VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2002

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

FURTHER MATHEMATICS
Written examination 2
(Analysis task)

Wednesday 6 November 2002

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
1	1	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
5	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 22 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

This task paper consists of a core and five modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions.

Core

Over recent years, the salaries of Patagonian cricketers have increased rapidly. The following data gives the average salaries in dollars of a large group of these cricketers over the period 1991–2000.

year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
average salary	45 000	47 000	50 000	58 000	70 000	78 000	93 000	105 000	126 000	142 000

a. This data will be used to predict future average salaries of Patagonian cricketers.

In this analysis, the **independent** variable is

1 mark

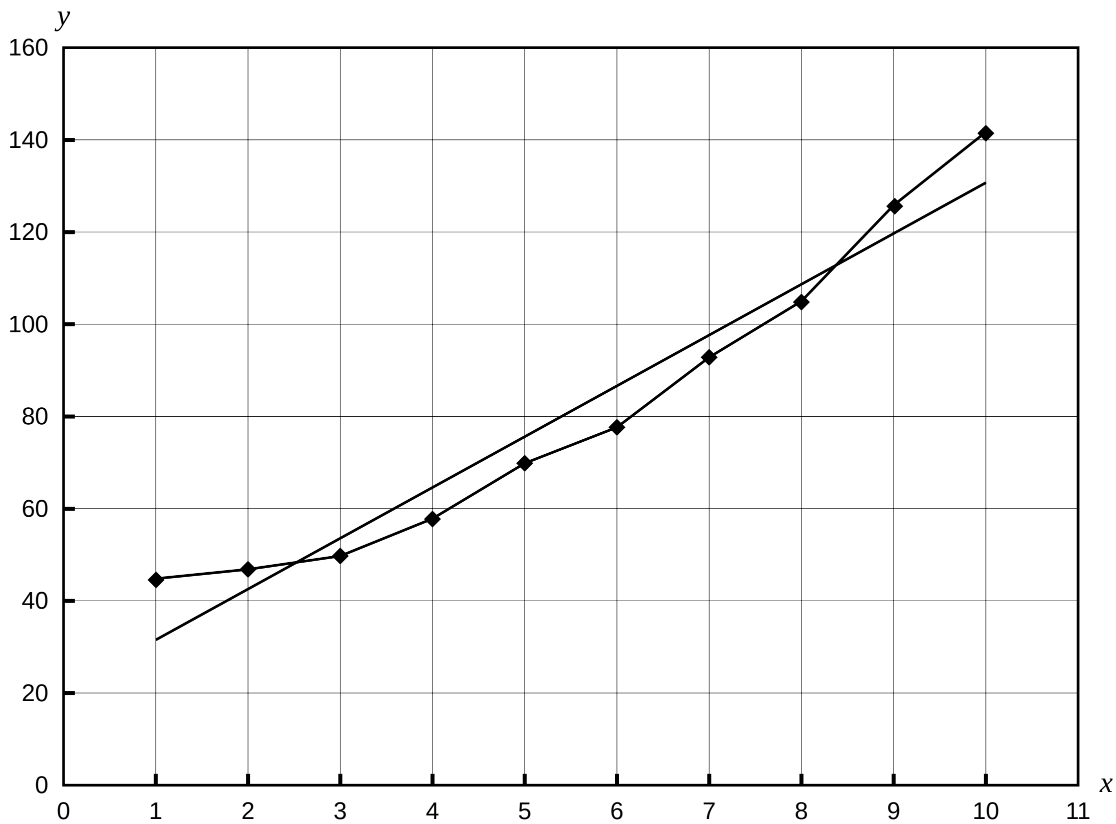
To begin the analysis, the years have been rescaled as $x = 1$ to $x = 10$ (1991 = 1, 1992 = 2, and so on), and average salary rescaled in thousands of dollars as the variable y.

year	1	2	3	4	5	6	7	8	9	10
average salary ($000s)	45	47	50	58	70	78	93	105	126	142

This rescaled data is displayed below as a time series plot. Also displayed is the least squares regression line which has been determined for this rescaled data.

