Solution to AEA Physics 2003 Q1

a) ***(Five marks)*** A *pulsar* is a rotating neutron star, they emit radio, light or x-ray pulses at regular intervals of up to 500 times per second. Pulses are detected because the star sends out two narrow radiation beams which rotate with the pulsar.

A *black hole* is essentially a collapsed massive star. If the core of a supernova exceeds about 3 solar masses even the neutrons formed cannot resist gravitational collapse. The core shrinks to become a black hole from which no particles or radiation can escape.

A similarity in the process of formation of black holes and pulsars is that both are formed when a star collapses.

A difference between a pulsar and black hole is that a black hole is formed only if the star has a mass of 3 solar masses.

Figure 1 - Chandra Observatory X-ray image of the Crab Nebula pulsar

b) ***(Three marks)***

(i) The origin of the Crab Nebula is the compression of the inner core of a supernova which gives rise to electrons and protons forming denser neutrons. The cause of this compression is usually an explosion.

(ii) The "beeps" from a pulsar are due to violent localised storms that emit radiation in well defined directions. Rotation of the star causes the detection of regular pulses of radiation.

c) **(*Six marks)***

(i) We are told the fraction energy loss is equal to 5×10^{-6}. Now as $E = hf$ and therefore $\Delta E \propto \Delta f$ we can say that 5×10^{-6} is the fractional frequency loss:

$$\frac{\Delta f}{f} = -5\times10^{-6}$$

The original frequency is given by:

$$f = \frac{c}{\lambda} = \frac{3\times10^{8}}{530\times10^{-9}} = 5.66\times10^{14}(Hz)$$

Therefore the change in frequency is given by:

$$\Delta f = -5.66\times10^{14}\times5\times10^{-6} = -2.83\times10^{-9}$$

(ii) The change in wavelength can be easily calculated. Consider the following.

$E = hf = \frac{hc}{\lambda}$ therefore $\Delta f \propto \frac{1}{\Delta\lambda} \propto \Delta\lambda^{-1}$ now taking the log of both sides gives:

$$\log(\Delta f) \propto \log(\Delta\lambda^{-1}) \propto -\log(\Delta\lambda)$$

Now taking the inverse log of both sides:

$$\Delta f \propto -\Delta\lambda$$

Now the fractional energy loss value can be used to calculate the change in λ:

$$\Delta\lambda = 2.65\times10^{-12}m$$

This is a positive change in wavelength (photon loses energy) and therefore it is known as a *red shift*, as red corresponds to the longest wavelength of the visible spectrum. Conversely a negative shift in wavelength (photon gains energy) is known as a *blue shift*, due to blue being of shorter wavelength than red.

(d) ***(Four marks)***

Density, ρ, is equal to an objects mass divided by volume. $\rho = \frac{m}{V}$

Assuming the neutron star is a perfect sphere, it has a density of:

$$\rho_N = \frac{m}{(4/3)\cdot\pi\cdot r^3} = \frac{2\times10^{-30}}{(4/3)\cdot\pi\cdot(12.5\times10^{3})^{3}} = 2.44\times10^{17}\,kg\cdot m^{-3}$$

The density of the nucleus of a carbon atom of relative mass 12 and radius $3.0\times10^{-15}\,m$ has a value of:

$$\rho_C = \frac{12\times1.66\times10^{-27}}{(4/3)\cdot\pi\cdot(3.0\times10^{-15})^3} = 1.76\times10^{17}\,kg\cdot m^{-3}$$

Note how the values are remarkably similar.

(e) ***(Four marks)***

In order to calculate the equatorial surface speed we first need to find the frequency of rotation of radiation emission of the star, there are two points which emit radiation, therefore:

$$f = \frac{1}{2}\times\frac{1}{1.33733} = 0.374\ (Hz)$$

Now, the surface speed of an object is given by:

$$v = r\omega = 15\times10^3\times2\pi\times0.374 = 3.52\times10^4\,ms^{-1}$$

The equatorial surface speed of the Earth is found to be:

$$v = 6.4\times10^6\times2\pi\times\left(\frac{1}{24\times3600}\right) = 465\ ms^{-1}$$

$\frac{3.52\times10^4\,ms^{-1}}{465\ ms^{-1}} \approx 76$ therefore the Earth has an equatorial surface speed 76 times slower than a pulsar.

