Advanced Extension Award:
Answers to 2004 paper

Akira Shibata

Contents

The examination tests the following requirements

1 Knowledge, Understanding and Skills

- 1.1 SI units
- 1.2 Mechanics
  - 1.2.1 Vectors
  - 1.2.2 Kinematics
  - 1.2.3 Dynamics
- 1.3 Momentum and energy
  - 1.3.1 Momentum concepts
  - 1.3.2 Energy concepts
  - 1.3.3 Molecular kinetic theory
- 1.4 Electricity
  - 1.4.1 Current
  - 1.4.2 Emf and potential difference
  - 1.4.3 Resistance
  - 1.4.4 DC circuits
  - 1.4.5 Capacitance
- 1.5 Atomic and nuclear physics
  - 1.5.1 Probing matter
  - 1.5.2 Ionising radiation
  - 1.5.3 Energy
- 1.6 Quantum physics
  - 1.6.1 Photons
  - 1.6.2 Mattes
- 1.7 Waves and oscillations
  - 1.7.1 Waves
  - 1.7.2 Oscillations
- 1.8 Fields
  - 1.8.1 Force fields
- 1.9 Magnetic effects of currents
  - 1.9.1 B-fields
  - 1.9.2 Flux and electromagnetic induction

2 Experiment and Investigation

- 2.1 Analysing evidence and drawing conclusions
- 2.2 Evaluating evidence and procedures

3 Mathematical Requirements

- 3.1 Arithmetic and computation
- 3.2 Handling data
- 3.3 Algebra
- 3.4 Geometry and trigonometry
- 3.5 Graphs

1 Question 1 : Total 25 marks

This question tests the following from AEA specification:

1 Knowledge, Understanding and Skills

- 1.2 Mechanics
  - 1.2.1 Vectors
  - 1.2.2 Kinematics
- 1.4 Electricity
  - 1.4.1 Current
  - 1.4.2 Emf and potential difference
- 1.8 Fields
  - 1.8.1 Force fields
- 1.9 Magnetic effects of currents
  - 1.9.1 B-fields
  - 1.9.2 Flux and electromagnetic induction

2 Experiment and Investigation

- 2.1 Analysing evidence and drawing conclusions

3 Mathematical Requirements

- 3.3 Algebra
- 3.5 Graphs

(a) 6 marks

(i) Force.

(ii) The direction perpendicular to, or, at right angle to.

(iii) The region in which a charge experiences a force **or** where there is a potential difference; often denoted as E, electric field is generally the derivative of the potential.

(iv) Measure of strength of magnetic field **or** magnetic flux per area; often denoted as B, it is measured in the units of Tesla. [1]

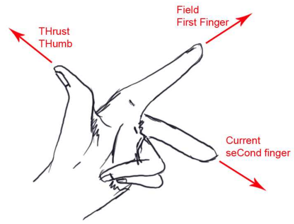
(v) Electromagnetic force on charge moving in a magnetic field,

$$F = qE + qv \times B,$$

where q is the charge and v is the velocity of the charge. The direction of the force due to any magnetic field can be found by using the left hand rule shown in

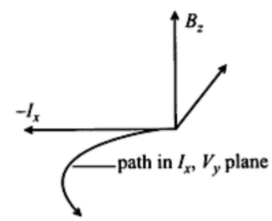
(vi) y is proportional to x if $y = mx$ is satisfied where m is a constant.

[1] Induction of strong magnetic field is beneficial to many applications and the current best artificial magnetic field is produced with superconducting magnets. This achieves typically around $5T$ over the volume of $1m^3$. On the surface of the earth, the magnetic field is around $10^{-5}T$.



(b) 6 marks

(i) NB: Be careful when you use the left hand rule, the second finger is for the direction of current which is opposite to the direction of the motion of the electron which has a negative charge. Right hand can be used for negatively charged particles in which case the second finger is for the electron direction of motion.



(ii) Due to the Lorentz force in negative y-direction, electrons accumulate on one side of the wire, resulting in a net negative charge on that side. This results in an electric potential difference between the positive y and negative y sides of the wire.

