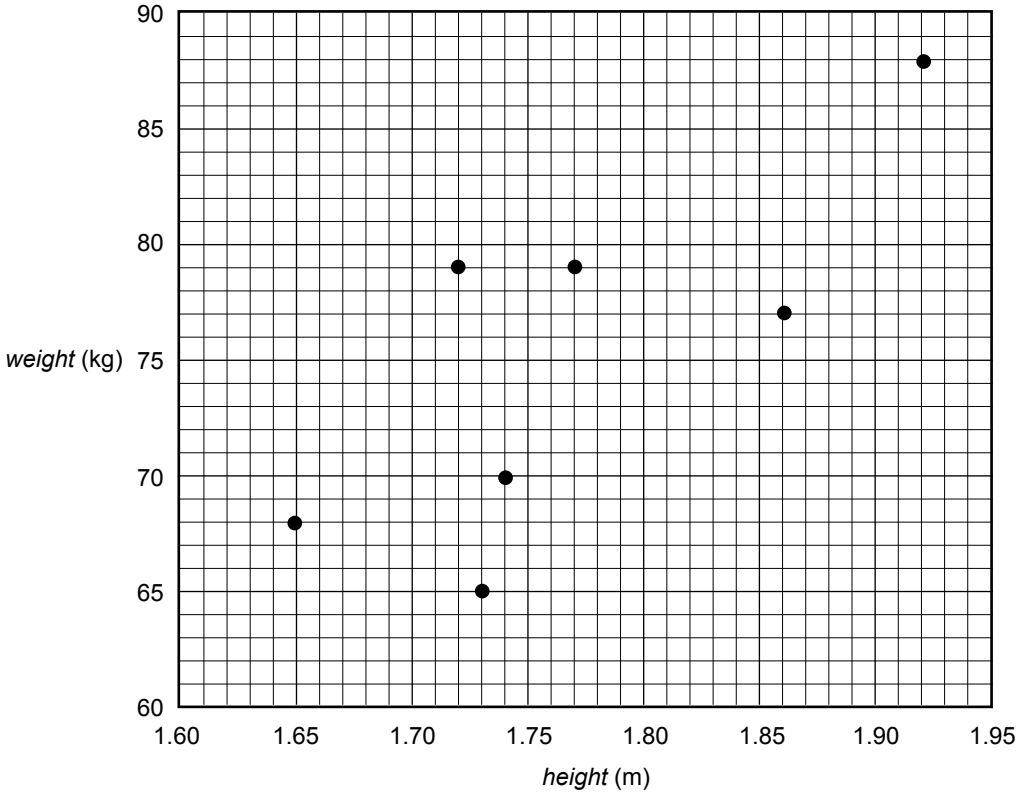
Table 1. Heights and weights of nine people

height (m)	*weight* (kg)
1.65	68
1.68	**63**
1.72	79
1.73	65
1.74	70
1.77	79
1.78	**81**
1.86	77
1.92	88

Table 1 gives the heights (m) and weights (kg) of a sample of nine people.

a. On the scatterplot below, the points representing the data for seven of these people has been plotted with *height* on the horizontal axis and *weight* on the vertical axis.

Plot points representing the data for the remaining two people (shown in bold in Table 1) on the scatterplot.



1 mark

b. Determine the equation of the least squares regression line that fits the data in Table 1. Use *height* as the independent variable and *weight* as the dependent variable. Complete the regression equation by writing the appropriate values in the boxes, correct to one decimal place.

$weight =$ ☐ $+$ ☐ $\times$ *height*

2 marks

c. The coefficient of determination for this data is 0.61

Complete the following sentence by filling in the missing information.

For this sample, 61% of the variation in the ☐ of the people can be explained by the variation in their ☐ .

1 mark

Question 2

Body Mass Index (*BMI*) is defined as

$$BMI = \frac{weight}{(height)^2}$$ where *weight* is measured in kilograms and *height* in metres.

a. Determine the *Body Mass Index* of a person who weighs 66 kg and who is 1.69 m tall. Write your answer correct to one decimal place.

1 mark

The *BMI* for each person in a sample of 17 males and 21 females is recorded in Table 2 below.