The equation of the least squares regression line is

$$y = 20.9 + 10.99x$$



Core – continued

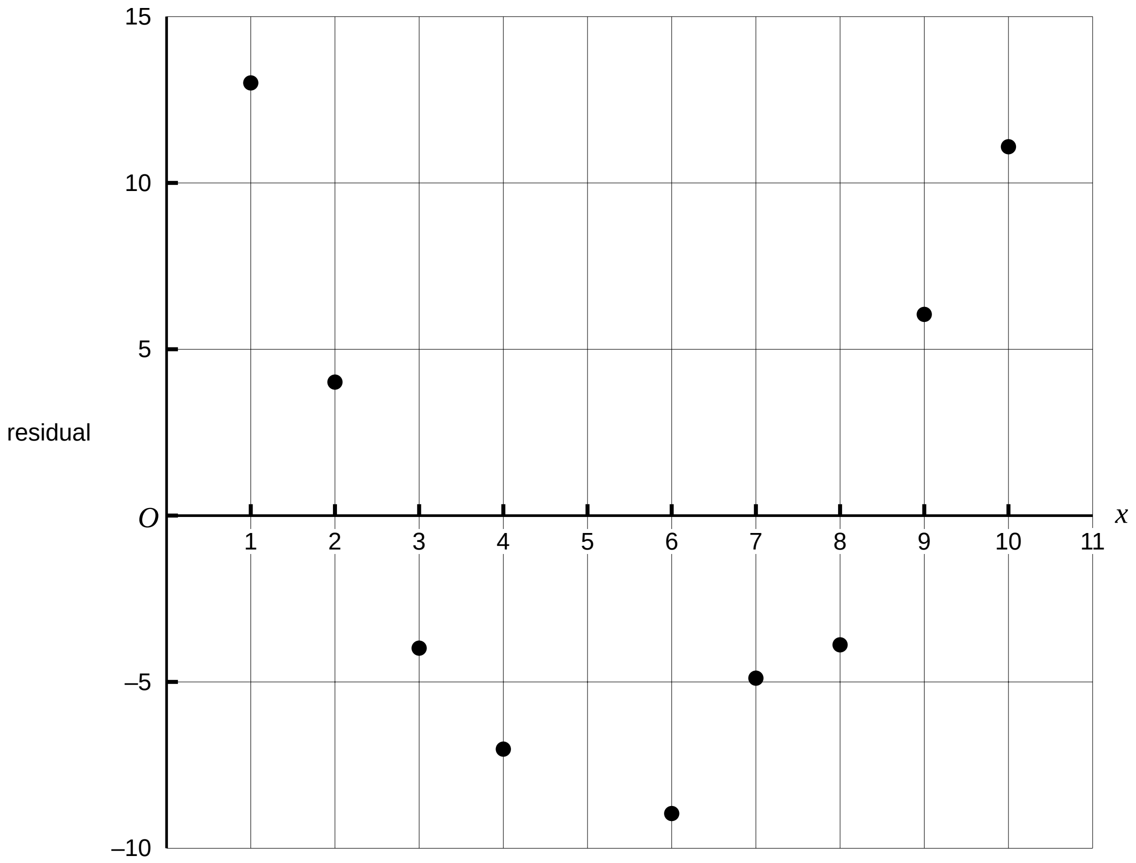
b. Using the regression equation $y = 20.9 + 10.99x$ to model the increase in these cricketers' average salaries, we would

i. say that the average salary increase for the cricketers was \$ ☐ per year.

ii. predict that their average salary in the year 2005 will be about \$ ☐ .

4 marks

From the time series plot, the increase in Patagonian cricketers' salaries over time appears nonlinear. This can be confirmed by constructing the corresponding residual plot as shown below.



c. Complete the plot by

i. calculating the value of the residual for 1995.

value = ☐

ii. plotting this residual as a point on the graph.

2 marks

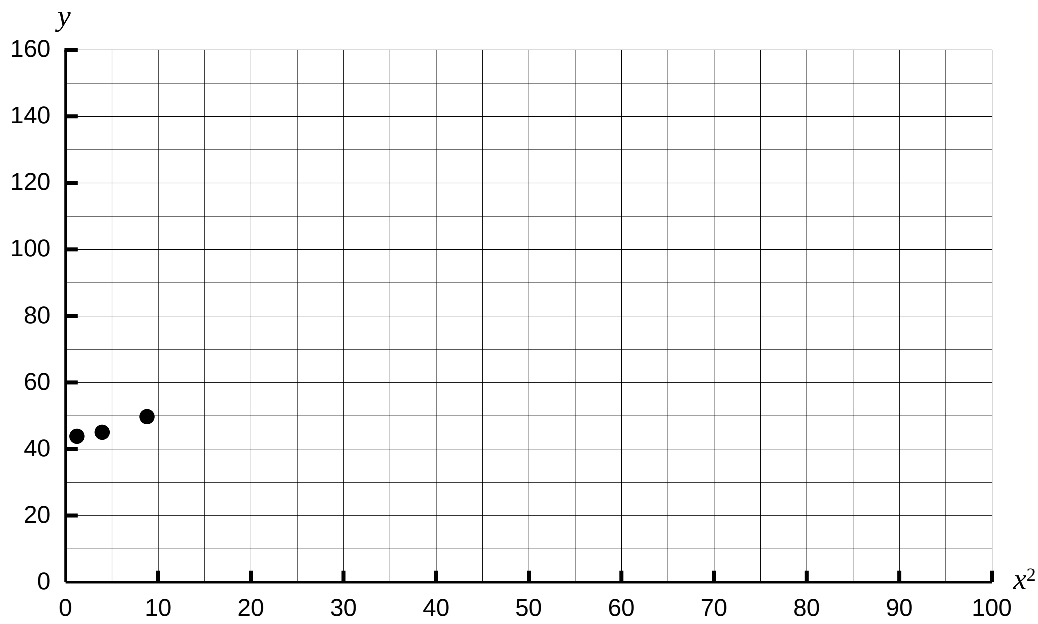
The time series, along with the residual plot, shows that the growth of salaries with time is nonlinear. Inspecting the time series, it would appear that it would be appropriate to use an x^2-transformation to transform the data to linearity.