(iii) As electrons pile up on one side, the net negative charge tends to repel further accumulation of electrons. Hence, there exists a point of equilibrium between the two forces and from then on, no net force acts on the motion of the electron. It passes straight along the conductor

(c) 5 marks

(i)

$$R_H = \frac{E_y A}{I_x B_z}$$

From the set-up of the experiment, I_x and B_z are positive. The Hall field E_y is the direction of the induced electric field due to the Hall effect. In case the current carriers are electrons, as in (b), and electrons accumulate into negative y direction, giving rise to the field in the negative y direction. The quantity A cannot be negative, so R_H is negative.

(ii) Some are positive and some are negative. The magnitude varies in different materials. Different magnitudes imply that the strength of the Hall field varies, depending how "free" the current carriers are, which may be related to the resistance of the material. The sign of R_H depends on the sign of the current carriers, electrons or holes. [2]

[2] R_H can be considered as a character of a given conductor. One of the most significant thing about the Hall effect is that it can distinguish between the materials in which currents are carried by positive or negative carriers by using a quantity such as R_H. This is an useful tool to determine the electric property of materials.

(d) 4 marks
In this simplified setting,

$$E_y = \frac{\Delta V_H}{B}.$$

Assuming that the equilibrium has been established between the Lorentz force and the Hall force,

$$B_z e v_x = e E_y,$$

and using the given expression,

$$I_x = nAv_x e, \frac{B_z I_z}{nab} = \frac{(e\Delta B_H)}{b} n = \frac{B_z I_z}{ea\Delta V_H}.$$

Or, in terms of Hall coefficient,

$$n = \frac{1}{eabR_H}.$$

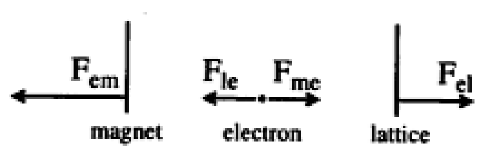
[3]

(e) 4 marks

(i) Newton's 3rd Law: the force exerted by one body on another is equal in magnitude but opposite in direction to the force exerted by the second body on the first. [4]

(ii) The 3rd law does not apply to the Lorentz force and the Hall force since both forces act on the same body, electron.

(iii) When force F_{me} exerted on electron by means of magnetic field from the magnet, electrons exert equal and opposite force F_{em} on the **magnet**. The force exerted on electron through the Hall field by the **wire** or the lattice, F_{le} is reacted by the equal and opposite force by electron onto the wire, F_{el}. Since $F_{le} + F_{me} = 0$, there remains a net force on the wire, F_{el}.



[3]Using a material with known R_H, one can construct a device for measuring magnetic field strength by fixing A and I_x and measuring the induced field E_x. This is a common application of this effect.

[4]Newton's Laws should be learnt by heart! 1. A body at rest remains at rest and a body in motion continues to move at constant velocity unless acted upon by an external force; 2. A force acting on a body causes an acceleration which is in the direction of the force and has a magnitude inversely proportional to the mass of the body ($F = ma$) and 3. Whenever a body exerts a force on another body, the latter exerts a force of equal magnitude and opposite direction of the former.

2 Question 2 : Total 11 marks

This question tests the following:

1 Knowledge, Understanding and Skills

- 1.2 Mechanics
  - 1.2.2 Kinematics
- 1.3 Momentum and energy
  - 1.3.2 Energy concepts

3 Mathematical Requirements

- 3.1 Arithmetic and computation
- 3.3 Algebra
- 3.5 Graphs

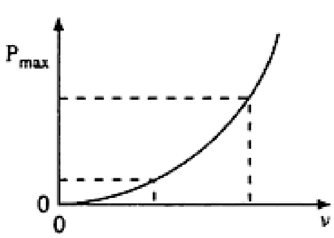
(a) 4 marks
Assume 100% efficiency, [5] i.e. all the available energy converted to electric energy. The available energy is in the form of kinetic energy of the air particles. Usual relation of $KE = \frac{1}{2}mv^2$ means unit volume of air has $\frac{KE}{V} = \frac{1}{2}\rho v^2$ (kinetic energy per unit volume), where V is the volume of the air. Since power is is energy per second, we are interested in the volume that goes pass the turbine per second, which is $\pi L^2 v$. Hence, the maximum possible power generated by the turbine is $P_{max} = \frac{1}{2}\pi L^2 \rho v^3$, as given.

