AEA

Specimen Paper and Mark Scheme

Edexcel Advanced Extension Award Mathematics (9801)

For First Examination

Summer 2002

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Paper Reference(s)

9801

Mathematics

Advanced Extension Award

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may NOT use calculators in answering this paper.

Instructions to Candidates

Full marks may be obtained for answers to ALL questions.
In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title, the paper reference (9801), your surname, other names and signature.

Information for Candidates

A booklet 'Mathematical Formulae including Statistical Formulae and Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

Printer's Log No.

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Turn over

1. (a) By considering the series

$$1 + t + t^2 + t^3 + \ldots + t^n,$$

or otherwise, sum the series

$$1 + 2t + 3t^2 + 4t^3 + \ldots + nt^{n-1}$$

for $t \neq 1$.

(5)

(b) Hence find and simplify an expression for

$$1 + 2\times 3 + 3\times 3^2 + 4\times 3^3 + \ldots + 2001\times 3^{2000}.$$

(1)

(c) Write down an expression for both the sums of the series in part (a) for the case where $t = 1$.

(1)

---

2. Given that $S = \int_0^{\frac{\pi}{2}} e^{2x} \sin x \, dx$ and $C = \int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx$,

(a) show that $S = 1 + 2C$,

(3)

(b) find the exact value of S.

(6)

---

3. Solve for values of θ, in degrees, in the range $0 \leq \theta \leq 360$,

$$\sqrt{2} \times (\sin 2\theta + \cos \theta) + \cos 3\theta = \sin 2\theta + \cos\theta.$$

(12)

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4. A curve C has equation $y = \mathrm{f}(x)$ with $\mathrm{f}'(x) > 0$. The x-coordinate of the point P on the curve is a. The tangent and the normal to C are drawn at P. The tangent cuts the x-axis at the point A and the normal cuts the x-axis at the point B.

(a) Show that the area of ΔAPB is

$$\frac{1}{2}[\mathrm{f}(a)]^2\left(\frac{[\mathrm{f}'(a)]^2+1}{\mathrm{f}'(a)}\right).$$

(8)

(b) Given that $\mathrm{f}(x) = \mathrm{e}^{5x}$ and the area of ΔAPB is e^{5a}, find and simplify the exact value of a.

(4)

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5. The function f is defined on the domain [–2, 2] by:

$$\mathrm{f}(x)=\begin{cases}-kx(2+x) & \text{if } \ -2\le x<0,\\ \ \ kx(2-x) & \text{if } \quad 0\le x\le 2,\end{cases}$$

where k is a positive constant.

The function g is defined on the domain [–2, 2] by $\mathrm{g}(x)=(2.5)^2-x^2$.

(a) Prove that there is a value of k such that the graph of f touches the graph of g.

(8)

(b) For this value of k sketch the graphs of the functions f and g on the same axes, stating clearly where the graphs touch.

(4)

(c) Find the exact area of the region bounded by the two graphs.

(5)

---

6. Given that the coefficients of x, x^2 and x^4 in the expansion of $(1+kx)^n$, where $n \geq 4$ and k is a positive constant, are the consecutive terms of a geometric series,

(a) show that $k = \dfrac{6(n-1)}{(n-2)(n-3)}$.

(6)

(b) Given further that both n and k are positive integers, find all possible pairs of values for n and k. You should show clearly how you know that you have found all possible pairs of values.

(5)

(c) For the case where $k = 1.4$, find the value of the positive integer n.

(2)

(d) Given that $k = 1.4$, n is a positive integer and that the first term of the geometric series is the coefficient of x, estimate how many terms are required for the sum of the geometric series to exceed 1.12×10^{12}. [You may assume that $\log_{10} 4 \approx 0.6$ and $\log_{10} 5 \approx 0.7$.]

(5)

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7. The variable y is defined by

$$y = \ln(\sec^2 x + \operatorname{cosec}^2 x) \quad \text{for} \quad 0 < x < \tfrac{\pi}{2}.$$

A student was asked to prove that

$$\frac{dy}{dx} = -4\cot 2x.$$

The attempted proof was as follows:

$$\begin{aligned} y &= \ln(\sec^2 x + \operatorname{cosec}^2 x) \\ &= \ln(\sec^2 x) + \ln(\operatorname{cosec}^2 x) \\ &= 2\ln\sec x + 2\ln\operatorname{cosec} x \\ \frac{dy}{dx} &= 2\tan x - 2\cot x \\ &= \frac{2(\sin^2 x - \cos^2 x)}{\sin x \cos x} \\ &= \frac{-2\cos 2x}{\frac{1}{2}\sin 2x} \\ &= -4\cot 2x \end{aligned}$$

(a) Identify the error in this attempt at a proof.

