VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

FURTHER MATHEMATICS

Written examination 2 (Analysis task)

Wednesday 5 November 2003

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
5	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 25 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

This page is blank

Instructions

This task paper consists of a core and five modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions.

Core

Question 1

Table 1 shows the number of telephone calls (both internal calls and external calls) made on a given day by a sample of 12 people working in a large company. Also given is the cost of each person's calls for the day.

Table 1

Person	Number of calls	Cost (dollars)
A	33	4.54
B	15	1.00
C	22	5.96
D	27	4.47
E	52	8.87
F	34	8.50
G	55	11.09
H	47	8.51
I	11	3.98
J	18	2.42
K	36	11.30
L	27	7.48

a. Determine the mean and standard deviation of the **cost** of the calls. Write your answers in dollars, correct to two decimal places.

mean = $ ☐ standard deviation = $ ☐

2 marks

b. For all workers in this company, the mean **number** of telephone calls made per day is 21.8.

For the sample of 12 workers in Table 1, determine the percentage of these workers who made more calls than the company mean of 21.8 calls.

percentage = ☐ %

1 mark

c. Use the data in Table 1 to

i. determine the equation of the least squares regression line that will enable call costs per person to be predicted from the number of calls they make. Write the missing coefficient correct to two decimal places in the space provided.

cost = 0.66 + ☐ × number of calls

ii. determine the value of Pearson's product moment correlation coefficient. Write your answer correct to four decimal places.

$r =$ ☐

1 + 1 = 2 marks

Core – Question 1 – continued

d. The value of Pearson's product moment correlation coefficient (calculated in **part c. ii.**) measures the strength and direction of the [ ] relationship between call cost and number of calls.

1 mark

e. The scatterplot shown in Figure 1 was constructed from the data displayed in Table 1. The point corresponding to person *K* has not been included. Complete the scatterplot by adding in the data point for person *K*, marking the point with a cross (×).

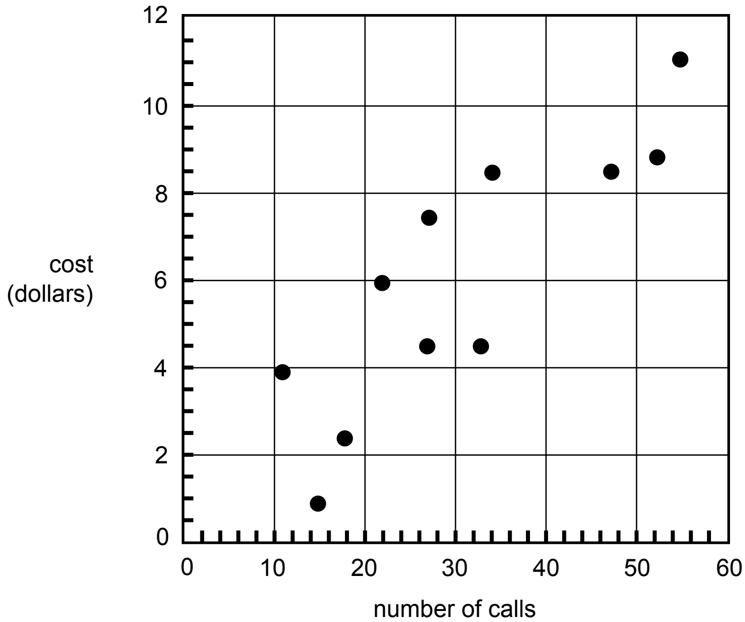


Figure 1

1 mark

f. We wish to predict the cost of calls from the number of calls made.

The dependent variable is [ ] .

1 mark

g. Complete the following sentences by filling in the boxes.

i. On average, for each extra call made, a worker's call costs increase by [ ] cents per call.

ii. To the nearest whole per cent, [ ] % of the variation in call costs can be explained by the variation in the number of calls.

1 + 1 = 2 marks

h. If we use the least squares regression line to estimate the call costs of a person who makes 34 calls, this would leave a residual value of [ ] dollars.

1 mark

Question 2

A supervisor believes that there is an increasing trend in the total monthly telephone call costs in the company. She gathers data using the previous year's figures as shown in Table 2.