Table 2. *BMI* of males and females

Body Mass Index	
males	**females**
31.4	27.0
30.1	26.9
26.8	25.2
25.7	24.6
25.5	24.2
25.5	24.2
23.6	23.4
23.3	23.4
22.5	22.8
22.4	22.5
22.3	22.4
22.0	21.8
21.8	21.5
21.6	21.4
21.1	20.9
20.9	20.6
20.6	20.3
	20.1
	19.9
	18.8
	17.5

b. Write the range of the *BMI* data for **males** in the sample.

1 mark

c. A *BMI* greater than 25 is sometimes taken as an indication that a person is overweight.
Use this criterion and the data in Table 2 to complete Table 3, the two-way frequency table below.

Table 3. Weight rating by gender

Weight rating	**Gender**	
	male	**female**
overweight		
not overweight		
Total	17	21

2 marks

d. Does the data support the contention that, for this sample, weight rating is associated with gender? Justify your answer by quoting appropriate percentages.

2 marks

e. The parallel boxplots in Figure 1 have been constructed to compare the distribution of *BMI* for males and females in this sample.

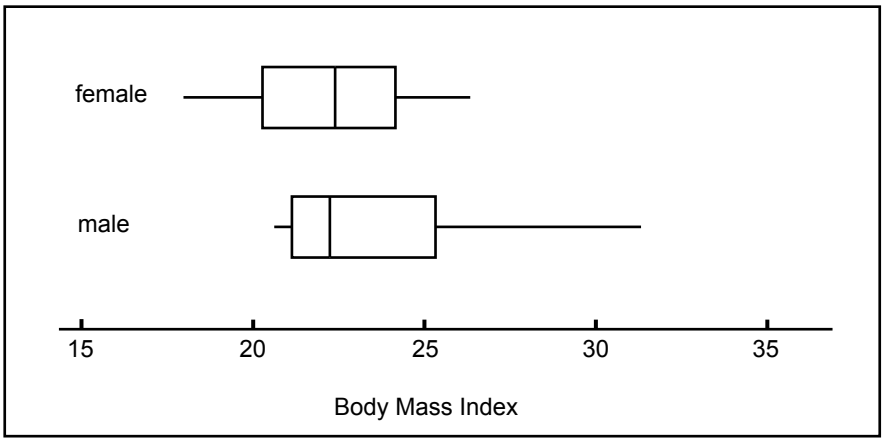


Figure 1. Parallel boxplots comparing *BMI* for males and females

i. Use the parallel boxplots to identify and name two **similar** properties of the *BMI* distributions for males and females.

ii. Use the information in Table 2 to determine the mean *BMI* for the males in this sample. Write your answer correct to one decimal place.

Mean *BMI* for males = ☐

iii. The median *BMI* for males is 22.5. Of the **mean** or **median**, which measure gives a better indication of the typical *BMI* for males? Explain your answer.

2 + 1 + 2 = 5 marks

Total 15 marks

END OF CORE

TURN OVER

Module 1: Number patterns and applications

Question 1

Australian Heating is a company that produces heating systems. The number of heating systems produced annually is modelled by an increasing geometric sequence. The number of heating systems produced in each of the first three years is shown in Table 1.

Table 1. Annual production of heating systems

Year	1	2	3
Number of heating systems produced	2000	2200	2420

a. Show that the common ratio, r, of this geometric sequence is 1.1

1 mark

b. What is the annual percentage increase in the number of heating systems produced each year?

%

1 mark

c. How many heating systems will be produced in year 5? Write your answer correct to the nearest whole number.

1 mark

d. The number of heating systems produced annually continues to follow this pattern. In total, how many heating systems will they produce in the first ten years of operation?

1 mark

e. The geometric sequence in Table 1 can also be generated by a difference equation of the form

$$P_{n+1} = bP_n + c$$

where $P_1 = 2000$ and P_n is the number of heating systems produced in the nth year.
Determine the values of b and c.

$b =$ $c =$

2 marks

Question 2

The purchase and installation of a basic heating system with **five** outlets costs $3500. Each additional outlet costs an extra $80.

a. Determine the cost of installing a heating system with **eight** outlets.

1 mark

b. A customer has $4400 to spend on a heating system and outlets. Determine the greatest number of outlets that can be bought with this heating system.

2 marks

c. Australian Heating recommends that a house with 20 squares of living area should have 12 heating outlets.

Using this recommended ratio, determine the **cost** of installing a heating system for a house having 35 squares of living area.