(1)

(b) Give a correct version of the proof.

(5)

(c) Find and simplify a general relationship between p and q, where p and q are variables that depend on x, such that the student would obtain the correct result when differentiating $\ln(p + q)$ with respect to x by the above incorrect method.

(8)

(d) Given that $p(x) = k \sec rx$ and $q(x) = \operatorname{cosec}^2 x$, where k and r are positive integers, find the values of k and r such that p and q satisfy the relationship found in part (c).

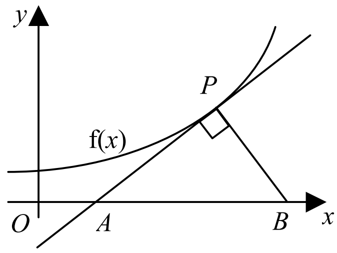
(4)

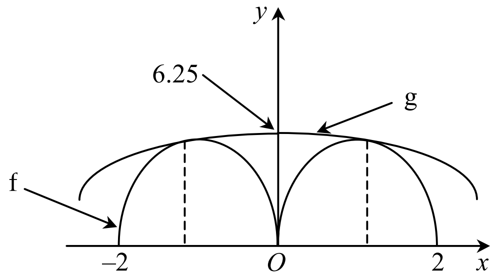
END

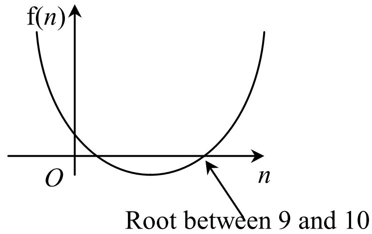
Marks for presentation: 7
TOTAL MARKS: 100

ADVANCED EXTENSION AWARD in MATHEMATICS
SPECIMEN PAPER – MARK SCHEME

No		Working	Marks
1	(a)	G.P. $a = 1, r = t,\quad S_n = \dfrac{t^{n+1}-1}{t-1}$	M1 A1
		$S = 1 + 2t + \ldots + nt^{n-1} = \dfrac{\mathrm{d}}{\mathrm{d}t}$ (1st series)	M1
		$\dfrac{\mathrm{d}}{\mathrm{d}t}(S_n) = \dfrac{(t-1)(n+1)t^n - (t^{n+1}-1).1}{(t-1)^2}$ Quotient rule	M1 A1
		$= \dfrac{nt^{n+1} - (n+1)t^n + 1}{(t-1)^2}$	(5)
	(b)	$t = 3, n = 2001$ $\Rightarrow \text{Sum} = \dfrac{2001.3^{2002} - 2002.3^{2001} + 1}{2^2} = \dfrac{4001.3^{2001} + 1}{4}$	B1 (1)
	(c)	$1 + t + \ldots + t^{n+1} \to n + 1$; $1 + 2t + \ldots + nt^{n-1} \to \dfrac{n(n+1)}{2}$	B1 (both) (1) **(7)**
2	(a)	$S = \int_0^{\frac{\pi}{2}} \mathrm{e}^{2x}\ \mathrm{d}(-\cos x)\mathrm{d}x = \left[-\cos x\, \mathrm{e}^{2x}\right]_0^{\frac{\pi}{2}} + 2\int_0^{\frac{\pi}{2}} \mathrm{e}^{2x} \cos x\ \mathrm{d}x$	M1 A1 (ignore limits)
		$S = (-0) - (-1) + 2C$ or $1 + 2C$	A1 cso (3)
	(b)	$C = \int_0^{\frac{\pi}{2}} \mathrm{e}^{2x}\mathrm{d}(\sin x) = \left[\sin x\mathrm{e}^{2x}\right]_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} \mathrm{e}^{2x} \sin x\mathrm{d}x$	M1 A1 (ignore limits)
		$C = (\mathrm{e}^{\pi}) - (0) - 2S$ i.e. $C = \mathrm{e}^{\pi} - 2S$	A1
		Solving $S = 1 + 2(\mathrm{e}^{\pi} - 2S)$	M1
		$= 1 + 2\mathrm{e}^{\pi} - 4S$	
		$5S = 1 + 2\mathrm{e}^{\pi}$ and $S = \dfrac{1}{5}(1 + 2\mathrm{e}^{\pi})$	M1, A1 (6) **(9)**