Table 2

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Cost ($)	180	201	221	198	180	233	205	199	185	197	291	206

To check this, the supervisor constructs the time series plot shown in Figure 2 using the previous year's data.

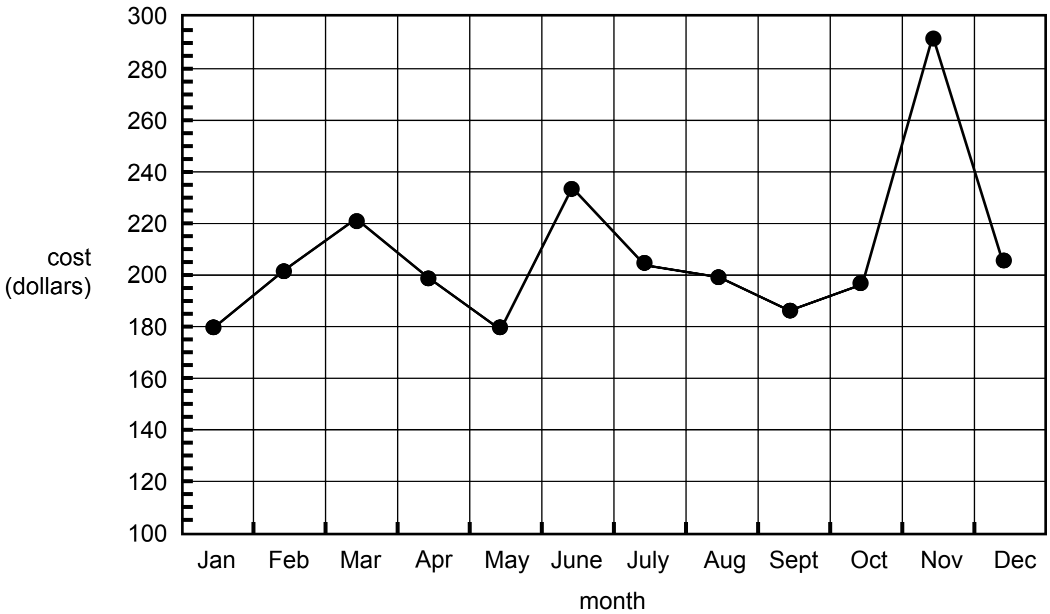


Figure 2

a. Use three-median smoothing to smooth the time series shown in Figure 2. Plot the smoothed time series on the same graph.

2 marks

b. Does the smoothed time series support the supervisor's belief that there has been an increasing trend in call costs in the company during the previous year? Briefly explain your answer.

1 mark

c. Name a feature (or features) of this particular time series which suggests that it is more appropriate to use **three-median** smoothing rather than **three-mean** moving average smoothing.

1 mark

Total 15 marks

END OF CORE

Module 1: Number patterns and applications

Question 1

On the O'Callaghans' farm there is only 10 000 litres of water left in their tank. They need to buy more water which is delivered by a water tanker. The water is pumped from the water tanker into the tank at a constant rate of 800 litres per minute.

a. How many litres of water are in the tank one minute after pumping starts?

[ ] litres

1 mark

b. How many litres of water are in the tank five minutes after pumping starts?

1 mark

c. If t_n litres denotes the amount of water in the tank n minutes after pumping starts, we can write

$$t_n = a + bn.$$

Determine the values of a and b.

$a =$ [ ] $b =$ [ ]

2 marks

d. The water tanker has a capacity of 20 000 litres. How long does it take to empty a full tanker at a rate of 800 litres per minute?

1 mark

Question 2

With a full tank, the O'Callaghans tend to use more water. As the level of water falls, they become more careful with their water usage. Each week they use 5% of the water that was in the tank at the start of that week. There are 30 000 litres of water in the tank at the **start** of the first week.

a. How many litres of water are in the tank at the **end** of the first week?

1 mark

b. How many litres of water are in the tank at the **end** of the fourth week? Write your answer correct to the nearest litre.

1 mark

c. At the **end** of which week would the amount of water in the tank **first** be less than 10 000 litres?

2 marks

Question 3

The O'Callaghans use river water to water their crops. As the weather becomes warmer they will use more river water. The amount of river water they use weekly increases in a geometric sequence.