(b) 2 marks
Putting the given values into the equation derived in (a),

$$P_{max} = \frac{1}{2}(13)^2 \times 1.2 \times (20)^3 = 2.5 \times 10^6 W (2.548 \times 10^6 W)$$

(c) 2 marks
Substitution of $v = 10ms^{-1}$ gives $3.2 \times 10^5 W$. $100 \times 10^6 / 3.1 \times 10^5 \sim 314$ *turbines*

(d) 3 marks
The power generated by turbines has **cubic** dependence on the speed of the wind. This non-linearity



indicates the advantage of having turbines in places where the wind speed is higher. Twice as high wind speed can reduce the number of turbines by factor of 8 to generate the same amount of energy.

[5] This is a very unrealistic assumption, typically power generators aim for maximum efficiency of ~ 50 to 60%. If you are interested in wind power generation, Wikipedia has an extensive summary on this topic.

3 Question 3 : Total 19 marks

This question tests the following:

1 Knowledge, Understanding and Skills

- 1.1 SI units
- 1.4 Electricity
  - 1.4.1 Current
  - 1.4.3 Resistance

2 Experiment and Investigation

- 2.1 Analysing evidence and drawing conclusions
- 2.2 Evaluating evidence and procedures

3 Mathematical Requirements

- 3.1 Arithmetic and computation
- 3.2 Handling data
- 3.3 Algebra
- 3.5 Graphs

(a) 4 marks

(i) The resistance of a material is proportional to the length of the material and inversely proportional to the cross-sectional area. Resistivity, ρ is the coefficient of this proportionality and hence this can be summarised as $R = l\rho/A$, where l is the length and A is the cross-sectional area of the material. Notice that R is in units of Ω and consequently, ρ is in Ωm.

(ii) Since exponential functions take unit-less values, C has the unit of temperature to cancel the unit of T, $Kelvin$. Hence B should have the same unit as $\frac{1}{\rho}$, which is $\Omega^{-1}m^{-1}$.

(iii) Just by substituting $\rho = \frac{l}{RA}$ for ρ,

$$\frac{1}{R} = \frac{AB}{L}e^{-(C/T)}R = \frac{L}{AB}e^{+(C/T)}. \tag{1}$$

(b) 15 marks

(i) The best choice of axis for plotting these values is the one that shows the relationship between R and ρ clearly. If we take the log of the equation 1,

$$\begin{aligned} ln(\frac{1}{R}\frac{L}{AB}) &= -\frac{C}{T} \\ ln(\frac{1}{R}) + ln(\frac{L}{AB}) &= -\frac{C}{T} \\ ln(\frac{1}{R}) &= -\frac{C}{T} + ln(\frac{L}{AB}) \end{aligned}$$

or alternatively,

$$
\begin{aligned}
ln(R\frac{AB}{L}) &= \frac{C}{T} \\
ln(R) + ln(\frac{AB}{L}) &= \frac{C}{T} \\
ln(R) &= \frac{C}{T} + ln(\frac{AB}{L})
\end{aligned}
$$

So if $1/R$ (or R) is plotted on a log scale, C can be related to the slope and B can be related to the y-intercept of the line.

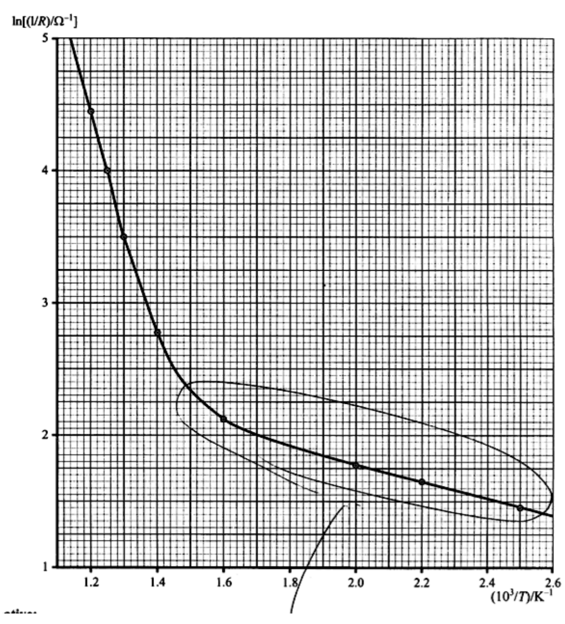
(ii) If $1/R$ is plotted, $-C$ is the gradient of the line, or in case R is plotted C is the gradient of the line.