This is the solution to AEA Physics 2003 Question 2:

We have the equation:

$$f^n = \frac{k}{V} \qquad (1)$$

In order to plot the above equation simply we need to manipulate the terms in order to obtain the equation in a linear, straight line form. Recall that a straight line has the mathematical form of:

$$y(x) = m \cdot x + c \qquad (2)$$

To linearise the above equation we use the mathematical function called logarithm.

Taking the log of both sides gives us:

$$\log(f^n) = \log(\frac{k}{V}) \qquad (3)$$

This expression simplifies to give:

$$n \cdot \log(f) = \log(k) - \log(V) \qquad (4)$$

Rearranging this equation in the form of a straight line gives us:

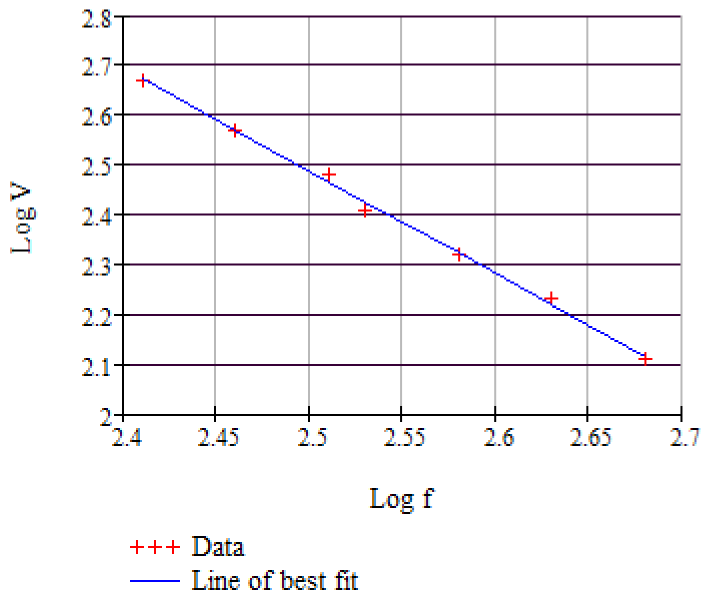
$$\log(V) = n \cdot \log(f) + \log(k) \qquad (5)$$

This enables us to plot log(V) vs log(f), which will have a gradient of n and an intercept of log(k).

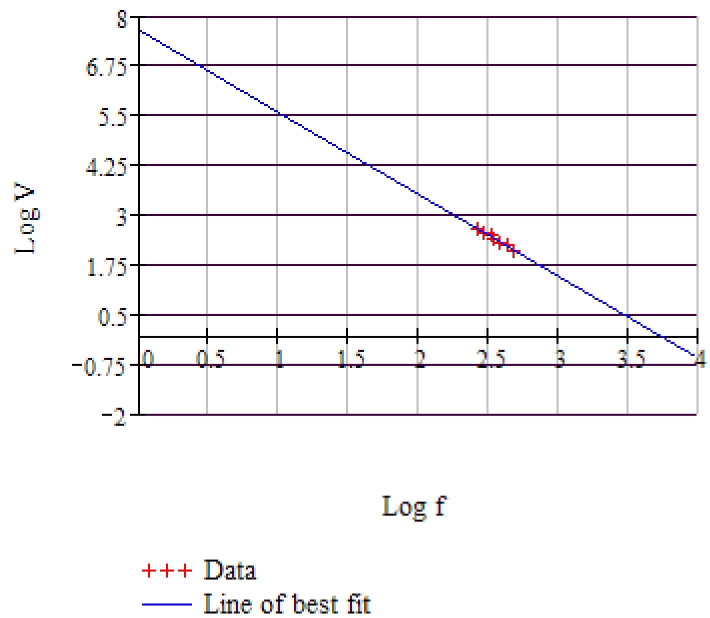
A suitable table of quantities would be the following (logarithms approximated to 2 decimal places):

Resonant frequency f/Hz	log f	Volume of air V/cm^3	log V
256	2.41	470	2.67
288	2.46	370	2.57
320	2.51	300	2.48
341	2.53	260	2.41
384	2.58	210	2.32
427	2.63	170	2.23
480	2.68	130	2.11

Plot on suitable axes we obtain the following graph:



If the line of best fit is extended then we can see immediately the *y* axes intercept:



However rather than redraw the graph on larger axes it is more convenient to find out the y axis intercept mathematically using the relation:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Therefore we have a gradient of

$$m = -\frac{2.585 - 2.185}{2.65 - 2.45}$$

$$m = 2.01$$

Where the gradient, *m* corresponds to the constant *n* in our equation $f^n = \frac{k}{V}$.