The original data has been reproduced in the table below and an extra row has been added for the transformed variable, x^2.

year (x)	1	2	3	4	5	6	7	8	9	10
year^2 (x^2)										
average salary (y) (\$000s)	45	47	50	58	70	78	93	105	126	142

d. **i.** Complete the table.

ii. Complete the time series plot below of the transformed data to show that the x^2-transformation has produced a more nearly linear plot. Use the grid below. (Note the first three points have been plotted.)

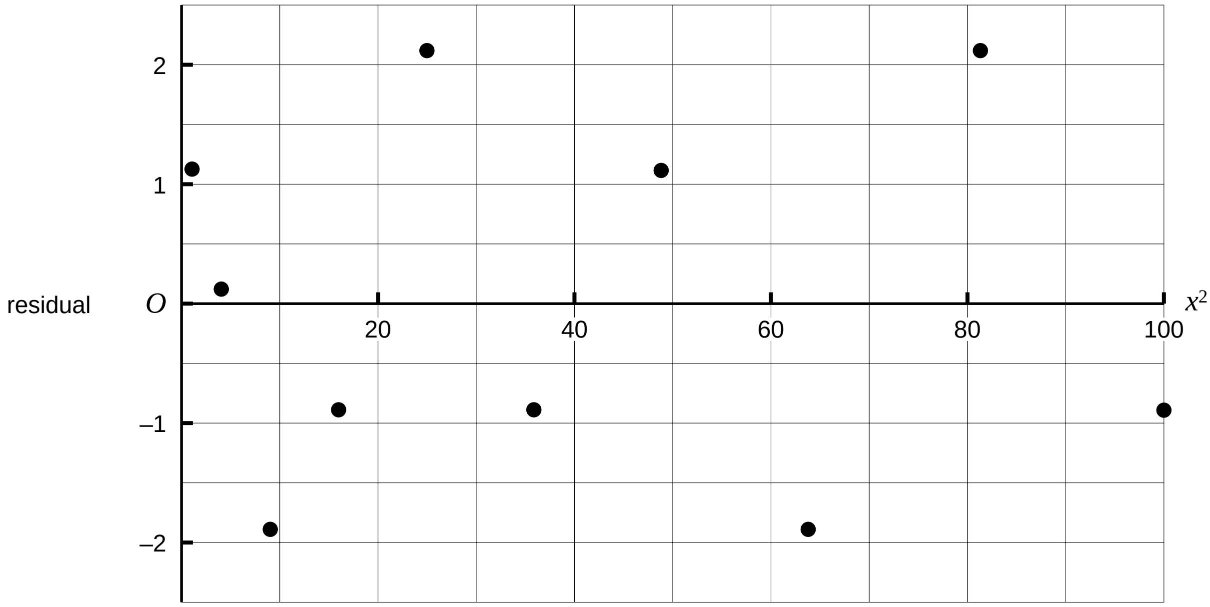


iii. Find the equation of the least squares regression line for the transformed data. Write the coefficients, correct to two decimal places, in the spaces provided.

y = [ ] + [ ] x^2

iv. Use this regression equation to predict the average salary of this group of Patagonian cricketers in 2005.

The residual plot that results from fitting a least squares regression line to this transformed data is shown below.



v. This plot suggests that the x^2-transformation has been successful in linearising the time series plot. What feature of this residual plot supports this conclusion?

1 + 2 + 2 + 2 + 1 = 8 marks

Total 15 marks

END OF CORE
TURN OVER

Module 1: Number patterns and applications

In the island world of Mysteria there is a race of powerful wizards. In the island's Wiz Academy young boys and girls study *Spell-ing* where they learn spells, *Potions* where they learn secret magic mixtures and *Mathemagic* where the mystery of numbers is revealed.

In the subject *Spell-ing*, each student must learn two spells in their first week and each week afterwards the number of **new** spells they must learn increases by three.

Question 1

a. How many new spells must a student learn in their third week?

1 mark

b. How many spells will a student have learnt altogether after 10 weeks if the number of spells they learn each week continues according to this rule?

1 mark

c. If they continue learning spells according to this rule, in which week will they learn 50 new spells?

1 mark

Question 2

In the subject *Potions* students learn to make a mixture which improves their athletic skills on the dance floor. This recipe calls for 3 parts by weight of dragonscale to 2 parts by weight of dogwort and 4 parts by weight of honey. How much dogwort is needed to make 27 grams of this mixture?

2 marks

Question 3

A serious illness affects the island's dragon population. The number, T_n , of sick dragons in week n obeys the difference equation

$$T_{n+1} = 2\,T_n - 11, \text{ for } n = 1, 2, \ldots, \text{ where } T_1 = k.$$

a. If the number of sick dragons in week 2 is 27, find the value of k, the number of sick dragons in week 1.

