

## AEA 2001 Specimen

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1. Find

$$\int x^2 (\ln x)^2 dx \quad (6)$$

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2. Given that  $a > -3$ , find the value of  $a$  such that

$$\int_{-3}^a \frac{x}{\sqrt{4+x}} dx = \frac{22}{3} \quad (6)$$

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3. Solve for values of  $x$ , in degrees, in the range  $0 \leq x \leq 360$ ,

$$5 \sin 2x + 2 \cos x(2 + \cos 2x) = 0 \quad (12)$$

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4. The following argument claims to show that  $1 = 9$ .

$$\cos^2 x = 1 - \sin^2 x$$

then  $1 + \cos x = 1 + \sqrt{1 - \sin^2 x}$

squaring  $[1 + \cos x]^2 = [1 + \sqrt{1 - \sin^2 x}]^2$

when  $x = 2\pi/3$   $[1 - 1/2]^2 = [1 + \sqrt{1 - 3/4}]^2$

$$1/4 = [1 + \sqrt{1/4}]^2$$

$$1 = 9.$$

- (a) Explain carefully what is wrong with this argument. (3)
- (b) Rewrite the argument to show clearly how  $(1 + \cos x)^2$  can be written in terms of  $\sin x$  for values of  $x$  in the range  $0 \leq x \leq 2\pi$ . (6)

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5. (a) Simplify

(i)  $\sin 7x \cos x + \sin x \cos 7x$ , (1)

(ii)  $\sin 7x \cos x - \sin x \cos 7x$ . (1)

(b) Find expressions, in terms of  $r$ , for  $P$  and  $Q$  so that

$$2 \sin x \cos(2r-1)x = \sin Px - \sin Qx. \quad (4)$$

(c) Prove that for positive integers  $n$ ,

$$\sin 2nx = 2 \sin x \sum_{r=1}^n \cos[(2r-1)x]. \quad (5)$$

(d) Solve, for  $0 < x < \pi$ , the equation

$$\cos x(\cos x + \cos 3x + \cos 5x + \cos 7x) = \frac{1}{2} \cot x. \quad (6)$$

(e) Find the exact value of

$$\int_{\pi/6}^{\pi/3} \frac{\sin 6x}{\sin x} dx. \quad (5)$$

- (f) State, giving a reason, what value  $\frac{\sin 2nx}{\sin x}$  takes as  $x \rightarrow 0$ . (2)

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6.

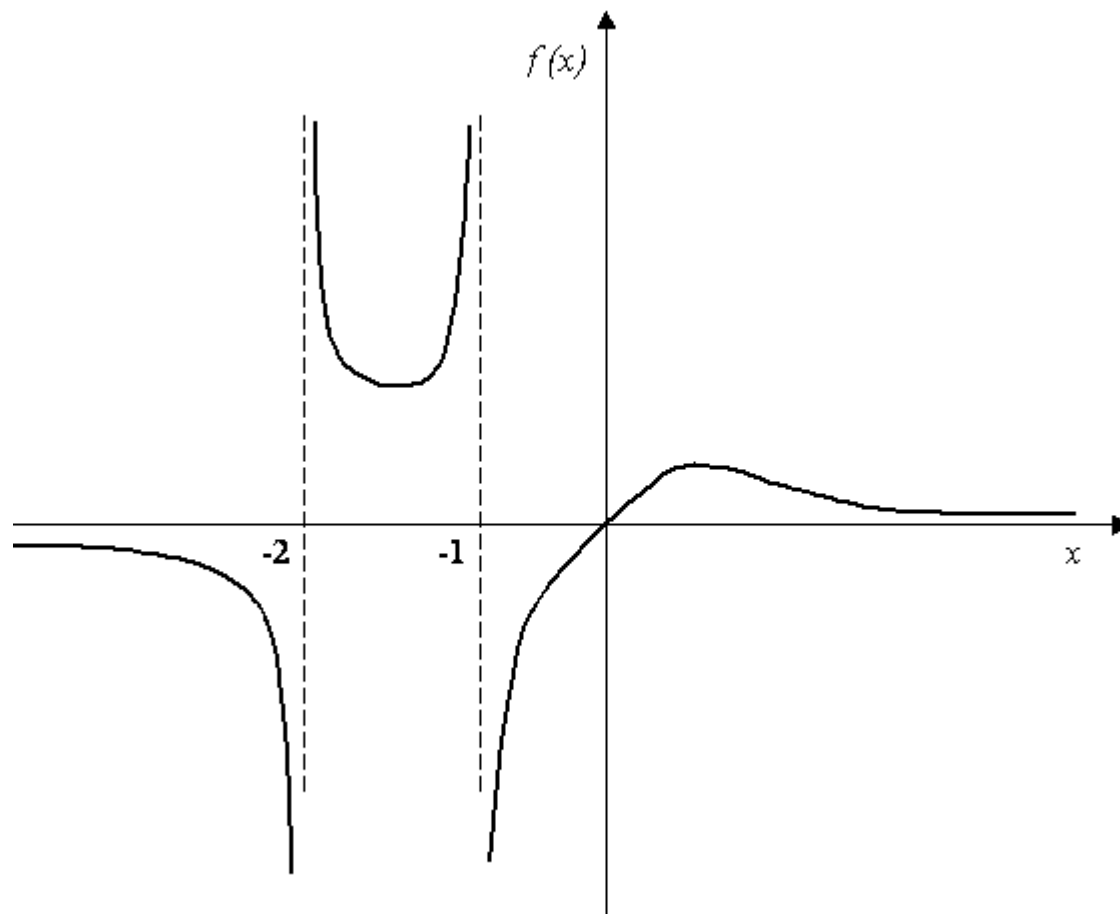


Figure 1 shows a sketch of the graph  $f$ , where

$$f(x) = \frac{x}{(x+1)(x+2)}, \quad x \in \mathbb{R}, \quad x \neq -1, \quad x \neq -2.$$

- (a) Find the exact values of the coordinates of the stationary points of  $f$ . Your answers should be in the form  $p + q\sqrt{r}$  where  $p$ ,  $q$ , and  $r$  are integers. (4)
- (b) Find the range of values of  $k$  for which  $f(x) = k$  has no real roots. (2)
- (c) Find the value of  $a$  such that

$$\int_a^{2a} f(x) dx = \ln 2. \quad (5)$$

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