2 marks

Question 3

The number, S_n, of heating systems sold in the nth year is generated by the difference equation

$$S_n = 1.2\ S_{n-1} - 200 \text{ where } n \leq 5 \text{ and } S_3 = 2224$$

a. Use the difference equation to determine how many heating systems were sold in the first year.

2 marks

b. What percentage of heating systems produced during the first three years was sold within the three years? Write your answer correct to one decimal place.

2 marks

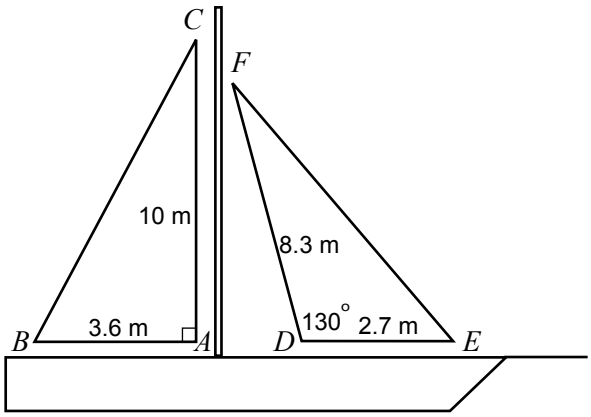
Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

A yacht has two flat triangular sails as shown in the diagram below.



The sail *ABC* is in the shape of a right-angled triangle. The height *AC* is 10 metres and the length *AB* is 3.6 metres.

a. Calculate angle *ABC*. Write your answer correct to the nearest degree.

1 mark

b. Calculate the length *BC*. Write your answer in metres, correct to one decimal place.

1 mark

The sail *DEF* has side lengths *DE* = 2.7 metres and *DF* = 8.3 metres. The angle *EDF* is 130°.

c. Calculate the length *EF*. Write your answer in metres, correct to one decimal place.

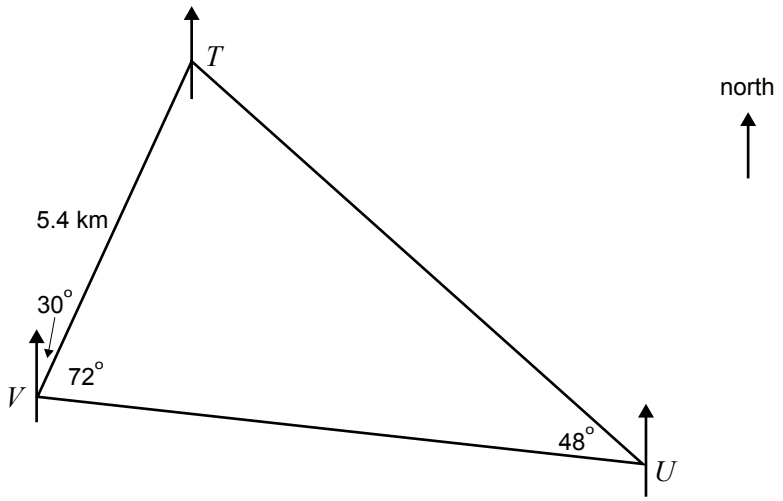
1 mark

d. Calculate the area of the sail *DEF*. Write your answer in square metres, correct to one decimal place.

1 mark

Question 2

A course for a yacht race is triangular in shape and is marked by three buoys T, U and V.



Starting from buoy V, the yachts sail 5.4 kilometres on a bearing of 030° to buoy T. They then sail to buoy U and back to buoy V. The angle TVU is 72° and the angle TUV is 48°.

a. Determine the bearing of V from U.

2 marks

b. Determine the distance TU. Write your answer in kilometres, correct to one decimal place.

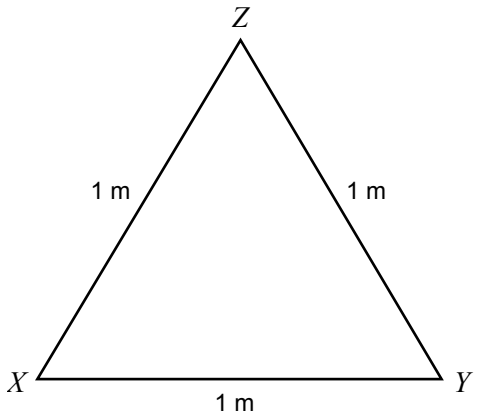
1 mark

c. Determine the shortest distance to complete the race. Write your answer in kilometres, correct to one decimal place.