2 marks

b. How many dragons are sick in week 6?

1 mark

c. Is the sequence generated by the rule for T_n arithmetic, geometric or neither of these? Justify your answer.

2 marks

Question 4

In *Mathemagic* students learn how the wizards have been enlarging the area of the island since they first arrived 60 years ago. The area of the island was initially 230 km^2 and in the first year they added 80 km^2. Each year since the first year, they have added only half as much area as they added the year before.

a. How much area did they add to the island in the fifth year?

1 mark

b. How much area did they add to the island in total in the six years after they arrived?

2 marks

c. What would be the island area eventually if they keep this pattern of increase going forever?

2 marks

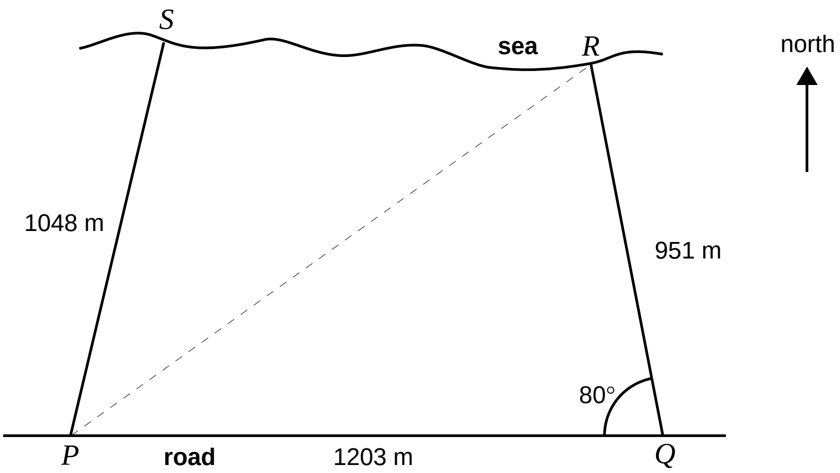
Total 15 marks

END OF MODULE 1
TURN OVER

Module 2: Geometry and trigonometry

Question 1

The diagram below shows a camping ground by the sea. The boundary *PQ* is 1203 metres long and runs beside an east-west road. The boundary *PS* is 1048 metres long and the bearing of *S* from *P* is 015º True. The boundary *QR* is 951 metres long and the angle *PQR* is 80º. The fourth boundary of the camping ground is along a cliff edge by the sea.



a. Find the area of triangle *PQR* in square metres, correct to the nearest square metre.

2 marks

b. Find the distance from *P* to *R*, correct to the nearest metre.

2 marks

c. Find the bearing of *R* from *P*, correct to the nearest degree.

2 marks

d. Find the area of triangle *PSR* in square metres, correct to the nearest 10 square metres.

2 marks

Module 2: Geometry and trigonometry – Question 1 – continued

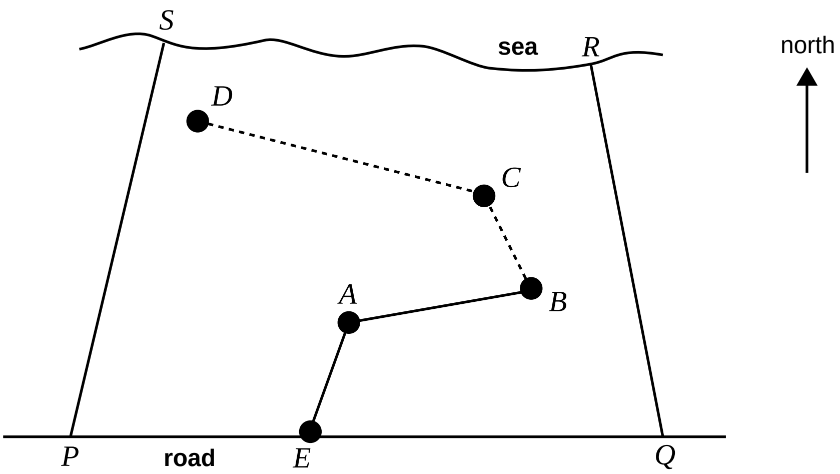
e. Hence find the approximate area of the camping ground. Give your answer correct to the nearest 10 square metres.

1 mark

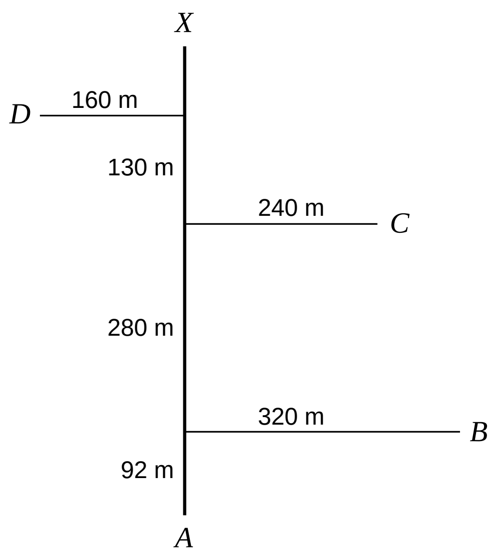
Working space

Question 2

In the camping ground there is a headquarters area with a centre at *A*, and there are three permanent camp sites with centres at *B*, *C* and *D*. A gravel road runs from the entrance *E* to *A* and from there to *B*. The only routes to *C* and *D* are along narrow bush tracks. The committee of management is considering constructing fire access roads as shown, from *B* to *C* and from *C* to *D*.