The intercept can be calculated::

$$c = \frac{y}{m \cdot x}$$

$$2 = \frac{a}{2.695}$$

Now we have sufficient information to calculate n and k:

$$\log k = \log V + n \log f$$

Taking any point on the graph we have:

$$\log k = 2.2 + 2 \times 2.645$$

$$\log k = 7.49$$

$$k = 3.1 \times 10^{7}$$

Advanced Extension Award:
Answers to 2003 paper

Chris Morgan

Contents

1 Question 3 : Total 12 marks

(a) 3 marks

Work must be done on a capacitor in order for it to charge. Electrical potential energy is stored as a result. The energy stored on a capacitor can be calculated from the following equivalent expressions:

$$\begin{aligned} U &= 1/2 \times \frac{Q^2}{C} && (1) \\ &= 1/2 \times CV^2 && (2) \\ &= 1/2 \times QV && (3) \end{aligned}$$

In this case we have a $10\mu F$ capacitor and a $12V$ battery. This gives us a final energy of

$$U = 1/2 \times 10 \times 10^{-6} \times 12^2 = 7.2 \times 10^{-4} J \quad (4)$$

In order to find the energy supplied by the battery during the charging process it is necessary to find the total charge stored in the capacitor.

$$Q = C \times V = 10 \times 10^{-6} \times 12 = 1.2 \times 10^{-4} C \quad (5)$$

The energy supplied by the battery can now be calculated.

$$E = Q \times V = 1.2 \times 10 - 4 \times 12 = 14.4 \times 10^{-4} J \quad (6)$$

(b) 3 marks

The gain of kinetic energy per second is given by:

$$\frac{1}{2} \times \frac{1200 \times 1.8^2}{60} = 32.4W \quad (7)$$

The force needed to drive the convey belt is found through consideration of the momentum change of the luggage.

Recall 1.0.1 *Newton's second law of motion: The rate of change of momentum of an object is proportional to the resulting force acting.*

$$resultant\ force \propto \frac{change\ in\ momentum}{time\ taken} \tag{8}$$

When the unit of force is defined in a suitable way i.e. the SI, the above proportion can be changed into an equation.

$$F = \frac{mv - mu}{t} \tag{9}$$

Therefore:

$$Force = \frac{1200}{60} \times 1.8 = 36N \tag{10}$$

And the power needed is given by:

$$Power = 36 \times 1.8 = 64.8W \tag{11}$$

(c) 6 marks

In part (a) the efficiency of energy transferal was equal to

$$Efficiency = \frac{Energy\ stored}{Energy\ supplied} = \frac{7.2 \times 10^{-4}}{14.4 \times 10^{-4}} = \frac{1}{2} \tag{12}$$

In part (b) the efficiency of energy transferal was equal to

$$Efficiency = \frac{32.4}{64.8} = \frac{1}{2} \tag{13}$$

The efficiency of energy transfer in part (a) was due to a resistance between the battery and capacitor. The potential difference (p.d.) of the battery consists of the p.d. of the capacitor plus the p.d. of the resistance. Therefore

$$Energy\ supplied = work\ done\ to\ store\ charge + work\ done\ to\ overcome\ resistance \tag{14}$$

The efficiency of energy transfer in part (b) was due to losses through friction. The total driving force consisted of a force to accelerate the luggage from rest plus a force to overcome friction. Therefore

$$Work\ done\ by\ driving\ force = gain\ of\ kinetic\ energy\ of\ objects \tag{15}$$
$$+\ work\ done\ to\ overcome\ friction\ on\ conveyor \tag{16}$$

$$work\ done\ to\ overcome\ friction = gain\ of\ kinetic\ energy\ of\ luggage \tag{17}$$

(d) 6 marks

(i) The number (or better,the density) of a particular species can be related to the number of Auger electrons. The more atoms there are, the more likely the atom-electron interaction occur and hence, more Auger electrons. i.e.

$$No\ of\ Auger\ electrons \propto No\ of\ atoms \tag{18}$$

($\propto$ is better though = is acceptable)

(ii) **image needs to be inserted here from the marking scheme.
It is important that the plot starts and ends with negative G. Label Auger energy clearly.

