Advanced Extension Award:
Answers to 2004 paper

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The examination tests the following requirements

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1 Question 1 : Total 25 marks

This question tests the following from AEA specification:

1 Knowledge, Understanding and Skills
  1.1 SI units
    1.2.2 Kinematics
  1.3 Momentum and energy
    1.3.2 Energy concepts
    1.3.3 Molecular kinetic theory

3 Mathematical Requirements
  3.3 Algebra

Relevant readings:

- Derivation of Van der Walls gas equation
- Equation of state

(a) 4 marks

(i) The volume of one mole of a given molecules.

(ii) Relating to physical observable measurable at large scale.

(iii) Supposed to fit the model description, may not be exactly real.

(iv) Collision in which no kinetic energy is lost (not converted to internal energy.)

(b) 3 marks

(i) The condition we normally live in. T $300K$ and p $1 \times 10^5 Pa$. [1]

(ii) Same law applicable to many different gases can imply that the assumption made in the ideal gas low is approximately true for most gases. Most importantly, "internal energy of an ideal gas depends only on temperature" means that energy in ideal gas only exist in the form of kinetic energy. For monatomic gas like argon, this is quite true while gases made of more complicated molecule can deviate from this more.

(iii) High pressure and high temperature. Low pressure and low temperature generally make gases more like ideal gas.

(c) 4 marks

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[1] $Pa = 1N/m^2$. eg. centre of earth: $4 \times 10^{11} Pa$, over-pressure in automobile tire: $2 \times 10^5 Pa$ and lowest vacuum achieved in lab: $10^{-12} Pa$.

(i) Kinetic energy of gas molecules, i.e. the movement of molecules (translational energy).

(ii) Real gas molecules have more degree of freedom than just one stated above. They can have Rotational motion or vibrational motion which contribute to the total energy of the gas. These effects are visible at higher temperatures.

(d) 8 marks

(i) Both pV_m and pb should have the same unit so the equation can hold. Hence b has has dimension of volume.

(ii) The b term is a correction to the ideal gas equation which corrects for the fact that molecules are not of infinitesimal size but they have a finite size. This term subtract the volume occupied by molecules making $(v-b)$ the effective volume available for molecules to move around. Considering molecules to be spherical, the volume occupied by molecules is $\frac{4}{3}\pi r^3$, where $r = \frac{1}{2}d$. So in terms of diameter of molecules, $b = \frac{1}{6}\pi d^3$ as a possible estimate for b.

(iii) Under the assumption that molecules being spherical, the actual volume excluded by the existance of molecules is more than that of their volume. There's dead space between molecules even when they are packed as close as possible, which should be subtracted from the total volume.

(iv) The gas would become liquid under high pressure. When molecules are put together into close distance, their attractive force become overwhelming compared to their kinetic energy and they go through a transition of phase. [2]

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[2] this equation, $p(V_m - b) = RT$ is a simplified version of what is known as Van der Waals gas equation. The full equation is $(p + \frac{a}{V_m^2})(V_m - b) = RT$. The first correction accounts for the attractive force between molecules (Van der Waals force) and the second accounts for the volume occupied by molecules

2 Question 2 : Total 18 marks

This question tests the following:

Relevant readings:

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(a) 3 marks

(b) 4 marks

(c) 3 marks

(d) 8 marks

3 Question 3 : Total 11 marks

This question tests the following:

1 Knowledge, Understanding and Skills
  1.3 Momentum and energy
    1.3.1 Momentum concepts
    1.3.2 Energy concepts
  1.8 Fields
    1.8.1 Force fields

3 Mathematical Requirements
  3.3 Algebra
  3.4 Geometry and trigonometry

Relevant readings:

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(a) 7 marks

(i) The electric force between two charges of mass m, charge q, placed r away from each other, F_{elec}, is given by $F_{elec} = qE$, where

$$E = \frac{q}{4\pi\epsilon_0 r^2}.$$

Hence

$$F_{elec} = 2.3 \times 10^{-28} \times \frac{1}{r^2}.$$

The gravitational force, on the other hand, is given by $F_{grav} = mg$, where

$$g = \frac{Gm}{r^2}$$

where G is the gravitational constant. Hence

$$F_{grav} = 5.5 \times 10^{-71} \times \frac{1}{r^2}.$$

So the ratio between the two is

$$\frac{F_{elec}}{F_{grav}} = 4.18 \times 10^{-42}.$$

(ii) The comparison made in (i) does suggest that gravitational force is much weaker than electric force and in fact it's quite true for most of such comparison made in microscopic physics. However, this is not the whole story. Gravitational force in the context of large distance, large mass scale physics tend to be much more dominant effect compared to electric force (eg. interstellar force). [3]

(iii) Electric force is only exerted on charged objects, while gravitational force is felt by any massive objects.