A traverse survey has been done to provide information for the committee. A field sketch (not to scale) from this survey is shown below. The baseline *AX* runs due north from *A*.



a. **i.** Find the straightline distance from *B* to *C*, correct to the nearest metre.

ii. Find the straightline distance from *C* to *D*, correct to the nearest metre.

1 + 2 = 3 marks

b. A historic pine tree is growing at a spot 450 metres due north of *A*. Determine whether this pine tree is on the straight line of the proposed road from *C* to *D*.

Justify your answer.

3 marks

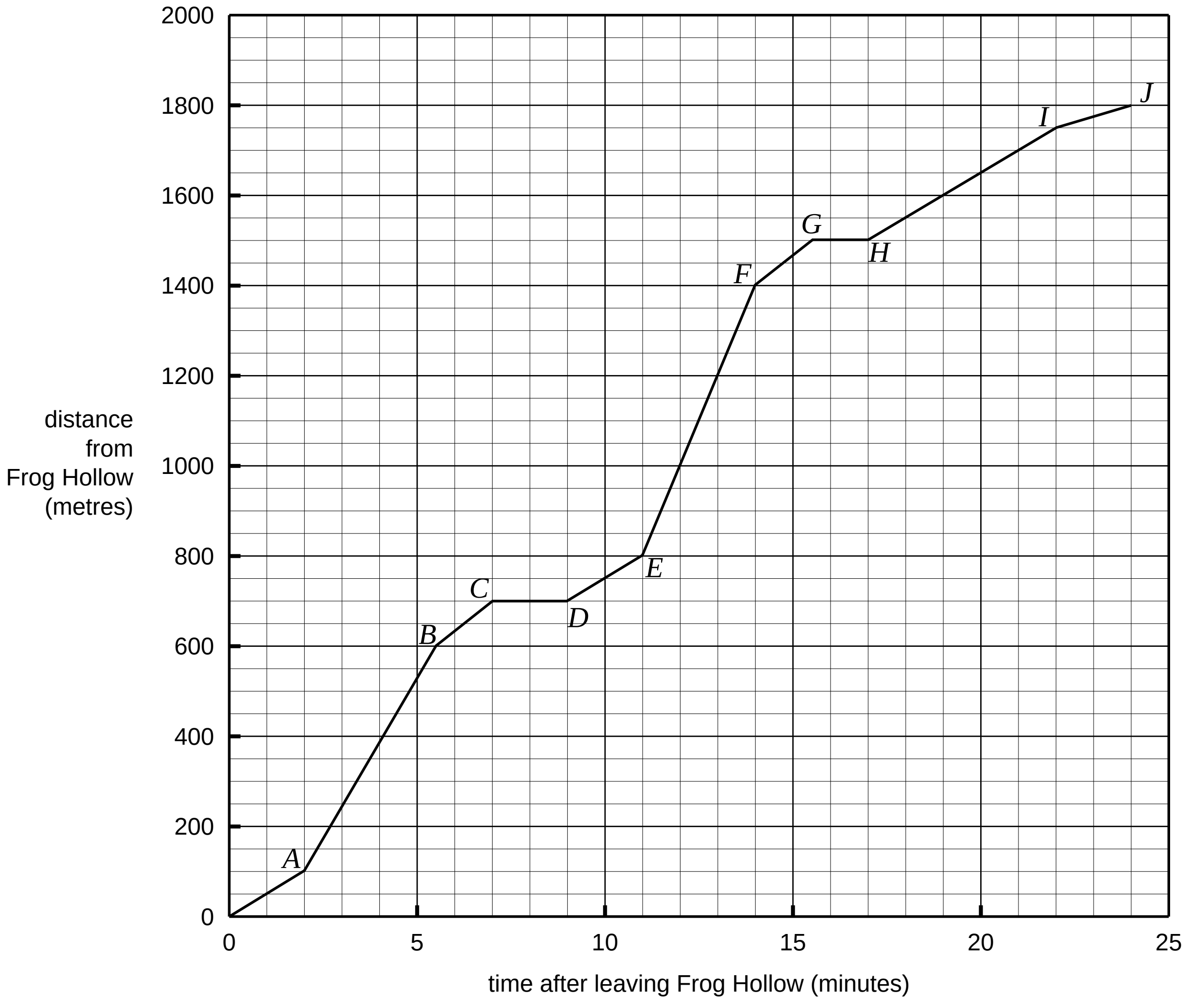
Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

Question 1

At the Gum Flat Fun Park there are many attractions. One that appeals especially to the younger visitors is the train Puffing Polly. The distance–time graph below represents a train trip for Puffing Polly from Frog Hollow to Eagle Hill, stopping at two stations on the way.



a. What is the total time for which Puffing Polly is stopped at the two stations on the way?