2 marks

Question 3

A navigational marker *XYZ* is in the shape of an equilateral triangle with side length of one metre. It is located in the vicinity of the yacht race.

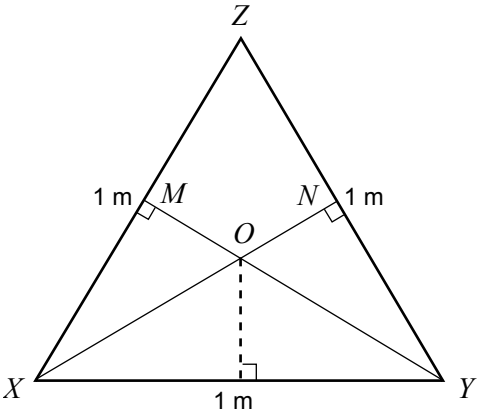


a. Write down the size of angle *XYZ*.

angle *XYZ* = ☐

1 mark

Point *O* is the centroid (centre) of the triangle. Points *M* and *N* are the midpoints of sides *XZ* and *YZ* respectively.

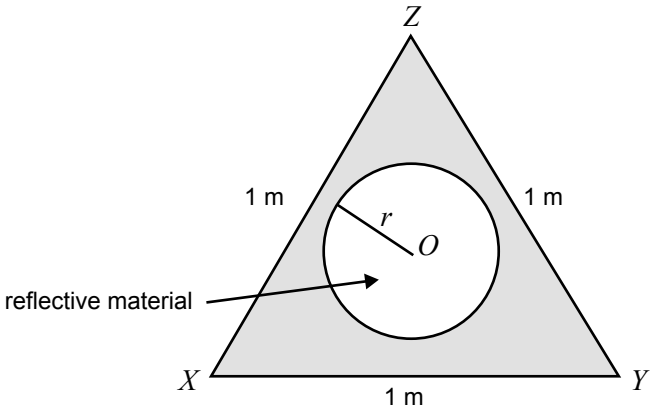


b. Calculate the shortest distance from point *O* to side *XY*. Write your answer in metres, correct to three decimal places.

2 marks

A piece of reflective material in the shape of a circle is attached to the centre of the navigational marker at the centroid O.

The ratio of the area of the shaded region of the navigational marker XYZ to the area of the reflective material is 2:1.



c. Determine the radius, r, of the circle. Write your answer in metres, correct to three decimal places.

3 marks

Total 15 marks

END OF MODULE 2

Working space

Module 3: Graphs and relations

Question 1

A clothing manufacturer finds that the cost, C dollars, of producing x shirts is given by the equation $C = 8x + 2400$.

a. Determine the cost of producing 400 shirts.