In the first week of summer they use 6000 litres of river water. Thereafter their water usage increases by 10% of the previous week's usage.

a. What is the common ratio, r, for this sequence of weekly river water usage?

$r = \boxed{}$

1 mark

b. The O'Callaghans follow this pattern of usage for five weeks. How many litres of river water, in total, will they have used in these five weeks? Write your answer correct to the nearest litre.

2 marks

The Gerbers live on a neighbouring property. If G_n denotes the amount of river water used by the Gerbers in the nth week of summer, then the sequence $G_1, G_2, \ldots$ follows the difference equation

$$G_{n+1} = 0.98\, G_n + 100, \qquad \text{where } G_1 = 8000$$

c. How many litres of river water will the Gerbers use in the third week of summer? Write your answer correct to the nearest litre.

1 mark

d. In which week of summer will the O'Callaghans first use more river water for that week than the Gerbers?

2 marks

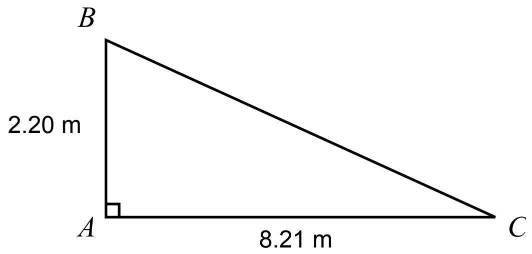
Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

Jane is landscaping her garden. A piece of shade cloth *ABC* has the dimensions as shown below.



a. Determine the length *BC* in metres. Write your answer correct to two decimal places.

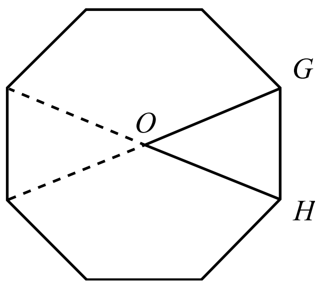
1 mark

b. Determine the angle *ACB*. Write your answer correct to the nearest degree.

1 mark

Question 2

A paved area is constructed in the shape of a regular octagon as shown below.



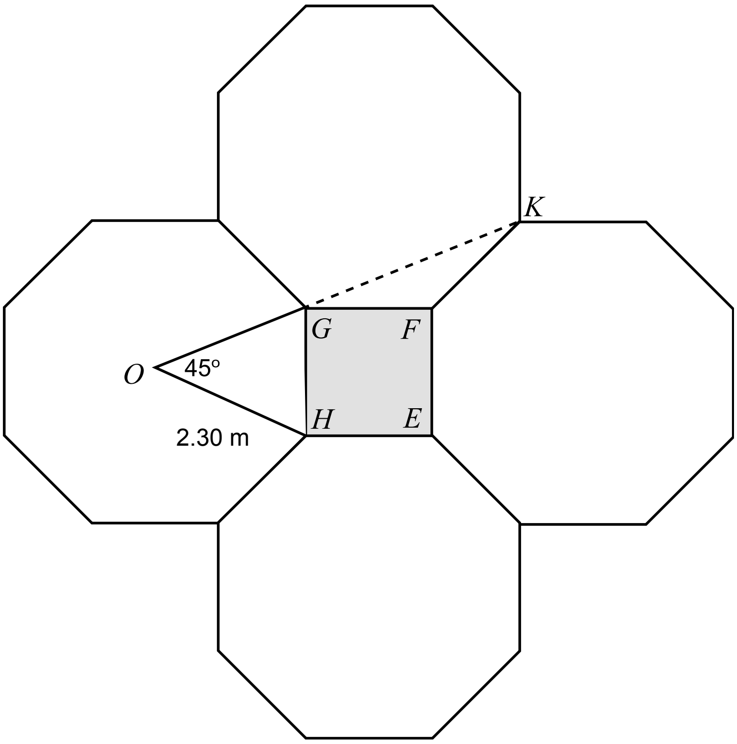
a. By calculation, show that the size of the angle *GOH* is 45º, where point *O* is the centre of the octagon.

1 mark

b. The length $OG = OH = 2.30$ metres. Calculate the area of the octagonal paved area. Write your answer correct to the nearest square metre.