1 mark

b. **i.** Which line segment of the graph represents the section of the trip when Puffing Polly is travelling fastest?

ii. Find its speed for this section of the trip, stating clearly the units used in your answer.

1 + 2 = 3 marks

Question 2

Nico is planning a visit to the Gum Flat Fun Park. He wants to spend his time enjoying the thrills of the most terrifying rides at the park, the Xceletron and the Yellevator.

He considers the cost of each ride, the total time it actually takes to have the ride (including the waiting time and so on) and the thrill time he expects to have on that ride.

This information is summarised in the following table.

Ride	**Cost per ride ($)**	**Time taken per ride (minutes)**	**Thrill time on ride (minutes)**
Xceletron	2.00	30	5
Yellevator	4.00	20	7

Let *x* represent the number of rides Nico has on the Xceletron and *y* represent the number of rides on the Yellevator. This gives the constraints

$$x \geq 0\ ,\ \ y \geq 0.$$

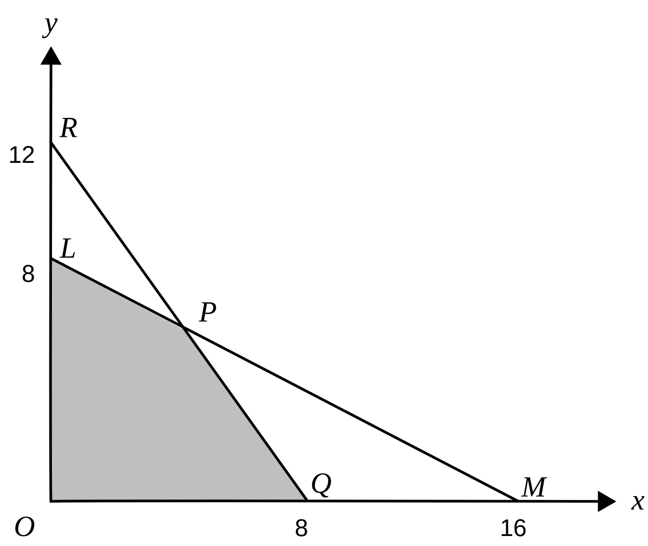
Nico expects that he will have $32 to spend on rides. This gives the money constraint

$$2x + 4y \leq 32.$$

Nico cannot spend more than 4 hours on the rides. Ignoring the time taken to travel between the rides, the time constraint is

$$30x + 20y \leq 240.$$

The feasible region defined by the above constraints is shaded on the following graph.



a. Write down the equation of the straight line that passes through *R* and *Q*.

1 mark

b. Find the coordinates of *P*, the intersection of lines *LM* and *RQ*.

2 marks

c. **i.** For Nico's visit to the fun park, let *T* minutes be the total thrill time. Write an expression for *T* in terms of *x* and *y*.

$T =$ ☐

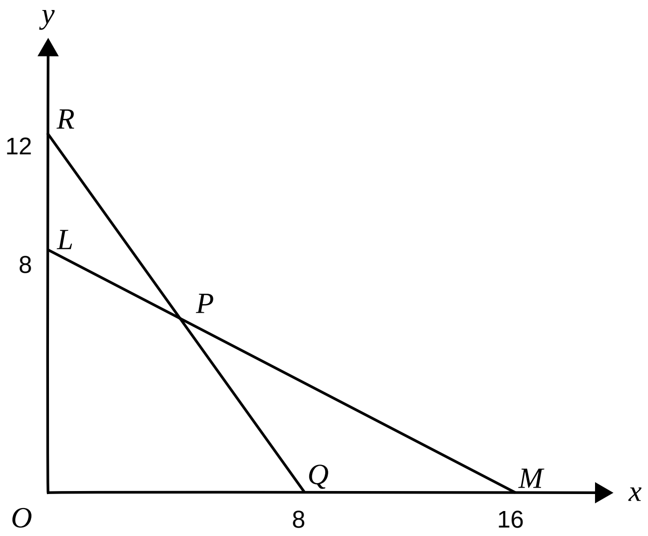
ii. Find the maximum total thrill time that Nico can have during his visit to the fun park.

1 + 2 = 3 marks

Nico's brother Pedro is also going to the fun park to have rides on the Xceletron and the Yellevator. Pedro also has $32 to spend on rides and cannot have more than 4 hours of rides.

d. Let x and y now represent the number of rides Pedro has on the Xceletron and on the Yellevator respectively. Pedro insists he must have at least 2 rides on the Xceletron.

On the graph following, draw the line corresponding to this new constraint and shade the feasible region for Pedro's case.