(b) 4 marks

Assume that the satellite's orbit is circular in the following. The period of the planetary motion is related to the speed of its rotation by

$$v = \frac{2\pi r}{T},$$

where v is the velocity of the satellite. The centripetal acceleration of the satellite is equal to the hypothetical gravitational force, hence

$$\frac{m_s v^2}{r} = \frac{H m_E m_s}{r},$$

where m_s is the satellite's mass and M is the Earth's mass. From the two equations, one can derive

$$r^{-2}T^2 = \frac{4\pi^2}{H m_E}.$$

(m and n can be other values as long as the right side has the same power. For eg, $r^{-1}T^1 = \sqrt{\frac{4\pi^2}{Hm_E}}$ is acceptable.) [4]

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[3] There is no direct reasons why electric force is weaker than gravitational field at this scale. Either type of forces do not change their intrinsic strength but the reality is that it is very difficult to create an object that is hugely in excess of one (+ or -) static charge. Molecules or atoms can be charged by ionization but once they are charged, like charges repel each other and they tend not to form larger objects with larger charge. Hence, large objects like stars are usually charge neutral and the electric field due to these objects is virtually zero. This should not be mistaken as there is no electromagnetic activity in such large objects. Stars can be magnetically highly active and all the light coming from stars are all due to internal electromagnetic activities.

[4] The period T derived from correct gravitational force is $T^2 = \frac{4\pi^2}{Gm_E}r^3$

4 Question 4 : Total 12 marks

This question tests the following:

Relevant readings:

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(a) 5 marks

(b) 6 marks

(c) 1 marks

5 Question 5 : Total 13 marks

This question tests the following:

- 1 Knowledge, Understanding and Skills
  - 1.1 SI units
  - 1.2 Mechanics
    - 1.2.1 Vectors
    - 1.2.2 Kinematics
    - 1.2.3 Dynamics
  - 1.3 Momentum and energy
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  - 1.7 Waves and oscillations
    - 1.7.2 Oscillations
- 3 Mathematical Requirements
  - 3.3 Algebra
  - 3.4 Geometry and trigonometry

Relevant readings:

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(a) 2 marks

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- f is the frequency in the unit of s^{-1}.
- k is the spring constant in the unit of $Nm^{-1} = kgs^{-2}$. (cf. $F = -kx$)
- m is the mass in kg.

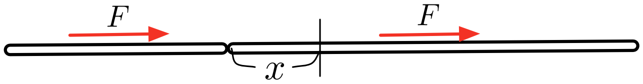
together you have s^{-1} on both sides.

(b) 11 marks

(i) Consider the force on spring due to extension $F = kx$. The frequency of the spring can be derived as follows:

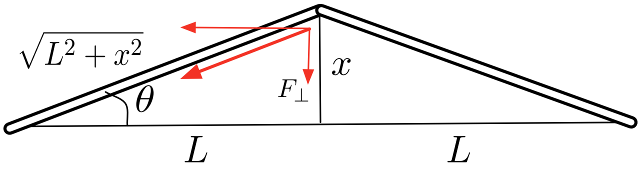
$$\begin{aligned} F &= kx \\ ma &= kx \\ a &= \frac{k}{m}x \\ a &= \omega^2 x \end{aligned}$$

where $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency and hence $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. In the case of two strings joined together, when the mass is moved along the direction of the string, the force on the mass is simply double that of single spring, ie. $F = 2kx$. Put this into the same procedure as above, $a = \omega^2 x$ and this shows that the motion is simple harmonic motion, but this time $\omega = \sqrt{\frac{2k}{m}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$. Therefore, the frequency increases by factor of $\sqrt{2}$ and this is equivalent to having one spring whose spring constant is $2k$.



(ii) 1. Assume that the displacement x is small compared to L. Then the displacement angle θ would also be small where the following approximation holds:

$$\sin\theta \approx \tan\theta \approx \theta.$$



Since $\tan\theta = \frac{x}{L}$, $\sin\theta \approx \frac{x}{L}$. The force in x direction due to one spring,

$$F_{\perp 1} = F\sin\theta \simeq kd\frac{x}{L},$$

where F is the force on the spring into the direction of the spring and d, is the extension of the string. The extension of the string is

$$d = \sqrt{L^2 + x^2} - L = L(1 + \frac{x^2}{L^2})^{\frac{1}{2}} - L \simeq L(1 + \frac{1}{2}\frac{x^2}{L^2}) - L = \frac{1}{2}\frac{x^2}{L}.$$

Hence,

$$F_{\perp 1} \simeq k\frac{1}{2}\frac{x^3}{L^2} = \frac{1}{2}(\frac{x}{L})^2 kx.$$

So for two springs,

$$F_\perp \simeq (\frac{x}{L})^2 kx$$

2. The mass would move in a simple harmonic motion like in (i) but the frequency is much lower in (ii) for the same x since the factor $\frac{x}{L}$ is very small.

6 Question 6 : Total 10 marks

This question tests the following:

Relevant readings:

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(a) 4 marks

(b) 6 marks

7 Question 7 : Total 15 marks

This question tests the following:

Relevant readings:

- none suggested

The answer to this question can vary largely. The following is one possible suggestion which includes many useful points to be included in your solution