2 marks

A square herb garden *EFGH* is surrounded by four regular octagonal paved areas as shown in the diagram below.



c. Calculate the side length *GH* of the square herb garden. Write your answer in metres, correct to two decimal places.

2 marks

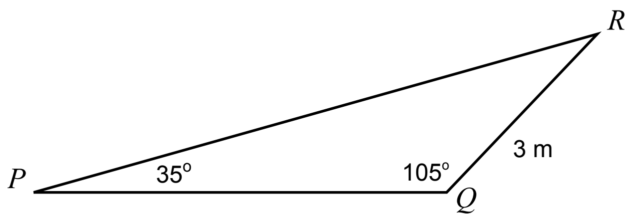
d. A straight wooden frame is to be built between points *O* and *K* for hanging baskets.

i. Calculate the length *GK*. Write your answer in metres, correct to two decimal places.

ii. Hence calculate the length *OK*. Write your answer in metres, correct to two decimal places.

2 + 1 = 3 marks

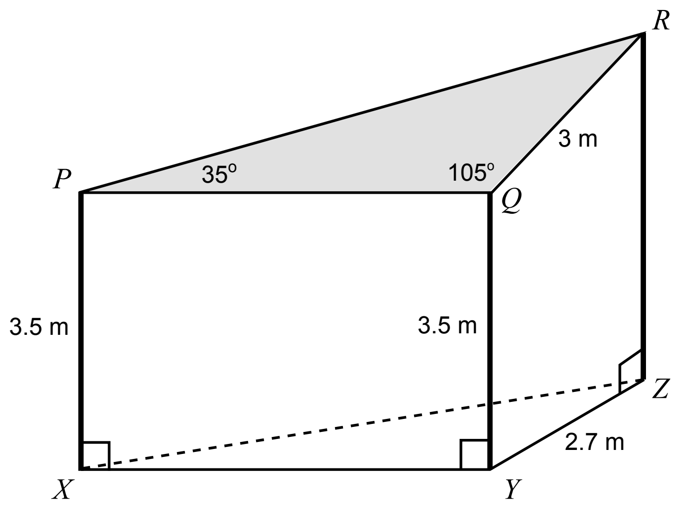
A second piece of shade cloth *PQR* is also triangular and has dimensions as shown in the diagram below.



e. Calculate the length *PR*. Write your answer in metres, correct to two decimal places.

1 mark

The second piece of shade cloth *PQR* is attached to three vertical poles located at *X*, *Y* and *Z* as shown in the diagram below. Poles *PX* and *QY* are each 3.5 metres long. The horizontal distance *YZ* is 2.7 metres.



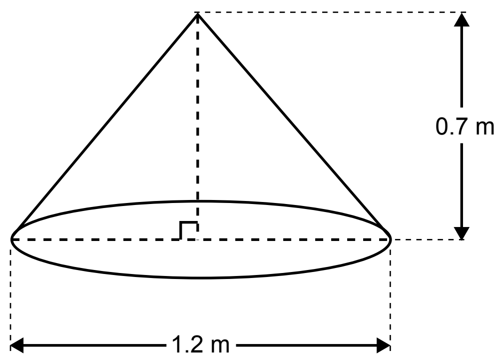
f. Calculate the length of the vertical pole *RZ*. Write your answer correct to the nearest **centimetre**.

2 marks

Question 3

Jane has soil delivered for her garden. There are two piles of soil, both in the shape of a right cone.

a. The first pile of soil has a base diameter of 1.2 metres and a height of 0.7 metres as shown in the diagram below.



Calculate the volume of soil in the first pile. Write your answer in cubic metres, correct to two decimal places.

1 mark

b. The second pile of soil has a base diameter of 2.4 metres and a height of 1.4 metres. What is the ratio of the volume of the first pile to the volume of the second pile?