1 mark

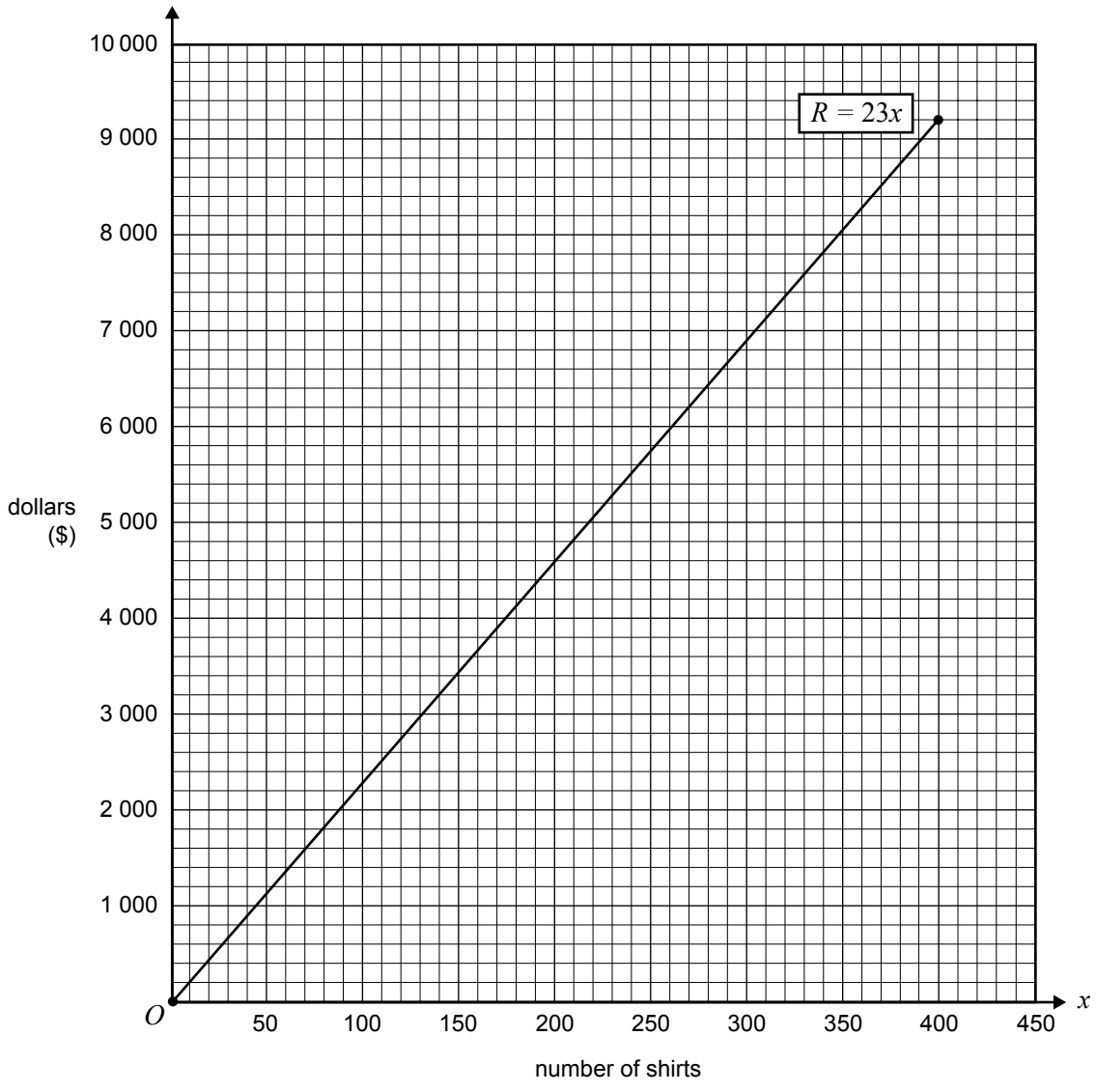
b. Determine the maximum number of shirts that can be produced for $3000.

1 mark

c. Assuming all the shirts are sold, the revenue, R dollars, from the sale of x shirts produced is given by an equation $R = 23x$.

A graph of the revenue equation $R = 23x$ for $0 \leq x \leq 400$ is drawn on the axes below.

On these same axes draw a graph of the cost equation $C = 8x + 2400$ for $0 \leq x \leq 400$.



2 marks

d. Determine the number of shirts that need to be produced and sold for the manufacturer to break even.

1 mark

e. Given the cost equation is $C = 8x + 2400$ and the revenue equation is $R = 23x$, write an equation for the profit, P dollars, from the production and sale of x shirts.

2 marks

f. Calculate the profit from the production and sale of 345 shirts.

1 mark

Question 2

The manufacturer also produces jackets. They receive an order for 250 jackets. The cost of producing the 250 jackets is $4800. Determine the selling price per jacket to achieve an overall profit of $3000.

