Advanced Extension Award: Answers to 2002 paper

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Abstract

Contents

1 Question 1 : Total 21 marks

This question tests the following from AEA specification:

- 1.3.1 Momentum concepts
- 1.3.2 Energy concepts
- 1.5.1 Probing matter
- 1.5.2 Ionising radiation
- 1.6.1 Photons
- 1.6.2 Matter
- 2.1 Analysing evidence and drawing conclusions
- 3.1 Arithmetic and computation
- 3.2 Handling data
- 3.3 Algebra
- 3.5 Graphs.

Relevant readings:

- Auger Electron Spectroscopy

(a) 1 marks
Atomic composition of the material making up the surface.
i.e. elements such as C, Fe, Cu, etc. They all have different characteristic energy levels.

(b) 2 marks
Elastic collision is a kind of interaction in which no kinetic energy is lost. In this case the momentum and the energy of electron and target atom can be computed by exploiting the conservation of these quantities.

In *inelastic* collisions, some kinetic energy goes into atomic or electronic excitation which causes all the phenomena that are under examination here. Of course total energy is always conserved but *kinetic* energy is not.

(c) 3 marks
A number of factors can be considered. Penetration of bombarded electrons should be limited to $\sim 10nm$ or the size of atoms. Crudely speaking, higher energy electrons penetrate deeper. This is because the cross section (or the probability) of electron scattering with the target is lower for higher energy particles.

Note: Strictly speaking the measurement is always under some uncertainties. One cannot guarantee that all the interactions occur in the first layer of atoms i.e. on the surface. This is also material dependent and calibration will be necessary.

(d) 6 marks

(i) The number (or better, the density) of a particular species can be related to the number of Auger electrons. The more atoms there are, the more likely the atom-electron interaction occur and hence, more Auger electrons. i.e.

$$No\_of\_Auger\_electrons \propto No\_of\_atoms \tag{1}$$

($\propto$ is better though = is acceptable)

(ii) **image needs to be inserted here from the marking scheme.
It is important that the plot stats and ends with negative G. Label Auger energy clearly.

(e) 6 marks

(i) From the diagram, $\Delta E = 3.00 \times 10^{-16} J$. Potential difference required can be obtained by $J = eV$ conversion, where e is the electron charge. Hence $V = 1875 Volts$.

(ii) $Energy\_released = \Delta E = E_{L_1} - E_K$
Reading off E_{L_1} and E_K from the diagram,

$$\Delta E = (-1.10 - (-3.00)) \times 10^{-16} = 1.90 \times 10^{-16} J. \tag{2}$$

(iii) The energy calculated in (ii) is used to knock off L_2 electron. So the energy of Auger electron is

$$\Delta E = (1.90 - 0.90) \times 10^{-16} = 1.00 \times 10^{-16} J. \tag{3}$$

(iv) The energy of the photon is calculated in (ii). To obtain the wavelength of this X-ray photon, use

$$E = hf \tag{4}$$

where h is the Planck constant and f is the frequency of the photon. Since

$$f = c/\lambda \tag{5}$$

where c is the speed of light and λ is the wavelength,

$$\lambda = \frac{hc}{E} \tag{6}$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{1.90 \times 10^{-16}} \tag{7}$$

$$= 1.05 \times 10^{-9} m \ (1.05 nm). \tag{8}$$

(f) 2 marks

The number of new case reported was initially zero and then increased, peaking at some point and then fall back to zero. Taking the derivative of this trend, i.e. "the increase of new cases", a plot similar to what was produced in **(d)(i)** can be made (starting point and end point are zero in this case.) One could deduce the conclusion that the outbreak had passed its most serious point by observing the decline in the rate of increase of new cases which is the first peak in the **(d)(i)** plot. At this point, the derivative of this plot (second derivative of total number reported and "the rate of increase of new cases") is zero although new cases were still reported at high figures.

2 Question 2 : Total 14 marks

This question tests the following:

- 1.7.1 Waves
- 2.1 Analysing evidence and drawing conclusions
- 2.2 Evaluating evidence and procedures
- 3.1 Arithmetic and computation
- 3.2 Handling data
- 3.3 Algebra
- 3.4 Geometry and trigonometry
- 3.5 Graphs

Relevant readings:

- none suggested

(a) 6 marks
Quantities of interest are $\Delta t_i = t_s truck - t_i^{detected}$ and $r_i = \frac{\Delta t_i}{v}$ where r_i is the distance from the point of impact, with i=A, B and C, and $v = 1.50 km/s$ is the speed of the wave. Tabulated values of these quantities are:

	Δt	r
A	61	91.5
B	75	112.5
C	111	166.5

One method of finding the point of impact is to draw circles of radius r from points A,B and C and the point of interception of these circles is the point in question.
*Diagram from marking scheme should be inserted here

Or more precise solution can be obtained analytically. i.e.

$$(x-100)^2 + (y-90)^2 = (91.5)^2 \tag{9}$$

$$(x-10)^2 + (y-40)^2 = (112.5)^2 \tag{10}$$

$$(x-120)^2 + (y-10)^2 = (166.5)^2 \tag{11}$$

10-9 gives

$$180x + 100y = 20684 \tag{12}$$

Similarly 10-11 gives

$$220x - 60y = -2266. \tag{13}$$

From the above, one obtains $x = 30.9km$ and $y = 151.2km$

In case graphical solution was attempted, the marks should be awarded to the answers with $\pm 10km$ from the above.

(b) 2 marks

Intensity of two-dimensional wave is measured by the amount of energy passing through unit length of circumference of circle centred on source per second. Let I_0 the power$(Energy/Time)$ at the source of the impact, the intensity I of the wave measured at a location at r km away from the source is,

$$I = \frac{I_0}{2\pi r} = \frac{K}{r}. \tag{14}$$

(c) 6 marks

The distance from the source can be calculated as in (a) and the intensity from (b). The following is the tabulated values of those quantities and are plotted in the figure below. **figure from marking scheme should be inserted below the table

	$\frac{1}{r}(10^{-3}km^{-1})$	I
A	10.9	15.7
B	8.9	12.7
C	6.0	8.1
D	4.7	6.1
E	3.6	4.3

As expected, straight line can be reasonably fit onto the points, although the line does not go through the origin. This means that the intensity will be negative at $r = \inf$ which is not physically acceptable. This may be explained by various reasons but "measurement error" is less likely candidate since all the points are almost on straight line. One of the better reasoning will be that the wave was weakened by attenuation due to some obstruction before it reached the sites which introduced extra term $(-\alpha)$ to equation 14.

3 Question 3 : Total 19 marks

This question tests the following:

- 1.3.2 Energy concepts
- 1.4.1 Current
- 1.4.2 Emf and potential difference
- 1.4.3 Resistance
- 1.4.4 DC circuits
- 3.1 Arithmetic and computation
- 3.3 Algebra

- 3.5 Graphs

Relevant readings:

- Resistors
- How voltage, current, and resistance relate
- Work
- Energy
- Mechanical Energy and Energy Conservation

(a) 3 marks

Power is defined as "rate of doing work". *Work* is defined as $Newton \times metre$, hence has the unit of Nm/s or J/s

Work done in moving charge q through a potential difference of V is qV ($Coulomb \times Volts = Joules$). Since current is defined as rate of flow of charge(Coulomb / Second), rate of doing work is IV (J/s).

Justification by dimensions only is acceptable.

(b) 6 marks

(i) Using $P = VI$ and $V = IR$, $P = V^2/R$. (Here, R is resistance in general and not the R specific to this question.) Total resistance is $R + r$ so,

$$P_{total} = \frac{\varepsilon^2}{R+r}. \tag{15}$$

Since power dissipated is proportional to the resistance,

$$P_{load} = P_{total} \times \frac{R}{R+r} = \frac{\varepsilon^2 \times R}{(R+r)^2} \tag{16}$$

(ii) From (i),

$$P_{load}/P_{total} = \eta = \frac{R}{R+r}. \tag{17}$$

At maximum power, the efficiency is,

$$\eta = \frac{R}{R+R} = 0.5. \tag{18}$$

Maximum possible efficiency is unity from the definition and this can be achieved by $R = \infty$ (in which case value of r is negligible) or $r = 0$. If $R = \infty$, then there will be no current and hence no power will be delivered. $r = 0$ implies no internal resistance in the source but this is not achievable. Hence theoretical maximum may not be reached.

(c) 10 marks

(i) The gravitational potential energy ($p.e.$) at a height h is $\frac{1}{2}mgh$. For an extended object, one can take the centre of mass and do the same as if all the mass is located at this point. Hence, at time t,

$$p.e.(t) = \frac{M}{L}yg\frac{y}{2} = \frac{1}{2}y^2\frac{M}{L}g. \tag{19}$$

Similarly, at $t + \Delta t$,

$$\begin{aligned} p.e.(t+\Delta t) &= \frac{M}{L}(y+\Delta y)g\frac{1}{2}(y+\Delta y) && (20)\\ &= \frac{Mg}{2L}(y+\Delta y)^2 && (21)\\ &= \frac{Mg}{2L}(y^2+2y\Delta y+(\Delta y)^2). && (22)\\ & && (23) \end{aligned}$$