(iii)

$T\ (K)$	833	800	769	714	625	500	455	400
$10^3/T\ (K^{-1})$	1.20	1.25	1.30	1.40	1.60	2.00	2.20	2.25
$R\ (m\Omega)$	11.8	18.3	30.2	62.1	119	169	192	232
$1/R\ (\Omega^{-1}$	84.7	54.6	33.1	16.1	8.40	5.92	5.21	4.31
$ln(1/R\ (\Omega^{-1}))$	4.44	4.00	3.50	2.78	2.13	1.78	1.65	1.46

Note, any more significant figure will not be of much use. Choice of R rather than $1/R$ may be better for the values given (requires less conversion.)

(iv) see fegure below.



(v) Since the x-axis is $1/T$, the low temperature region is on the right with shallower slope. In *Kelvin*, values in the range -600 to -680 are acceptable (or 600 to 680 if $ln(R)$ was plotted.)

(vi) The graph can be divided in two regions with different proportionality constants with a non-linear region in between. One possibility is that the semiconductor changed its property at a certain temperature (around $650K$), though both of these states obey equation (1). [6]

[6] Note that the resistance of metal conductors typically **increase linearly** when the temperature is increased, as opposed to the **exponential decrease** in semiconductors. Naively, this can be pictured as follows: in metal conductors,

4 Question 4 : Total 13 marks

This question tests the following:

1 Knowledge, Understanding and Skills

- 1.2 Mechanics
  - 1.2.2 Kinematics
- 1.3 Momentum and energy
  - 1.3.1 Momentum concepts
- 1.6 Quantum physics
  - 1.6.2 Mattes
- 1.8 Fields
  - 1.8.1 Force fields

3 Mathematical Requirements

- 3.1 Arithmetic and computation
- 3.3 Algebra

(a) 7 marks

(i) The centripetal force, F, due to circular motion is $F = \frac{mv^2}{r}$. If this force is provided by the gravitational force, $F = G\frac{M_E M_S}{r^2}$. Hence, equating the two equations, $v = (\frac{GM_S}{r})^{\frac{1}{2}}$, and

$$p = mv = M_E(\frac{GM_S}{r})^{\frac{1}{2}}.$$

(ii) The de Broglie wavelength, $\lambda = h/p$. Substituting in the appropriate values,

$$\lambda = h/p = \frac{6.63 \times 10^{-34}}{5.98 \times 10^{24}}(\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{1.49 \times 1011})^{\frac{1}{2}} = 3.71 \times 10^{-63} m.$$

On the other hand, the circumference of the Earth's orbit is,

$$2\pi r = 2\pi \times 1.49 \times 10^{11} = 9.36 \times 10^{11} m$$

The analogy is not valid here since there is no relevance between the de Broglie wavelength and the the circumference of the Earth's orbit. The Earth is not a quantum mechanical object! [7]

---

increasing temperature causes the lattice to thermally vibrate which tends to be in the way of electron's movement. In semiconductors, increasing temperature can increase the availability of free electrons or the charge carriers which overcomes other effects, resulting in a larger conductivity. The change in the availability of free electrons in conducting metals is much smaller. This simple picture, however, does not explain the discontinuous behaviour observed in this question. It is often interesting to study such discontinuity.

[7] "Why is this analogy not perfect?" throws an interesting question. Why is the earth not a quantum mechanical object whose de Broglie wavelength is a meaningful quantity to talk about? 10^{-63} is a *very* small distance. We do not know anything about the physics at such a small scale and such a wavelength is not easily observable. Electrons on the other hand, display interesting quantum-mechanical property in doable experiments. A recent experiment succeeded in measuring the wavelength of an object as large as carbon-60 (football-like structure of 60 carbon atoms), also known as buckyball.