2 marks

Question 3

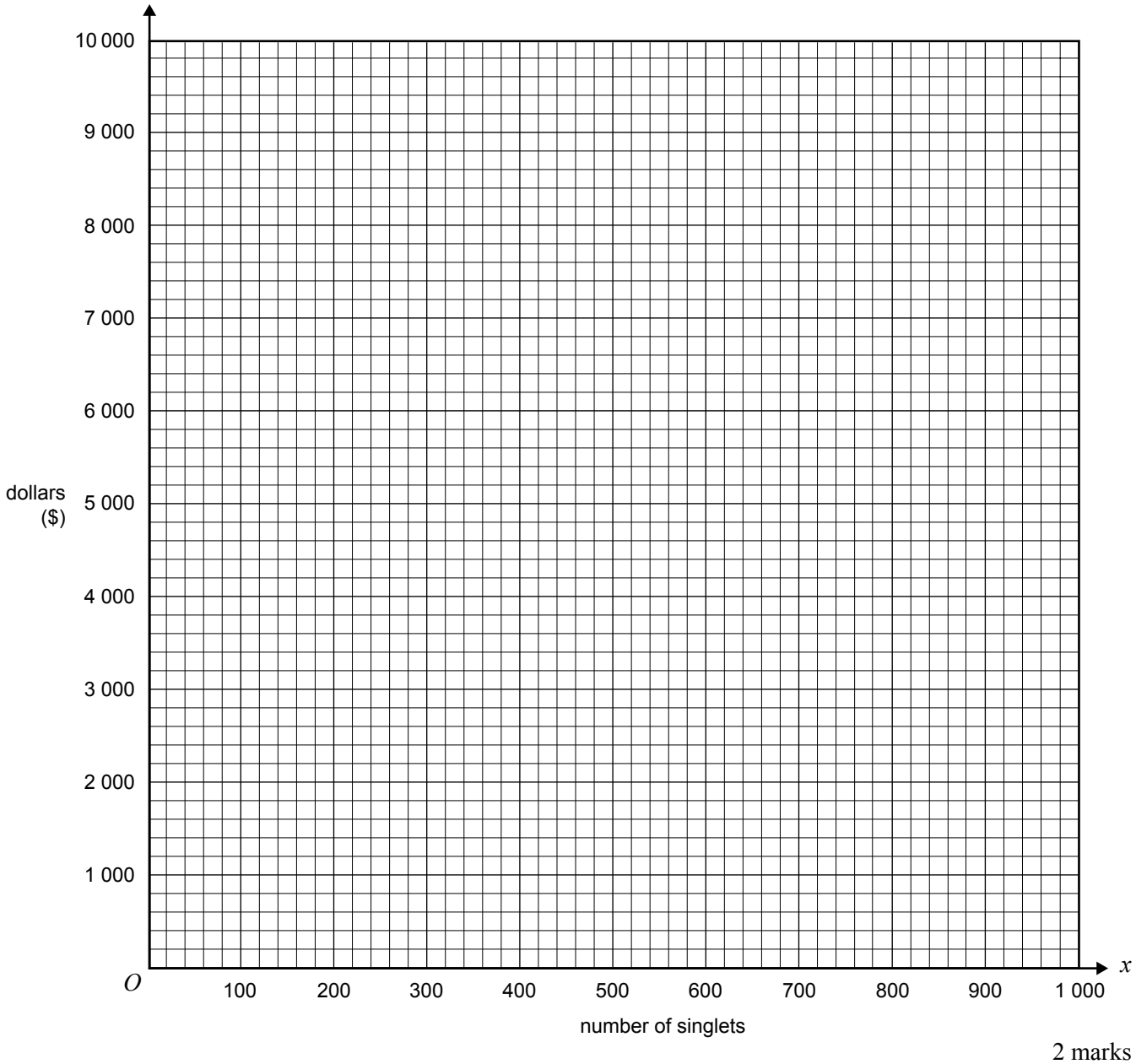
Singlets are produced and sold in larger quantities. The revenue, R_S dollars, generated from the sale of x singlets is given by the equation

$$R_S = \begin{cases} 10x & x \leq 500 \\ 6x + 2000 & x > 500 \end{cases}$$

a. Calculate the revenue, R_S, generated by the sale of 620 singlets.

1 mark

b. Sketch a graph of the revenue, R_S, for $0 \leq x \leq 1000$ on the axes below.



2 marks

c. If the cost, C_S dollars, of producing x singlets is $C_S = 4x + 1500$, determine the number of singlets that would need to be produced and sold to obtain a profit of $2000.

2 marks

Total 15 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1

Remy borrows $650 to buy a digital camera. He fully repays this loan with six monthly repayments of $120.

a. Determine

i. how much it costs Remy to pay off the loan.

ii. the total amount of interest Remy pays.

1 + 1 = 2 marks

b. For this loan

i. determine the annual simple interest rate. Write your answer correct to one decimal place.

ii. the effective interest rate is 37% per annum. Explain why the effective interest rate is greater than the simple interest rate.

1 + 1 = 2 marks

c. The $650 price of the camera includes 10% GST (Goods and Services Tax).

Determine the price of the camera before the GST was applied. Write your answer correct to the nearest cent.

1 mark

d. Remy uses his camera for work and he wants to depreciate its value over five years. He can either use a flat rate method of depreciation at the rate of 12% per annum or a reducing balance method of depreciation at the rate of 15% per annum.

Which method gives the greater total depreciation over five years? Explain your answer.