Neglecting the small term $((\Delta t)^2)$,

$$p.e.(t+\Delta t) - p.e.(t) = \Delta E_pot = \frac{Mg}{L}y\Delta y \tag{24}$$

For kinetic energy($k.e.$), using $k.e. = \frac{1}{2}mv^2$, at time t,

$$k.e.(t) = \frac{M}{2L}yv^2 \tag{25}$$

and at time $t + \Delta t$

$$k.e.(t+\Delta t) = \frac{M}{2L}(y+\Delta y)v^2. \tag{26}$$

Hence,

$$\Delta E_kin = \frac{M}{2L}v^2\Delta y. \tag{27}$$

The power P supplied is the rate of increase of energy($p.e. + k.e.$), i.e.

$$\begin{aligned} P &= \frac{\Delta E_{pot} + \Delta E_{kin}}{\Delta t} && (28)\\ &= \frac{Mv}{L}(gy + \frac{1}{2}v^2) && (29) \end{aligned}$$

(ii) **Please put the diagram from marking scheme here.
From $t = 0$ to L/v (until the chain is fully lifted), the power increases due to the increase in kinetic energy, which is due to the amount of mass off the ground. At $t = L/v$, the increase in kinetic energy stops and there is a drop in power supplied. From then on, the power provided is only due to the increase in potential energy which is constant provided the speed of lifting stays constant (true in this case).

4 Question 4 : Total 9 marks

This question tests the following:

- 1.4.1 Current
- 1.4.2 Emf and potential difference

- 1.4.5 Capacitance
- 1.8.1 Force fields
- 3.1 Arithmetic and computation
- 3.2 Handling data
- 3.3 Algebra

Relevant readings:

- Capacitors

(a) 5 marks

(i) Note that $X_1 - X_2$ is the potential difference (or the voltage dropped across this capacitor) across C_1 and $Y_1 - Y_2$ is that across C_2. Since the capacitors are in series, the charge in C_1, Q_1 is equal to the charge in C_2, Q_2. Using $Q = CV$, $Q_2 = C_2(Y_1 - Y_2)$ and $Q_1 = Q_2$, $Q_1 = C_2(Y_1 - Y_2)$. Since the value of C_2 is known, Q_1 can be measured using the potential difference across this capacitor.

(ii) Using $Q_1 = C_1V_1$ and $Q_2 = C_2V_2$ and from $Q_1 = Q_2$, one can deduce

$$\frac{V_1}{V_2} = \frac{C_2}{C_1} \tag{30}$$

which implies that in case $V_1 \gg V_2$, $C_1 \ll C_2$

(b) 4 marks

(i) Using the relationship $E = \frac{V}{d}$, where E is the electric field, V is the voltage across the capacitor and d is the distance between the plates,

$$E = \frac{2}{50 \times 10^{-9}} = 4 \times 10^{7} Vm^{-1}. \tag{31}$$

One may comment on this as "very large" though this is largely dependent on context.

(ii) Using $Q = CV$ as given previously,

$$Q = 9 \times 10^{-12} \times 2 = 1.8 \times 10^{-11} C \tag{32}$$

the electrons on one side of the plate will move over to the other plate through the wire. The number of electron can be given by the total charge divided by the charge on one electron, i.e.

$$no\_of\_electron = \frac{1.8 \times 10^{-11}}{1.6 \times 10^{-19}} = 1.1 \times 10^{8} \tag{33}$$

5 Question 5 : Total 11 marks

This question tests the following:

- 1.2.1 Vectors
- 1.4.1 Current
- 1.4.4 DC circuits
- 1.9.1 B-fields
- 3.1 Arithmetic and computation
- 3.3 Algebra
- 3.4 Geometry and trigonometry

Relevant readings:

- Right Hand Rules
- Magnetic Field and Lorenz Force
- Force and Tension

(a) 4 marks
Magnitude of force on a current-carrying infinitesimal wire of length s, F is given by,

$$F = BIs. \tag{34}$$

The direction is given by using the "right hand rule" or more formally considering

$$\vec{F} = \vec{B} \wedge q\vec{v} \tag{35}$$

where $\vec{B}$ is pointing in to the paper and $\vec{v}$ is clockwise and hence the direction of the force is outwards from the centre of the loop. Therefore, this force applied to all parts of the wire, the wire will form a circle.