No	Working	Marks
3	$\left(\sqrt{2}-1\right)\left(\sin 2\theta+\cos\theta\right)+\cos 3\theta=0$	M1
	$\left(\sqrt{2}-1\right)\cos\theta\left(1+2\sin\theta\right)+\cos 3\theta=0$	
	$\cos 3\theta\equiv\cos 2\theta\cos\theta-\sin 2\theta\sin\theta$	M1
	$\equiv\cos\theta\left[\cos 2\theta-2\sin^2\theta\right]$	M1
	$\equiv\cos\theta\left[1-4\sin^2\theta\right]$	A1
	So: $\cos\theta\left(\left(\sqrt{2}-1\right)\left(1+2\sin\theta\right)+\left(1-2\sin\theta\right)\left(1+2\sin\theta\right)\right)=0$	M1
	i.e. $\cos\theta\left(1+2\sin\theta\right)\left(\sqrt{2}-1+1-2\sin\theta\right)=0$	M1
	i.e. $\sqrt{2}\cos\theta\left(1+2\sin\theta\right)\left(1-\sqrt{2}\sin\theta\right)=0$	M1
	$\cos\theta=0\Rightarrow\theta=90°,\ 270°$	A1 (both)
	$1+2\sin\theta=0\Rightarrow\sin\theta=-\frac{1}{2}\Rightarrow\theta=210°,\ 330°$	A1, A1
	$1-\sqrt{2}\sin\theta=0\Rightarrow\sin\theta=\frac{1}{\sqrt{2}}\Rightarrow\theta=45°,\ 135°$	A1, A1 **(12)**
4 (a)	 Equation of tangent: $y-f(a)=f'(a)(x-a)$	M1
	$\Rightarrow A$ is $\left(a-\dfrac{f(a)}{f'(a)},0\right)$	A1
	Gradient of normal is $-\dfrac{1}{f'(a)}$;	M1
	Equation of normal: $y-f(a)=-\dfrac{1}{f'(a)}(x-a)$	M1
	$\Rightarrow B$ is $\left(a+f(a)f'(a),0\right)$	A1
	Area ΔAPB is $\frac{1}{2}AB.f(a)=\frac{1}{2}f(a)\left[f(a)f'(a)+\dfrac{f(a)}{f'(a)}\right]$	M1, A1
	$=\frac{1}{2}\left[f(a)\right]^2\left(\dfrac{\left[f'(a)\right]^2+1}{f'(a)}\right)$	A1 c.s.o. (8)
(b)	$f'(a)=5e^{5a}$ $\quad\therefore e^{5a}=\frac{1}{2}\left(e^{5a}\right)^2\left(\dfrac{25\left(e^{5a}\right)^2+1}{5e^{5a}}\right)$	M1, A1
	$10\left(e^{5a}\right)^2=\left(e^{5a}\right)^2\left(25e^{10a}+1\right)$	
	$e^{10a}=\frac{9}{25}\Rightarrow a=\frac{1}{10}\ln\left(\frac{9}{25}\right)=\frac{1}{5}\ln\frac{3}{5}$	M1 A1 c.s.o. (4) **(12)**

No		Working	Marks
5	(a)	By symmetry consider RHS: $\frac{25}{4} - x^2 = 2kx - kx^2$	M1
		i.e. $(k-1)x^2 - 2kx + \frac{25}{4} = 0$	M1
		Equal roots $\Rightarrow$ $4k^2 = 4 \times \frac{25}{4} \times (k-1)$	M1
		i.e. $4k^2 - 25k + 25 = 0$	A1
		i.e. $(4k-5)(k-5) = 0$	M1
		i.e. $k = 5$ or $\frac{5}{4}$	A1
		$x = \frac{-b}{2a} = \frac{2k}{2(k-1)} = \frac{k}{k-1}$ and $\because x < 2$, we need $k = 5$	M1, A1 (8)
	(b)	 Symmetric touch × 2 touch at $\left(\pm\frac{5}{4}, \frac{75}{16}\right)$	B1 B1 B1, B1 (4)
	(c)	Area $= 2 \times \int_0^{\frac{5}{4}} \left(6.25 - x^2 - \left[10x - 5x^2\right]\right) \mathrm{d}x$	M1
		$= 2\left[6.25x - 5x^2 + \frac{4}{3}x^3\right]_0^{\frac{5}{4}}$	M1 A1
		$= 2\left[\left(\frac{25}{4} \times \frac{5}{4} - 5 \times \frac{25}{16} + \frac{4}{3} \times \frac{125}{64}\right) - (0)\right]$	M1
		$= 2 \times \frac{125}{48} = \frac{125}{24} = 5\frac{5}{24}$	A1 (5) **(17)**