1 mark

Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

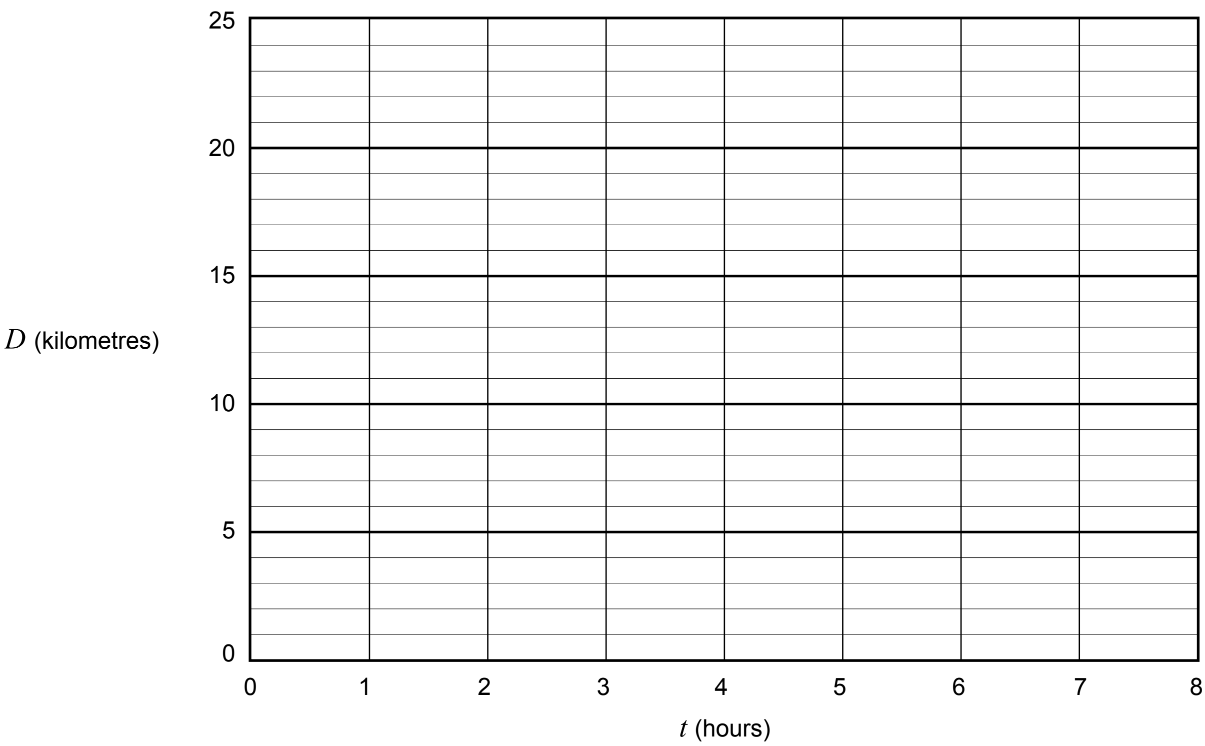
Question 1

Two bushwalkers, Malinda and Christos, set out to walk from Fishbone Creek to Snake Gully, a distance of 20 km. They start at the same time and follow the same route.

a. Malinda walks at a constant speed of 4 km/h for the entire journey and takes no rest periods. How far does she travel in 1.5 hours?

1 mark

b. The distance walked by Malinda from Fishbone Creek, in kilometres, is given by the equation $D_m = 4t$ for $0 \le t \le 5$, where t is the time in hours since she began walking. Draw and label the graph of D_m against t on the set of axes below.



2 marks

Christos started walking at the same time as Malinda and followed the same route. At the start he walked at a constant speed of 6 km/h. However, after walking at this speed for two hours he developed sore feet. Rather than stopping, he slowed down to a constant speed of 2 km/h for the remainder of the trip.

c. Let D_c represent the distance walked by Christos. Draw and label the graph of D_c against t on the set of axes in part **b**.

2 marks

d. Malinda eventually catches up to Christos. How many hours after they start walking does this happen?

1 mark

e. The equations below give the distance, D_c in kilometres, walked by Christos at any time t hours.

$$D_c = \begin{cases} at & 0 \le t \le 2 \quad \text{hours} \\ bt + h & 2 < t \le d \quad \text{hours} \end{cases}$$

Determine the values for a, b, h and d.