(b) 5 marks
*please insert the diagram from the marking scheme here. The tension T is the force applied in the direction of the wire. The above shows the part of the wire under consideration. The length of this part of wire is

$$s = r\theta \tag{36}$$

where r is the radius of the formed circle and θ is the corresponding angle. In this limit of small θ, $sin\theta \simeq \theta$. Tension is the component of the force along the direction of the wire. So using trigonometry to find the projection of F onto the direction of wire;

$$F = 2Tsin\frac{1}{2}\theta \simeq T\theta. \tag{37}$$

But from (a), $F = BIs = BIr\theta$. Hence

$$T = BIr = \frac{BIL}{2\pi}. \tag{38}$$

(c) 2 marks
Force is now in the other direction (inwards) and since the wire is very flexible, every part of the wire heads towards the opposite side of the circle to finally organize itself to be a circle again. The end of the loop will be twisted once. Though this is a more realistic answer, it is also possible provided that a perfect circle was formed first, the wire stays the same since the tension is exactly the same everywhere on the loop and hence will come to equilibrium without moving at all.

6 Question 6 : Total 14 marks

This question will test the following:

- 1.3.3 Molecular kinetic theory
- 3.1 Arithmetic and computation
- 3.2 Handling data
- 3.3 Algebra

Relevant readings:

- Unit Systems
- Choice of Units
- How to Change Units
- Ideal Gasses
- Pressure

(a) 5 marks
Remembering,

$$\begin{aligned} N &= kgms^{-2} && (39)\\ J &= Nm && (40)\\ W &= Js^{-1} && (41)\\ & && (42) \end{aligned}$$

$$\begin{aligned} m &= JN^{-1} && (43)\\ s &= JW^{-2} && (44)\\ kg &= Ns^2m && (45)\\ &= NJ^2W^{-2}J^{-1}N && (46)\\ &= N^2JW^{-2} && (47) \end{aligned}$$

This unit system certainly is usable to describe mechanical system but it is difficult to use since standard units of energy, force and power, even when properly defined, can not be easily reproduced. Not like trying to measure something with ruler but one may need a whole apparatus to make measurements of force etc.

The suggested scheme of using mass, volume and density is not suitable as mechanical units since it is essential to have a unit of time to describe mechanical systems and none of the three is related to such quantity.

(b) 9 marks

(i) Considering a car of one ton, the load is $10^3 \times 9.8 \sim 10^4 N$ and one wheel supports $\sim 0.25 \times 10^4 N$. Supposing the air pressure inside car tyre is $2atm \simeq 2 \times 10^5 Pa$, and using $p = \frac{F}{A}$,

$$A \sim \frac{0.25 \times 10^4}{2 \times 10^5} \sim 1 \times 10^{-2}. \quad (48)$$

Answer in range $0.5 \times 10^{-2} m^2$ to $2 \times 10^{-2} m^2$ is acceptable. No more than 2 s.f. should be used, this is an order-of-magnitude estimate.

(ii) Typical human lung capacity is 0.5 - 5 liters. Chose $1\ litre = 1 \times 10^3 m^3$ (good choice, this will simplify the calculation later.) Using $pV = nRT$, where $p = 1atm \simeq 10^5 Pa$, $T \simeq 300K$,

$$\begin{aligned} n &= \frac{pV}{RT} && (49) \\ &\simeq \frac{10^5 \times 10^{-3}}{8.3 \times 300} && (50) \\ &\simeq 4.0 \times 10^{-2} mol && (51) \\ &\simeq 4.0 \times 10^{-2} \times 6.0 \times 10^{23} && (52) \\ &= 2.4 \times 10^{22} molecules && (53) \end{aligned}$$

Answer in range 1×10^{22} to 1×10^{23} is acceptable. Same rule for significant figures apply.

7 Question 7 : Total 12 marks

This question tests the following:

- 1.2.3 Dynamics
- 1.3.3 Molecular kinetic theory
- 2.1 Analysing evidence and drawing conclusions
- 3.2 Handling data

Relevant readings:

- none suggested

The following is a possible answer to the question.

Since the balloon is filled with less dense gas than air, typically helium, the balloon will acquire upwards force and initially accelerate. This acceleration will be obstructed by the air friction and may also be affected by the upward or downward motion of the wind. The disturbance from friction is dependent on the speed of the balloon and at one point the acceleration will come to a halt when the accelerating force and frictional force reach equilibrium. Also, the speed of the balloon will be decreased by the change of the surrounding pressure as it goes up and less upthrust will be felt by the balloon and it will finally cease to rise.

The size of the balloon is dependent on the difference between the pressure inside the balloon and outside. As the balloon rises to less dense atmosphere and since the balloon is made of rubber, it will expand and the force on the balloon surface will increase.

The gas inside balloon consists of large number of small particles. These particles are moving randomly and their speed depends on the temperature and subsequently this will reduce the pressure inside the balloon.

Overall, these points are interrelated and precise motion of the balloon is hard to determine without more detailed investigations.