No	Working	Marks
6 (a)	$(1+kx)^n = 1+nkx+\frac{n(n-1)}{2}k^2x^2+\ldots+\frac{n(n-1)(n-2)(n-3)}{4!}k^4x^4+\ldots$	M1 A2/1/0
	Let r = ratio of geometric series $r = \frac{n(n-1)}{2}\times\frac{k^2}{nk} = \frac{n(n-1)(n-2)(n-3)k^4\times 2}{4!\ n(n-1)k^2}$	M1
	i.e. $\frac{(n-1)k}{2} = \frac{(n-2)(n-3)k^2}{12}$ i.e. $k = \frac{6(n-1)}{(n-2)(n-3)}$	M1, A1 cso (6)
(b)	$n=4 \Rightarrow k = \frac{6.3}{2.1} = 9$; $n=5 \Rightarrow k = \frac{6.4}{3.2} = 4$	B1, B1
	$n = 6 \Rightarrow k = 2.5$, $n = 7 \Rightarrow k = 1.8$, $n = 8 \Rightarrow k = 1.4$, $n = 9 \Rightarrow k = \frac{8}{7}$	M1
	$k < 1 \Rightarrow 6n - 6 < n^2 - 5n + 6$ i.e. $0 < n^2 - 11n + 12$	M1
	So $k < 1\ \forall n \geq 10$ 	A1 (5)
(c)	$k = 1.4 \Rightarrow 7(n^2-5n+6) = 30n-30$ $7n^2-65n+72=0$	M1
	$(7n-9)(n-8)=0$ $\therefore n=8$ is the integer solution	A1 (2)
(d)	$k = 1.4$, $n = 8$, $a = nk = 11.2$, $r = \frac{(n-1)k}{2} = \frac{7}{2}\times\frac{7}{5} = 4.9$	B1, B1
	$S_m > 1.12\times10^{12} \Rightarrow \frac{11.2(4.9^m-1)}{3.9} > 1.12\times10^{12}$	M1
	$4.9^m - 1 > 3.9\times10^{11}$ $4.9^m > 3.9\times10^{11}+1 \approx 3.9\times10^{11}$	A1
	$m > \frac{\log 3.9 + 11}{\log 4.9} \approx \frac{11.6}{0.7} = 16.\ldots$ So need $m = 17$	A1 (5) **(18)**

No		Working	Marks
7	(a)	In line 1 the student assumes that ln $(a + b)$ = ln a + ln b [Although it can be shown that for these functions this is true, it is not a general statement.]	B1 (1)
	(b)	$y = \ln\left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right)$	
		$= \ln\left(\frac{1}{\cos^2 x \sin^2 x}\right)$	M1
		$= -2\ln(\sin x \cos x)$	M1
		$= -2\ln\frac{1}{2} - 2\ln(\sin 2x)$	M1
		$\therefore \frac{dy}{dx} = \frac{-2}{\sin 2x} \times 2\cos 2x = -4\cot 2x$	M1,A1 (5)
	(c)	Require $\frac{p'+q'}{p+q} = \frac{p'}{p} + \frac{q'}{q}$	M1, M1
		$= \frac{p'q + pq'}{pq}$	A1
		i.e. $\frac{d}{dx}[\ln(p+q)] = \frac{d}{dx}[\ln(pq)]$	M1, A1, A1
		$\ln(p+q) = \ln(pq) + \ln A$	M1
		$(p+q) = A(pq)$	A1 (8)
	(d)	$p(1 - Aq) = -q$	
		$p = \frac{q}{Aq - 1}$	M1
		$q = \text{cosec}^2 x \quad \Rightarrow \quad p = \frac{\text{cosec}^2 x}{A\,\text{cosec}^2 x - 1} \times \left(\frac{\sin^2 x}{\sin^2 x}\right)$	M1
		$p = k\sec rx \quad \Rightarrow \quad k\sec rx = \frac{1}{A - \sin^2 x}$	M1
		$A = \frac{1}{2} \quad \Rightarrow k\sec rx = \frac{2}{\cos 2x}$	
		$\therefore k = r = 2$	A1 (4) **(18)**

ADVANCED EXTENSION AWARD in MATHEMATICS
SPECIMEN PAPER – MARK SCHEME

Presentation

1 (a) 1 mark may be awarded for a good and largely accurate attempt at the whole paper.