2 marks

e. Pedro has worked out his own expression for the total thrill time, T, for his rides at Gum Flat Fun Park. For Pedro

$$T = 4x + 8y$$

Find how many rides of each type Pedro could choose if he wants to maximise his total thrill time.

Give every possible solution, and explain why there cannot be any other solutions.

3 marks

Total 15 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1

Sally's credit union passbook looked like this in June 2002.

Date	Particulars	Deposits	Withdrawals	Balance
01 July 2001	Brought Forward			2400.00
15 Dec 2001	Deposit	1200.00		3600.00
02 Feb 2002	ATM Withdrawal			3000.00
14 May 2002	Interest	85.50		
20 June 2002	ATM Withdrawal		450.00	2635.50

a. Complete this table by filling in

i. the amount withdrawn on 2 February 2002.

ii. the account balance for 14 May 2002.

1 + 1 = 2 marks

b. Interest on this account was paid at a rate of 0.3% per month, based on the minimum monthly balance. How much interest did Sally earn for the month of December 2001?

1 mark

Question 2

On 1 July 2002, Sally invested \$4000 in a new term deposit that offered a total of \$416 interest after two years.

a. What was the annual simple interest rate offered for this term deposit?

1 mark

b. An alternative option for Sally had been to invest with a bank at a rate of 4.8% per annum compounding annually. To calculate the total amount in this account after two years with this option, Sally wrote down an equation that looked like this

$$\text{total amount} = 4000 \times c \times c$$

What number should Sally have used for c?

1 mark

c. What annual compounding interest rate, correct to two decimal places, would Sally have needed to earn \$416 interest in two years on a \$4000 investment?

2 marks

Question 3

Sally bought her car five years ago for $23 600 and it is now worth $7000.

Calculate, correct to one decimal place, the percentage annual rate of depreciation of the value of Sally's car over five years

a. on a flat rate basis.

2 marks

b. on a reducing balance basis.

2 marks

Question 4

Sally wants to borrow $20 000 for four years. Interest is calculated quarterly on the reducing balance at an annual rate of 8%.

a. Sally decides to use the annuities formula following for calculating this loan.

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

For Sally's loan

i. $R =$ ☐

ii. $n =$ ☐

1 + 1 = 2 marks

b. Sally can afford to repay this loan at $1500 per quarter.

Will this enable her to repay the loan in four years? Explain.

2 marks

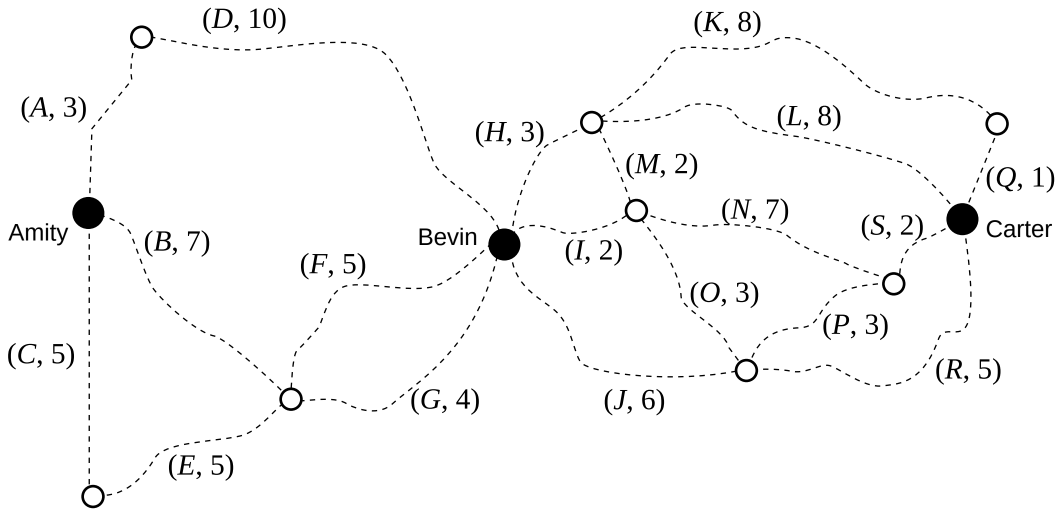
Total 15 marks

END OF MODULE 4
TURN OVER

Module 5: Networks and decision mathematics

The map below shows the roads that connect the towns of Amity, Bevin and Carter. The towns and the major intersections (indicated by open circles) form the nodes of this network of roads.

Labels on roads indicate their names and lengths in kilometres. For instance, (E, 5) indicates Road E is 5 km long.



Question 1

a. The length of the shortest path from Amity to Bevin is ☐ kilometres.

1 mark

b. Draw this path on the map above.

1 mark

Question 2

The Amity Cycling Club conducts a race **beginning** at Amity with checkpoints at every node in this network.

The race covers the full length of every road on the network in any order or direction chosen by the riders. A rider may pass through each checkpoint more than once, but must travel along each road exactly once.

a. One competitor claims this cannot be done. By referring to the **degrees of the nodes** in this network of roads, explain why it is possible to travel every road once only when cycling according to the club's rules.