(e) 6 marks

(i) From the diagram, $\Delta E = 3.00 \times 10^{-16} J$. Potential difference required can be obtained by $J = eV$ conversion, where e is the electron charge. Hence $V = 1875 Volts$.

(ii) *Energy released* $= \Delta E = E_{L_1} - E_K$
Reading off E_{L_1} and E_K from the diagram,

$$\Delta E = (-1.10 - (-3.00)) \times 10^{-16} = 1.90 \times 10^{-16} J. \quad (19)$$

(iii) The energy calculated in (ii) is used to knock off L_2 electron. So the energy of Auger electron is

$$\Delta E = (1.90 - 0.90) \times 10^{-16} = 1.00 \times 10^{-16} J. \quad (20)$$

(iv) The energy of the photon is calculated in (ii). To obtain the wavelength of this X-ray photon, use

$$E = hf \quad (21)$$

where h is the Planck constant and f is the frequency of the photon. Since

$$f = c/\lambda \quad (22)$$

where c is the speed of light and λ is the wavelength,

$$\lambda = \frac{hc}{E} \quad (23)$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{1.90 \times 10^{-16}} \quad (24)$$

$$= 1.05 \times 10^{-9} m \ (1.05 nm). \quad (25)$$

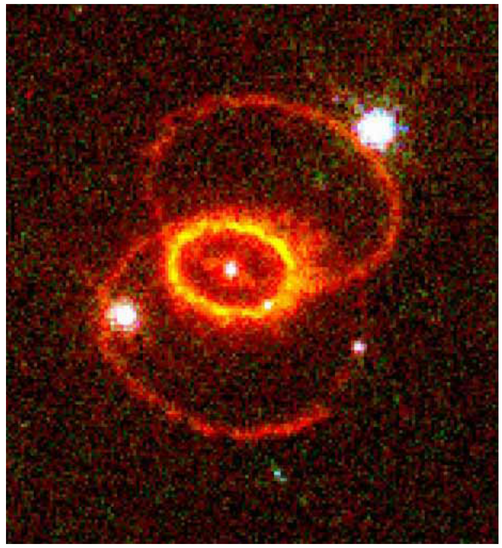
2 Question 4 : Total 16 marks

(a) 8 marks

(i) In order to tackle this question we must firstly assume that cosmic ray activity has been constant for a period of time much greater than 5700 years.

Recall 2.0.2 *Any type of particle that bombards the Earths atmosphere are known as* ***cosmic rays****. Supernova have recently been proved to be a source of cosmic rays, although they were theoretically predicted long before. The main types of cosmic ray are:*

- *Galactic Cosmic Rays (GCRs). Originating form outside the solar system but generally from within our Milky Way. GCRs are atomic nuclei from which all the surrounding electrons have been stripped away during their high-speed passage through the galaxy.*
- *Anomalous Cosmic Rays. Thought to come from interstellar space at the edge of the heliopause*
- *Solar Energetic Particles (SEPs). Atoms which are associated with solar flares. These atoms move away from the sun due to plasma heating, acceleration and other forces.*



A source of cosmic rays - supernova SN1987A. Found in the Large Magnetic Cloud galaxy. Image was taken from the Hubble Telescope in 1994, this and other Hubble images are available here.

This results in the C_{14} half life being of a very short length scale in relation to the cosmic rays. Thus the rate of isotope decay R_d can be said to be equal to the rate of isotope production R_p in comparison to the time span of cosmic rays.

$$R_d = R_p \tag{26}$$

If we have the rate of production of one isotope equal to the rate of production of another isotope then the ratio of the two must be constant.