1 (b) For a novel or neat solution to any question award, in each of up to 3 questions. 2 marks if the solution is fully correct and 1 mark if the solution is sound in principle but a minor algebraic or numerical slip occurs.

ADVANCED EXTENSION AWARD
IN
MATHEMATICS

GUIDANCE FOR CENTRES

INTRODUCTION

Advanced Extension Awards (AEAs) are being introduced for 18 year olds in England, Wales and Northern Ireland in the summer of 2002 in a range of subjects.

This Guidance is designed to provide information about the AEA in Mathematics to those who teach and advise Advanced level students. It incorporates much of what is included in the “Guidance for Students”.

The Guidance should be read in conjunction with the specimen examination paper and mark scheme for Mathematics. Together, these documents will help teachers to guide students in their preparation for the examination. The Guidance explains and amplifies key points in the specification and the examination paper.

WHAT ARE ADVANCED EXTENSION AWARDS?

AEAs are qualifications intended to challenge the most able GCE Advanced level (A level) students. They are designed to provide opportunities for candidates to demonstrate a greater depth of understanding than that required at A level and to demonstrate the ability to think critically and creatively. Consequently, AEAs will help to differentiate between the most able candidates, particularly in subjects with a high proportion of A grades at A level.

AEAs are based on A level subject criteria rather than individual specifications. They are intended to be accessible to all candidates who are expected to achieve a grade A in the A level for a particular subject, irrespective of where they have studied or which A level specification (syllabus) they have followed. They will not require any additional teaching or resources. It is intended that significantly more students should have the opportunity to take AEAs than take existing Special Papers but it is acceptable for centres to enter as many or as few candidates as they wish.

The requirements for the AEA in Mathematics are set out in the test specification drawn up by the regulatory authorities (QCA, ACCAC and CCEA). It has been agreed that there will be only one AEA examination for each subject. The examination for the AEA in Mathematics is administered by Edexcel but will be available to candidates in all centres. Representatives from each of the awarding bodies (AQA, CCEA, Edexcel, OCR and WJEC) will be involved in the process of setting examination papers and standards for the AEA in Mathematics.

THE STRUCTURE OF THE AEA EXAMINATION IN MATHEMATICS

There will be a single three-hour written examination. It will consist of about seven questions.

Questions may be multi-step with confidence building parts or unstructured. Some may be of an unusual nature that might include topics from GCSE and logic based items. Questions may be open-ended.

Seven percent of the marks will be assigned for style and clarity of mathematical presentation. The examiners will seek to reward elegance of solution, insight in reaching a solution, rigour in developing a mathematical argument and excellent use of notation.

Use of resources

The use of scientific or graphical calculators will NOT be allowed nor will computer algebra systems.

Candidates will be required to remember the same formulae as for GCE Advanced level Mathematics. They will also be expected to be familiar with the Mathematical Notation agreed for GCE Advanced level Mathematics.

HOW WILL CANDIDATES BE GRADED?

Assessment materials and mark schemes will lead to awards on a two-point scale: Distinction and Merit, with Distinction being the higher. Candidates who do not reach the minimum standard for Merit will be recorded as ungraded.

Performance level descriptors have been developed (see below) to indicate the level of attainment that is characteristic of Distinction and Merit. They give a general indication of the required learning outcomes at each level. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objective overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

Candidates who achieve **Distinction** will demonstrate understanding and command of most of the topics tested.

Candidates who achieve a **Merit** will demonstrate understanding and command of many of the topics tested.

Candidates:

- handle complex mathematical expressions accurately;
- exhibit insight and clarity of thought;
- adopt effective and imaginative mathematical strategies to produce logically coherent and elegant solutions to problems;
- set out formal proofs, generalise and pick out special cases;
- detect and correct faulty logic;
- cope with unfamiliar and unstructured questions.

ADVANCED EXTENSION AWARDS
IN
MATHEMATICS

GUIDANCE FOR STUDENTS

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WHAT IS THE AEA IN MATHEMATICS?

The content of the AEA in Mathematics is based on similar requirements to those for A level.

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Further copies of this publication are available from
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Edexcel
Success through qualifications