3 marks

Question 2

Anna borrows $12 000 at 7.5% interest, per annum, compounding monthly. The loan is to be fully repaid over four years by equal monthly repayments. The monthly repayments can be determined with the annuities formula

$$A = PR^n - \frac{Q\left(R^n - 1\right)}{R - 1}$$

a. What values of A, n and P should be substituted into the annuities formula to determine the monthly repayments?

$A =$ ☐ $n =$ ☐ $P =$ ☐

2 marks

b. Determine the monthly repayment for this loan. Write your answer correct to the nearest cent.

1 mark

c. Determine the total amount of interest paid on the loan after four years.

1 mark

d. After six equal repayments have been made, how much has Anna paid off the loan? Write your answer correct to the nearest dollar.

1 mark

e. At the end of six months the interest rate increases to 8.0% per annum. Anna still has to completely pay out the balance of the loan within the original period of the loan.

i. Determine the values of n and P for the remaining period of the loan.

$n =$ ☐ $P =$ ☐

ii. Determine the new monthly repayments that now apply. Write your answer correct to the nearest dollar.

1 + 1 = 2 marks

Total 15 marks

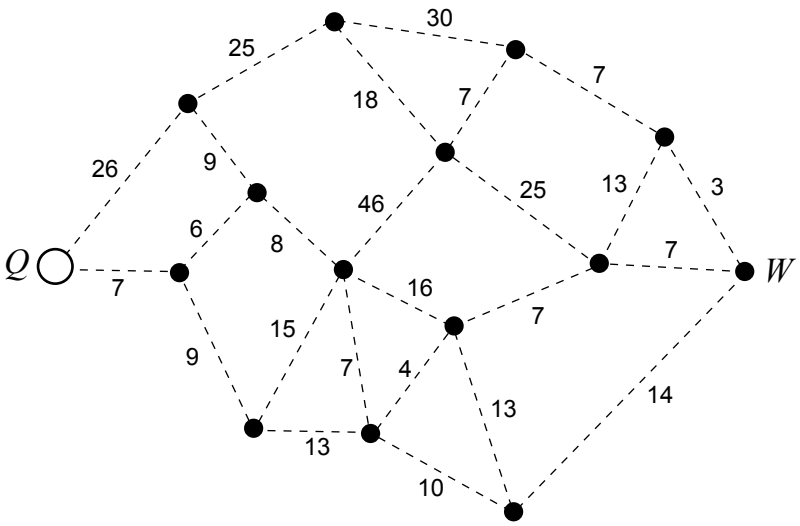
END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

The network diagram shows the distances, in kilometres, along a series of roads that connect a quarry, Q, with worksites shown as nodes.

a. One of these worksites is labelled as W.



i. On the diagram above, clearly draw in the shortest path from the quarry to W.

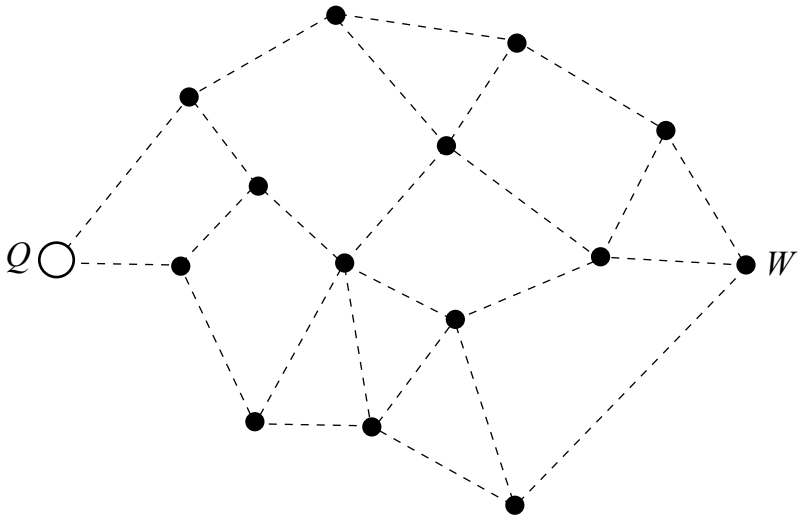
ii. Determine the length, in kilometres, of the shortest path between the quarry Q and the worksite W.

1 + 1 = 2 marks

b. The engineer at the quarry wants to visit all worksites in the network. Beginning at Q, he wants to pass through each worksite only once before returning to the quarry.

i. What mathematical term describes the route the engineer wants to take?