(ii) The ratio of C_{12} to C_{14} will be equal in living matter and the atmosphere but not in dead matter. This is due to the fact that all living matter takes in carbon through food and respiration in the form of CO_2 but dead matter does not. Therefore the proportion of C_{14} is gradually reduced by radioactive decay. Therefore the ratio of C_{12} to C_{14} will gradually change over time in dead matter.

(iii) The age of a piece of timber found on an archeological site can be determined through a process known as carbon dating. Firstly the radioactivity of the timber from the archeological site is measured.

This is then compared with the radioactivity of a piece of living wood containing an equal mass of carbon. The difference in measured radioactive count can then be used to calculate the elapsed time through knowledge of the half life.

(b) 8 marks

The number of disintegrations from 5.0g of timber is found to be 21 per minute. The age of the specimen is found by firstly determining the total number of carbon atoms in the sample. This is done using the atomic mass of carbon, which is 12g. This number, 12g, is the mass of one mole of carbon. One mole of carbon contains Avogadro's number, 6.022×10^{23}, atoms of carbon.

Therefore in our sample the number of carbon atoms is given by:

$$\frac{5}{12} \times 6.022 \times 10^{23} = 2.51 \times 10^{23} \tag{27}$$

We can now use the ratio given previously to find the number of C_{14} atoms when the timber ceased to be alive, this will be called N_0

$$\frac{2.51 \times 10^{23}}{1.25 \times 10^{12}} = 2.0 \times 10^{11} = N_0 \tag{28}$$

Recall 2.0.3 ***Nucleonic decay and half-life***
For a collection of radioactive nuclei the more undecayed nuclei the frequently a decay is likely to occur.

$$activity \propto number\ of\ undecayed\ nuclei \tag{29}$$

Activity is defined as $-dN/dt$. The minus sign indicating that a decrease in N gives positive activity.

Adding in a suitable constant, λ, known as the radioactive decay constant and defining the number of undecayed nuclei after a time t by N we can rewrite the proportionality as:

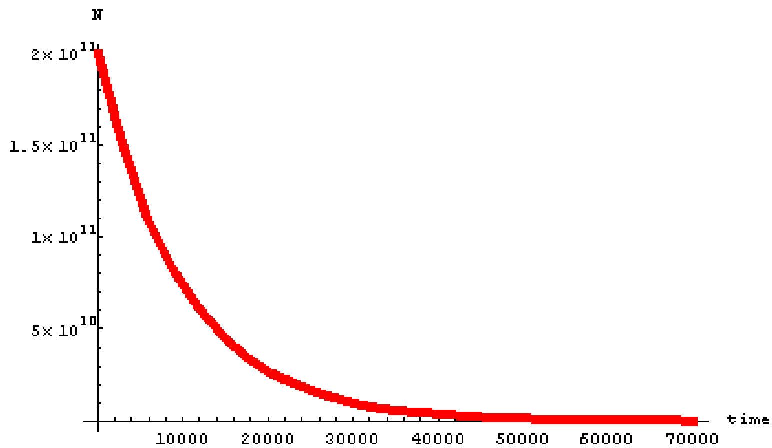
$$-\frac{dN}{dt} = \lambda N \tag{30}$$

This equation may be integrated to yield:

$$N = N_0 e^{-\lambda t} \tag{31}$$

This is known as the ***radioactive decay law****.*

Observe this relation graphically.



The ***half-life*** *of a material has two equally valid definitions.*

(1) The average time taken for the number of undecayed nuclei to halve in value.

(2) The average time taken for the activity to halve.

The half-life, $t_{1/2}$, is defined, according to (1), as $N = \frac{N_0}{2}$. If this is substituted for N in equation (31) then t represents $t_{1/2}$. Taking logs and rearranging yields:

$$t_{1/2} = \frac{ln2}{\lambda} \tag{32}$$

In order to get a clearer picture of what is happening equations (30) and (31) can be combined to give:

$$-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} \tag{33}$$

Which demonstrates that the activity ($-dN/dt$) decreases exponentially with time.

