AEA 2001 Specimen

1. Find

$$\int x^2 (\ln x)^2 \, dx \qquad \textbf{(6)}$$

Hints Short answer Full working

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2. Given that $a > -3$, find the value of a such that

$$\int_{-3}^{a} \frac{x}{\sqrt{4+x}} \, dx = \frac{22}{3} \qquad \textbf{(6)}$$

Hints Short answer Full working

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3. Solve for values of x, in degrees, in the range $0 \le x \le 360$,

$$5\sin 2x + 2\cos x(2 + \cos 2x) = 0 \qquad \textbf{(12)}$$

Hints Short answer Full working

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4. The following argument claims to show that 1 = 9.

$$\cos^2 x = 1 - \sin^2 x$$

then $$1 + \cos x = 1 + \sqrt{1 - \sin^2 x}$$

squaring $$[1 + \cos x]^2 = \left[1 + \sqrt{1 - \sin^2 x}\right]^2$$

when $x = 2\pi/3$ $$\left[1 - \tfrac{1}{2}\right]^2 = \left[1 + \sqrt{1 - \tfrac{3}{4}}\right]^2$$

$$\tfrac{1}{4} = \left[1 + \sqrt{\tfrac{1}{4}}\right]^2$$

$$1 = 9.$$

(*a*) Explain carefully what is wrong with this argument. **(3)**

(*b*) Rewrite the argument to show clearly how $(1+\cos x)^2$ can be written in terms of $\sin x$ for values of x in the range $0 \le x \le 2\pi$. **(6)**

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5. (*a*) Simplify

(*i*) $\sin 7x \cos x + \sin x \cos 7x$, **(1)**

(*ii*) $\sin 7x \cos x - \sin x \cos 7x$. **(1)**

(*b*) Find expressions, in terms of r, for P and Q so that

$$2\sin x \cos(2r-1)x \;=\; \sin Px - \sin Qx.$$ **(4)**

(*c*) Prove that for positive integers n,

$$\sin 2nx \;=\; 2\sin x \sum_{r=1}^{n} \cos[(2r-1)x].$$ **(5)**

(*d*) Solve, for $0 < x < \pi$, the equation

$$\cos x(\cos x + \cos 3x + \cos 5x + \cos 7x) \;=\; \tfrac{1}{2}\cot x.$$ **(6)**

(*e*) Find the exact value of

$$\int_{\pi/6}^{\pi/3} \frac{\sin 6x}{\sin x}\,dx.$$ **(5)**

(*f*) State, giving a reason, what value $\frac{\sin 2nx}{\sin x}$ takes as $x \to 0$. **(2)**

Hints Short answer Full working

6.

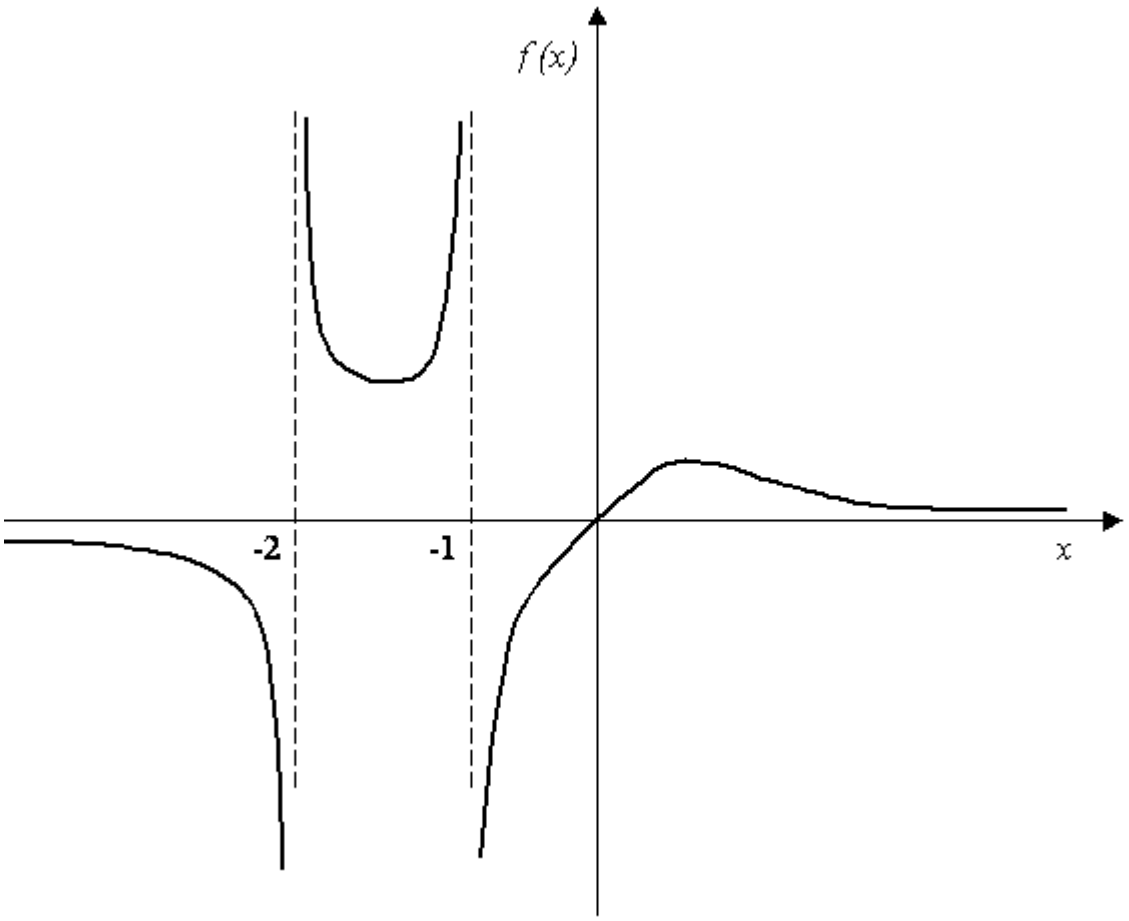

Figure 1 shows a sketch of the graph *f*, where

$$f(x) = \frac{x}{(x+1)(x+2)}, \quad x \in \mathbb{R}, \ x \neq -1, \ x \neq -2.$$

(*a*) Find the exact values of the coordinates of the stationary points of *f*. Your answers should be in the form $p + q\sqrt{r}$ where *p*, *q*, and *r* are integers. **(4)**

(*b*) Find the range of values of *k* for which *f*(*x*) = *k* has no real roots. **(2)**

(*c*) Find the value of *a* such that

$$\int_{a}^{2a} f(x)\,dx = \ln 2.$$

(5)

Hints Short answer Full working