(b) 6 marks

One of the numerous possibilities is: the analogy between radioactive decay and discharge of capacitor in R, C circuit. Equation

$$A = A_0 e^{-\lambda t}$$

holds for radioactive decay where A is the number of radioactive isotopes at time t, A_0 is the initial number of isotopes at time 0 and λ is the decay constant that is related to the mean life of the isotope. In analogy to this, the charge stored in capacitor, Q decreases as

$$Q = Q_0 e^{-t/\tau}$$

where τ is the time taken for the charge to decrease by $1/e$ of the initial charge.

The analogy is very useful and the ideas developed in one can be applied to the other. The number of radioactive isotope is directly analogous to the amount of charge on capacitor. The characteristic decay of radioactive isotope stems from the fact that one cannot tell when a give isotope decays and the process is probabilistic. This can be used to deduce the behaviour of the electrons in RC circuits that causes exponential decay.

5 Question 5 : Total 10 marks

This question tests the following:

1 Knowledge, Understanding and Skills

   1.7 Waves and oscillations

      1.7.1 Waves

      1.7.2 Oscillations

2 Experiment and Investigation

   2.1 Analysing evidence and drawing conclusions

   2.2 Evaluating evidence and procedures

3 Mathematical Requirements

   3.1 Arithmetic and computation

   3.2 Handling data

   3.3 Algebra

   3.4 Geometry and trigonometry

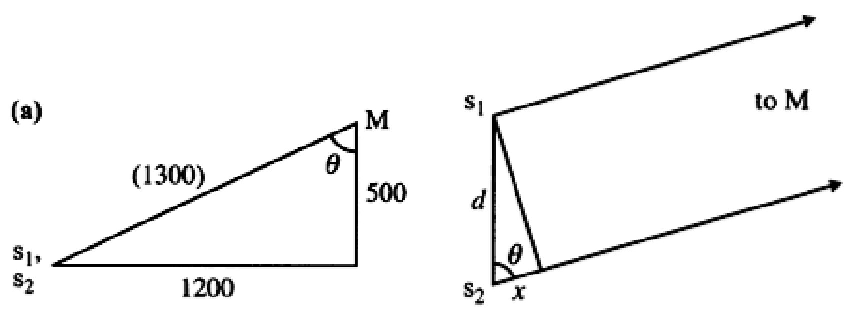
   3.5 Graphs

Relevant readings:

- Right Hand Rules

(a) 5 marks

Since $0.8m$ is a very small distance compared to $1200m$ and $500m$, they can be seen to be at the



same point from a global point of view. The angle θ is given by

$$\theta = tan^{-1}(\frac{1200}{500}) \simeq 67.4^o.$$

Since the point of measurement is far away from the speakers, the sound from two speakers come from about the same direction, parallel to each other. In this case, the path difference between these two paths is x as shown in the diagram and θ is the same angle as the one found earlier and hence,

$$x = dcos\theta \simeq 0.31m.$$

The first minimum occurs when $x = \lambda/2$ where λ is the wavelength of the sound wave. Hence the velocity of the sound wave v is,

$$v = f\lambda = 2fx \simeq 2 \times 560 \times 0.31 \simeq 347ms^{-s}.$$

N.B. This experiment is analogous to Young's double-slit experiment, though the expression $\lambda = \frac{xs}{D}$, does not apply here since this is only a good approximation when D, the distance from the slit to the screen ($= 1200m$ here), is much larger than x, the distance between the light and the dark band ($= 500m$ here). [8]

(b) 5 marks

The frequency is now increased at a constant rate, so $f = kt$, where k is the proportionality constant. The time taken by the sound wave to travel from the speakers to M is D/v. If the waves are out of phase (minimum reading, or silence) at time T, that frequency had been reached at the point of the speakers at time $T - D/v$. Hence,

$$x = \frac{\lambda}{2} = \frac{v}{2k(T - D/v)}$$

$$v^2 - 2kxTv + 2kxD = 0.$$

[8]Thomas Young's double-slit experiment is of prime importance in Physics and should be known thoroughly. It is one of the most "beautiful" experiment in physics as described here. As well as classical wave phenomena such as the one in this question, the experiment is very popularly used to explain quantum mechanical phenomena such as single photon interference or even single electron interference.