$a =$ $\boxed{}$

$b =$ $\boxed{}$

$h =$ $\boxed{}$

$d =$ $\boxed{}$

3 marks

CONTINUED OVER PAGE

Question 2

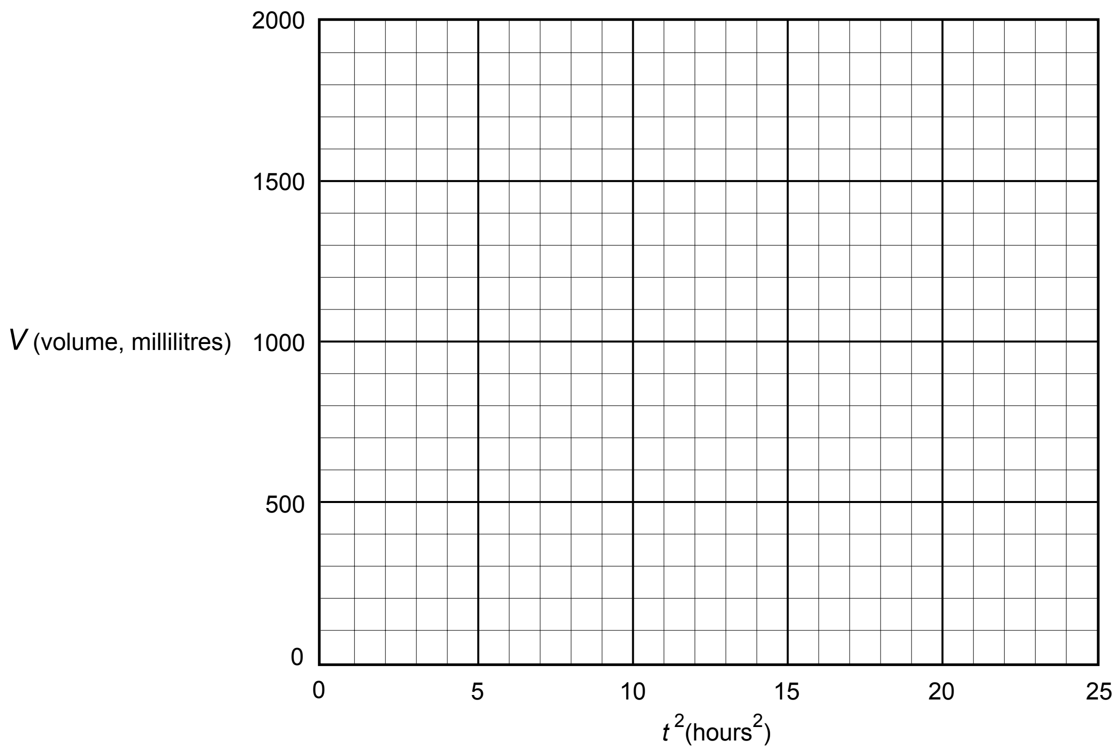
Malinda began the walk with 2000 millilitres of water in her bottle. It was a hot day and she sipped small amounts of water from her bottle frequently to ensure that she would not dehydrate.

The table below shows the total volume, V, of water, in millilitres, that Malinda had drunk after t hours.

t (hours)	0	2	4	5
t^2 (hours^2)	0			
V (millilitres)	0	300	1200	1875

a. **i.** Complete the table above.

ii. On the set of axes below, plot the four points (t^2, V) from the table you have completed above.



1 + 1 = 2 marks

b. Assume that this data is modelled by the relation $V = kt^2$. Use the graph or table to determine the value of k.

1 mark

c. Use your answer from **part b.** to determine the volume of water Malinda drank during the first three hours. Write your answer correct to the nearest millilitre.

1 mark

d. At some point during the walk, Malinda had drunk half the water in her bottle.

i. How long after the start of the walk did this happen? Write your answer in hours, correct to two decimal places.

ii. Determine how far apart Malinda and Christos were at this time. Write your answer in kilometres, correct to one decimal place.

1 + 1 = 2 marks

Total 15 marks

Module 4: Business-related mathematics

Question 1

Brad wants to buy a coffee machine for his café. Crazy Bill's normally sells them for $3450, but they have a special discounted price of $3100 for this week.

a. What is the percentage discount? Write your answer correct to one decimal place.

%

1 mark

b. Crazy Bill's offers to sell the machine for the discount price of $3100. The terms of the sale are $200 deposit and $275 per month for 12 months.

i. What is the total cost of the machine on these terms?

ii. What is the annual flat rate of interest charged? Write your answer correct to one decimal place.