Using equation (32) a value can be obtained for λ

$$\lambda = \frac{ln2}{t_{1/2}} = \frac{0.693}{5.7 \times 10^3} = 1.22 \times 10^{-4}\ y^{-1} \tag{34}$$

The initial activity is therefore

$$\lambda N_0 = 2.44 \times 10^7 y^{-1} = \ 46.4 min^{-1} \tag{35}$$

In order to determine the age of the specimen we must rearrange equation (31) in order to obtain $t = ln\frac{N_0}{N_1}$. And therefore

$$t = \frac{1}{1.22 \times 10^{-4}} ln\frac{46.4}{2.1} = 6500\ years \tag{36}$$

3 Question 5 : Total 11 marks

(a) 2 marks In order to calculate the resistance of the track if undamaged we must first find the length and area of track track between P and S.

Length

$$l = 2 \times \frac{270}{360} = 7.1 \times 10^{-2}\ meters \tag{37}$$

Area $= 6 * 10^{-3}\ m \times 1.5 * 10^{-3}\ m$

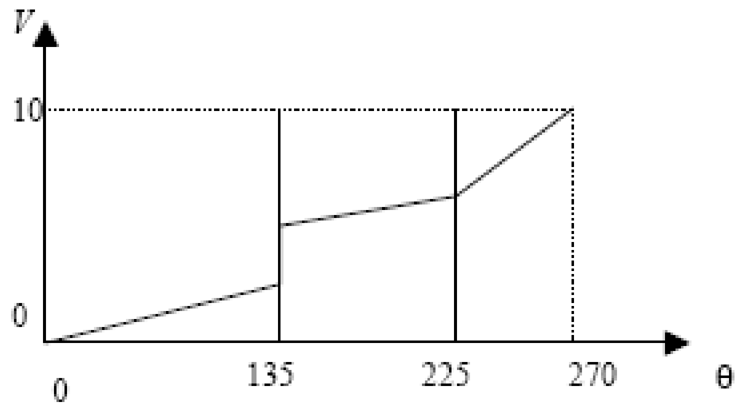
Now using the formulae for resistivity we obtain

$$R = \frac{\rho \, l}{A} = 3.9 \times 10^2 \Omega \tag{38}$$

(b) 9 marks

The important thing to realise in this question is that as the battery is 'perfect' , i.e. it has no internal resistance, then the initial potential at P will be zero and the final potential at S will be 10V. Also the resistance of a piece of conductive material is inversely proportional to its area. The larger the area the conductive or less resistive the material is.

This should enable a graph like the following to be drawn.



Graph showing V increasing with θ with features due to dimensions of carbon track.

Important features are:

- The graph starts at zero.
- Linear increase until135 ° .
- Almost vertical step at 135 ° as resistance increases sharply.
- Similar linear increase as before (track is made of same material) until 225 °.
- Straight line of greater slope until 270 ° as the track has a greater resistance due to a smaller cross section.
- Final potential is 10V.

4 Question 6 : Total 11 marks

These questions are designed to test your skill at deductive reasoning and estimation. (As well as basic physics!)

(a) 5 marks

The key to answering this question successfully is to calculate the work done and power used in order to obtain the time ($time = \frac{work\ done}{power}$). The power from a domestic mains power supply can be reasonably estimated to be 2kW. In order to calculate the work done we must firstly obtain a reasonable estimate for the combined masses of both the engineer and hoist. 150kg would be reasonable, consult the Mark Scheme !make into a link! in order to find out the boundaries allowed on this answer, you will find they are quite broad. Now an estimate of the height of a lamp post of around 8m is required. We are now able to calculate the change in potential energy ($PE = mgh$) which gives the following answer.

$$PE = mgh = 150 \times g \times 8 = 12 \times 10^3 J \tag{39}$$

and therefore the time is found to be

$$t = \frac{12 \times 10^3}{2 \times 10^3} = 6s \tag{40}$$

(b) 6 marks

For this question the diameter of a turbine must be estimated. A reasonable answers would range between $10m$ to $66m$.

Wind turbines convert the kinetic energy (KE) of the wind into electricity. The KE arriving per sec can be found through

$$KE = 1/2mv^2 = 1/2(\rho A\nu)\nu^2 = 13 \times 10^4 W \ to \ 2MW \tag{41}$$

Therefore depending on the diameter of turbine used, the number of turbines should be between 15 and 1.

Some wind turbines.