___

___

2 marks

b. Under the club's rules for the race, the finish line must be at the intersection of the roads labelled

☐

1 mark

c. One competitor begins his race along roads *A–D–I–M–H* in that order.

To continue the race, which road should this competitor avoid at the end of road *H*?

Justify your answer.

___

___

___

2 marks

d. For 2003, the club wants to start the race at Amity and finish at Carter over the shortest route that still requires riders to ride the full length of every road in this network.

The rules will be modified to allow riders to travel twice along one of the roads.

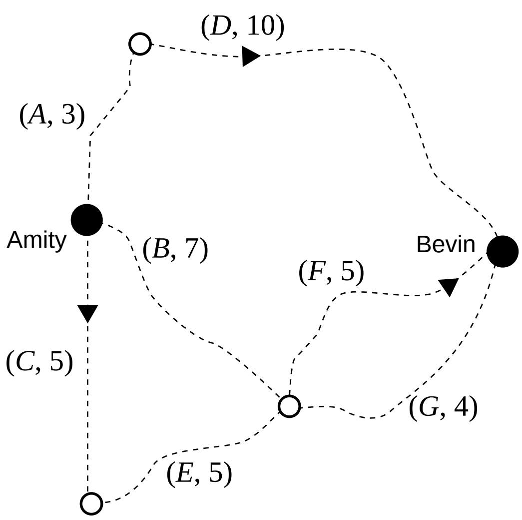
Which road must be travelled twice in 2003?

Road [ ]

1 mark

e. A suggestion for the proposed race in 2003 is to permit riders to travel along roads only in a specified direction between Amity and Bevin.

For this section of the race, the suggested directions for roads *C*, *D* and *F* are as shown by arrows on the map section below. On this map section, clearly draw in the correct direction for each of the other roads.

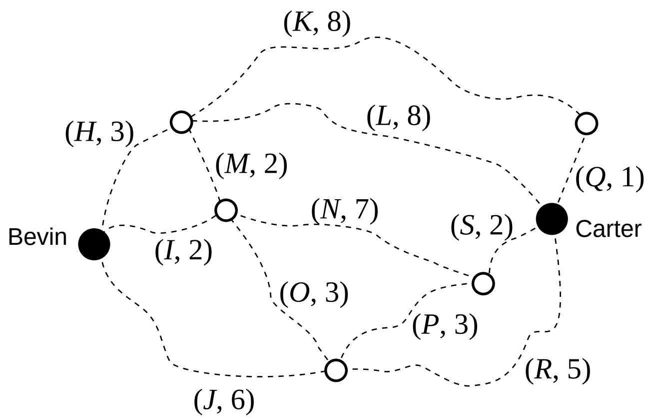


2 marks

Question 3

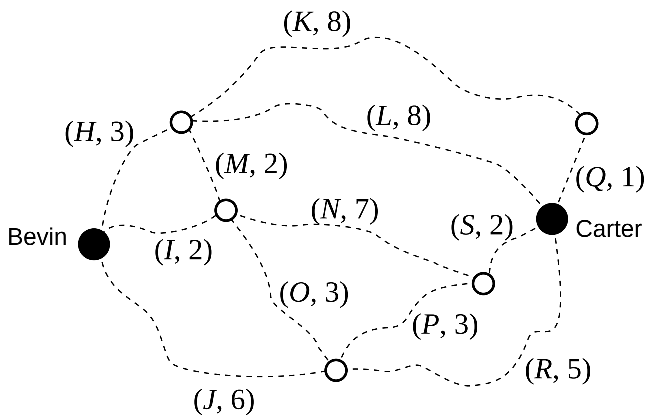
The Water Authority wants to lay water mains along the roads in order to put a fire hydrant at every node on the network shown on this map section. It decides that a minimal spanning tree for this network is suitable.

a. On the map section below, draw in a minimal spanning tree for this network.



2 marks

b. Each week, Andrew, who lives in Bevin, must travel through this network to inspect each of the fire hydrants and then return to Bevin.



i. Write down, in order, the road sections that Andrew must travel to complete a circuit of shortest length, beginning at Bevin. He does this by travelling along a circuit that prevents him from travelling along any road more than once.

Shortest circuit is [ ]

ii. The total length of this shortest circuit is [ ] kilometres.

1 + 2 = 3 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \frac{r}{100}$

Geometry and trigonometry

area of a triangle: $\frac{1}{2} bh$

area of a triangle: $\frac{1}{2} bc \sin A$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3} \pi r^3$

volume of a cone: $\frac{1}{3} \pi r^2 h$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Graphs and relations

Straight line graphs

gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y - y_1 = m(x - x_1)$ gradient-point form

$y = mx + c$ gradient-intercept form

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ two-point form

Number patterns and applications

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, \; r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, \; |r| < 1$

linear difference equations:

$$t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1} - 1)}{a - 1}, \; a \neq 1$$

$$= a^n t_0 + b\frac{(a^n - 1)}{a - 1}$$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

END OF FORMULA SHEET