1 + 2 = 3 marks

c. Brad sees the same coffee machine for sale at Discount King, also for $3100. The terms of the sale there require no deposit and monthly repayments over two years at an interest rate of 9% per annum, calculated monthly on the reducing balance.

The monthly repayments can be determined using the annuities formula: $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$.
The loan is paid out in two years.

i. What values for n, P and A should be substituted into the annuities formula to determine the monthly repayments?

$n =$ ☐ $P =$ ☐ $A =$ ☐

ii. What is the monthly repayment for this loan? Write your answer in dollars, correct to two decimal places.

iii. What is the total cost of the machine from Discount King on these terms? Write your answer correct to the nearest dollar.

2 + 1 + 1 = 4 marks

d. Whose terms, Crazy Bill's or Discount King's, offer the lowest total cost for the coffee machine? Justify your answer by calculating the difference in total money paid.

1 mark

Question 2

Brad buys the coffee machine with an initial value of \$3100. He considers two methods of depreciating the value of the coffee machine.

a. Suppose the value of the machine is depreciated using the reducing balance method over three years and reducing at a rate of 15% per annum.

What is the depreciated value of the machine after three years? Write your answer correct to the nearest dollar.

2 marks

b. Alternatively, suppose that the machine is depreciated using the unit cost depreciation method.

Brad sells 15 000 cups of coffee per year and the unit cost per cup is 3.0 cents.

Determine the depreciated value of the machine after three years. Write your answer correct to the nearest dollar.

2 marks

c. Brad wants the depreciated value of the machine after three years to be the same when calculated by both methods of depreciation. What would the unit cost per cup have to be for this to occur? Write your answer in cents, correct to one decimal place.

2 marks

Total 15 marks

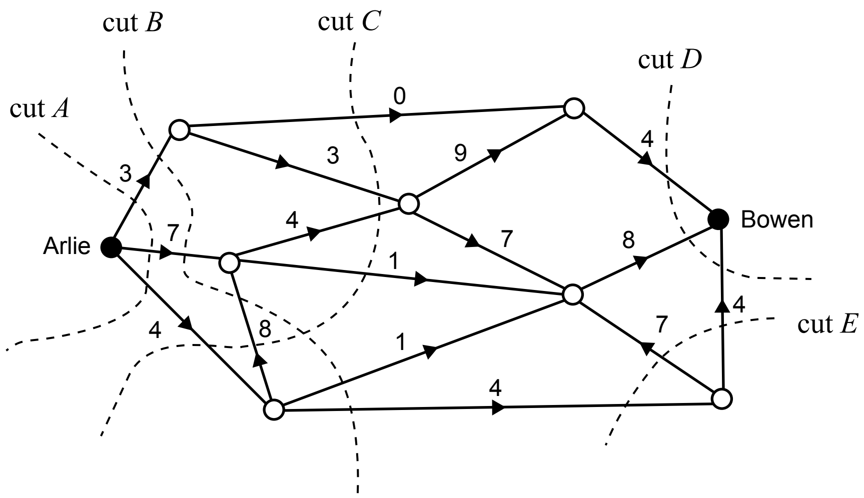
END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

A train journey consists of a connected sequence of stages formed by edges on the following directed network from Arlie to Bowen.

The number of available seats for each stage is indicated beside the corresponding edge, as shown on the diagram below.



The five cuts, A, B, C, D and E, shown on the network, are attempts to find the maximum number of available seats that can be booked for a journey from Arlie to Bowen.

a. Write down the capacity of cut A, cut B and cut C.

cut A = ☐ cut B = ☐ cut C = ☐

3 marks

b. Cut E is not a valid cut when trying to find the minimum cut between Arlie and Bowen. Why?