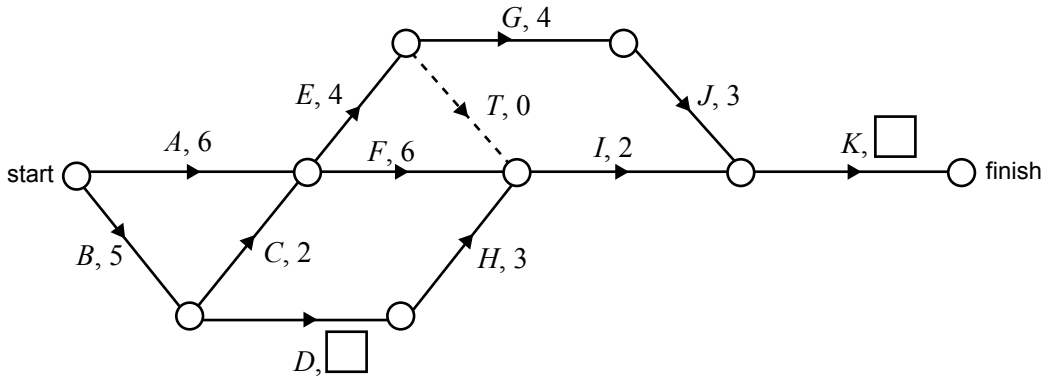
ii. On the diagram below, clearly draw in a complete route that the engineer could take to visit each worksite only once before returning to the quarry.



1 + 2 = 3 marks

Question 2

All the activities and their durations (in hours) in a project at the quarry are shown in the network diagram below. The **least time** required for completing this entire project is 30 hours.



For each activity in this project, Table 1 shows the Completion time, the Earliest starting time and the Latest starting time.

a. Complete the missing times in Table 1.

Table 1. Activity table

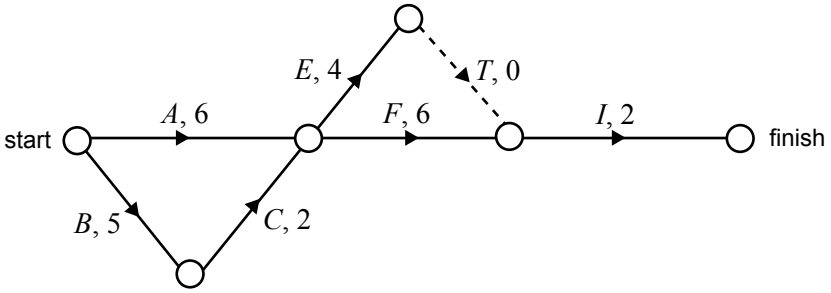
Activity	Completion time (hours)	Earliest starting time (hours)	Latest starting time (hours)
A	6	0	
B	5	0	0
C	2	5	5
D		5	9
E	4	7	7
F	6	7	
G	4	11	11
H	3	9	13
I	2	13	16
J	3	15	15
K		18	18

4 marks

b. Write down the critical path for this project.

1 mark

To speed up the project, several activities can be dropped from the project. The diagram below shows the activities that must remain in this modified version of the project and their usual completion times.



c. Determine the shortest time in which this modified project can be completed.

1 mark

The completion time of some of the remaining activities in the **modified project** can be reduced at a cost.

Table 2 shows the reduced times (least possible time to complete an activity after maximum reduction of time). The cost of this reduction, per hour, is also shown.

Table 2. Reduced times and costs

Activity	Usual completion time (hours)	Reduced time (hours)	Cost of reduction per hour ($)
A	6	3	50
B	5	4	100
C	2	2	-
E	4	2	20
F	6	4	50
I	2	2	-

d. For this modified project, determine

i. the activities that should be reduced in time to minimise the completion time of the project.

ii. the maximum time, in hours, that can be saved by this reduction.

iii. the minimum cost to achieve this time saving.

2 + 1 + 1 = 4 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, *where* $R = 1 + \frac{r}{100}$

Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Graphs and relations

Straight line graphs

gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation:

$y - y_1 = m(x - x_1)$	gradient-point form
$y = mx + c$	gradient-intercept form
$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$	two-point form

Number patterns and applications

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, |r| < 1$

linear difference equations: $t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1} - 1)}{a - 1}, a \neq 1$

$= a^n t_0 + b\frac{(a^n - 1)}{a - 1}$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

END OF FORMULA SHEET