6 Question 6 : Total 7 marks

This question will test the following:

1 Knowledge, Understanding and Skills

- 1.1 SI units
- 1.2 Mechanics
- 1.3 Momentum and energy
  - 1.3.2 Energy concepts
  - 1.3.3 Molecular kinetic theory

3 Mathematical Requirements

- 3.1 Arithmetic and computation
- 3.2 Handling data
- 3.3 Algebra

Relevant readings:

- Pressure

Estimate the following:

- $v = 50\ m.p.h \simeq 20\ ms^{-1}$;
- the mass of a car $M \simeq 500 \sim 2000\ kg$;
- the increase of temperature needed for water to boil $\Delta\theta \simeq 70 \sim 90\ K$;
- the specific heat capacity of water $c_{water} = 4.2 \times 10^3\ Jkg^{-1}K^{-1}$.

The amount of water that can be boiled by transferring the kinetic energy of a car moving at $50m.p.h$ is

$$\frac{1}{2}Mv^2 = mc_{water}\Delta\theta,$$

where m is the mass of water to be boiled. Estimates in the range $0.3kg < m < 1kg$ are acceptable.[9]

[9] Even if the number obtained is not in this range, most credits in this question are given to sensible estimation of quantities and how they are related. Not many people remember the specific heat capacity of water!

7 Question 7 : Total 15 marks

This question tests the following:

1 Knowledge, Understanding and Skills

- 1.5 Atomic and nuclear physics
  - 1.5.1 Probing matter
- 1.6 Quantum physics
  - 1.6.1 Photons
  - 1.6.2 Mattes

The answer to this question can vary largely. The following is one possible suggestion which includes many useful points to be included in your solution

In reply to your questions you asked me the other day, let me try and tell you what quantum theory of Physics is and why electrons are important to us.

Quantum theory, formally known as Quantum Mechanics is one of the two pillars that form the very basis of modern physics which developed enormously in the 20th century, the other being the theory of Relativity. Experiments in the early 20th century revealed several new phenomena that demanded physicists to develop and accept radical concepts on the nature of light and matter. Examples of these experiments includes photoelectric effect (ejection of electrons from metal surface in response to light shone on it), line spectra obtained from various materials (discrete spectra of light absorbed/emitted from materials) and Compton scattering (scattering of light from materials which resulted in altered wavelength). What was at the heart of the conflict between theories were the generally-accepted ideas about particles and waves. Particles existed with locality: you can talk about the position of the particle, velocity, momentum, etc. In contrast, waves are conceptually very different kind of object; their existence extend over space and the description was made through quantities such as wavelength and amplitude. What was found was that light, which was largely believed to be a kind of wave, showed some behaviours that made physicists believe that it has particle properties in certain situations. What's more, elementary particles such as electrons, which were believed to be a kind of particle not wave, acted as if they were waves.

This astonishing evidences were eventually formulated into Quantum Mechanics under the principle of wave-particle duality and the theory successfully explained the mysterious behaviours observed in the experiments using quantisation of quantities such as energy. A huge number of experiments have been conducted since then and their results supported the theory to an impressive accuracy. Modern elementary particle physics, for example, is hugely based on this theory. One of the most important applications of the theory is semiconductor devices, from which modern computers, laser devices and numerous others are derived.

The importance of Electrons is closely related to Quantum Mechanics, though people had used them in various applications even before the development of the theory. What's known as electricity is current of electrons flowing through electric conductors. The discovery of electrons as particle was made by J.J. Thompson in his vacuum tube experiments and subsequently their properties were studied. It was found that electrons all have the same amount of "electric charge" which caused these particles to be deflected in presence of a magnetic field. This is the basis of TV tube. TV images are produced by electrons hitting surface of the screen and the movement of electrons is regulated by the magnets at the other end of the tube.

Electric power is one of the most important form of energy we use today. It is generated using the principle of electromagnetic induction which is the manipulation of electric and magnetic fields: by rotating large magnets in turbines, other forms of energies (heat, wind etc) can be converted into electric energy. The importance of electric energy and electric devices should not be hard to understand. From light bulbs to the Internet, it has influenced every part of our life and is now an essential part of everyday life.