1 mark

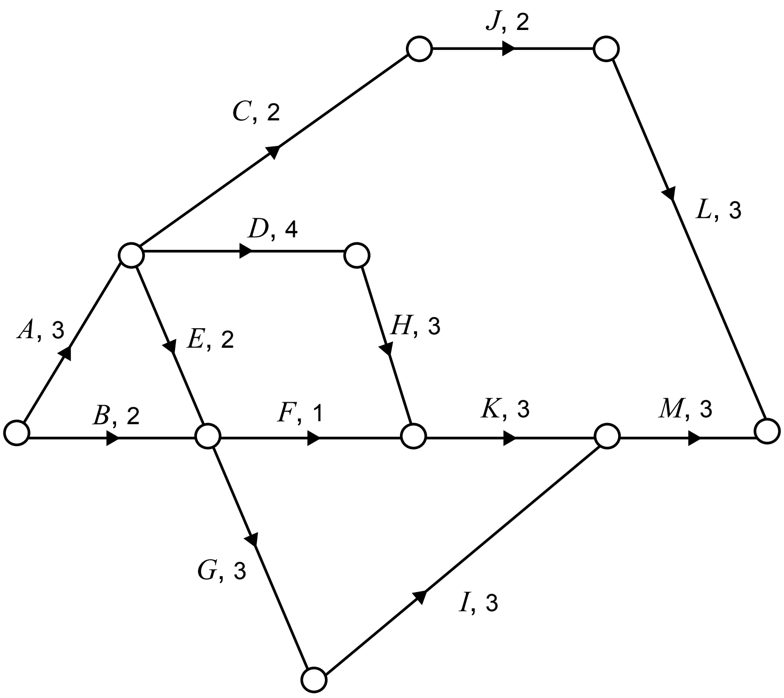
c. Determine the maximum number of available seats for a train journey from Arlie to Bowen.

1 mark

Question 2

The Bowen Yard Buster team specialises in backyard improvement projects. The team has identified the activities required for a backyard improvement. The network diagram below shows the activities identified and the **actual** times, in hours, needed to complete each activity, that is, the duration of each activity.

The table below lists the activities, their immediate predecessor(s) and the **earliest starting times** (EST), in hours, of each of the activities. Activity ***X*** is not yet drawn on the network diagram.



	Immediate predecessor(s)	EST
A	–	0
B	–	0
C	*A*	3
D	*A*	3
E		3
F	*B, E*	5
G	*B, E*	5
H	*D*	7
I	*G*	
J	*C*, ***X***	8
K	*F, H*	10
L	*J*	10
M	*I, K*	
X	*D*	7

a. Use the information in the network diagram to complete the table above by filling in the shaded cells.

3 marks

b. Draw and label activity ***X*** on the network diagram above, including its direction and duration.

1 mark

c. The path *A–D–H–K–M* is the only critical path in this project.

i. Write down the duration of path *A–D–H–K–M*.

duration = ☐ hours

ii. Explain the importance of the critical path in completing the project.

____

____

1 + 1 = 2 marks

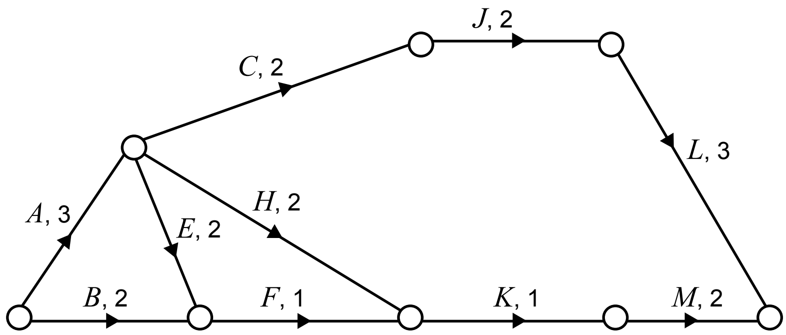
d. When the weather is poor, activity *B* takes more than two hours to complete. What is the maximum time that could be allowed to complete activity *B* without delaying the completion of the entire project?

maximum time = ☐ hours

1 mark

Question 3

To save money, Bowen Yard Busters decide to revise the project and leave out activities *D*, *G*, *I* and ***X***. This results in a reduction in the time needed to complete activities *H*, *K* and *M* as shown below.



a. For this revised project network, what is the **earliest starting time** for activity *K*?

earliest starting time = ☐ hours

1 mark

b. Write down the critical path for this revised project network.

critical path = ☐

1 mark

c. Without affecting the earliest completion time for this entire **revised** project, what is the **latest starting time** for activity *M*?

latest starting time = ☐ hours

1 mark

Total 15 marks

END OF QUESTION AND ANSWER BOOK