Pearson Edexcel

Mark Scheme (Results)

Summer 2022

Pearson Edexcel AEA
In Mathematics (9811) Paper 01

Summer 2022
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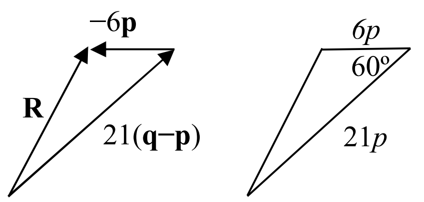
General Marking Guidance

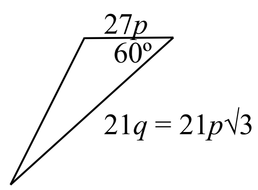
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question	Scheme		Marks	AOs
1	Writes $f(x) = e^{x^2 \ln x}$ or $y = x^{(x^2)} \Rightarrow \ln y = x^2 \ln x$	Applies ln (or log) in an appropriate manner	**B1**	2
	$f'(x) = \left(2x\ln x + \frac{x^2}{x}\right)e^{x^2 \ln x}$ or $\frac{1}{y}\frac{dy}{dx} = 2x\ln x + \frac{x^2}{x}$	Differentiates $x^2 \ln x$ applying the product rule	**M1**	1
	Fully correct differentiated expression (both sides) and must be in base e, though allow recovery from "log" if base e is implied later.		**A1**	2
	$f'(x) = 0 \Rightarrow 2x\ln x + x = 0 \Rightarrow \ln x = -\frac{1}{2} \Rightarrow x = ...$	Sets f'(x) or $\frac{dy}{dx} = 0$ and solves for x (allow any base).	**dM1**	2
	$x = \frac{1}{\sqrt{e}}$ oe	Correct answer.	**A1**	1
	Accept answers from a correct attempt at differentiating $x^2 \ln x$ even if the "1/y" was incorrect. Ignore reference to $x = 0$.			
			(5)	
			(Total 5 marks)	

Question	Scheme		Marks	AOs
2(a)	E.g. centre point is $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ so $\overrightarrow{OR} = 2\times(\overrightarrow{OQ} - \overrightarrow{OP})$; $\overrightarrow{OS} = \overrightarrow{OQ} - 2\overrightarrow{OP} + \overrightarrow{PQ}$; $\overrightarrow{OT} = \overrightarrow{OQ} - 2\overrightarrow{OP}$	Attempts to find at least one of the other vectors in terms of **p** and **q** with at least one correct expression (may be in direction vectors) for a key vector.	**M1**	3
	$\overrightarrow{OR} = 2(\mathbf{q} - \mathbf{p})$; $\overrightarrow{OS} = 2\mathbf{q} - 3\mathbf{p}$; $\overrightarrow{OT} = \mathbf{q} - 2\mathbf{p}$	Two correct ; all three correct	**A1;** **A1**	2;2
	$\mathbf{R} = \mathbf{p} + 2\mathbf{q} + 3\times 2(\mathbf{q} - \mathbf{p}) + 4(2\mathbf{q} - 3\mathbf{p}) + 5(\mathbf{q} - 2\mathbf{p}) = \ldots$	Attempts the sum using their expressions with at least one force involving both **p** and **q**	**M1**	1
	$= -27\mathbf{p} + 21\mathbf{q}$	Correct answer	**A1**	3
			(5)	
(b)	E.g. $\mathbf{R} = 21(\mathbf{q} - \mathbf{p}) - 6\mathbf{p}$ and $\mathbf{q} - \mathbf{p}$ and **p** have same length, *p* say (regular hexagon sides), with angle 60º between as per diagram (below)	Formulates a correct strategy – may be other ways, e.g finding length of **q** relative to **p** first.	**M1**	3
	So $\lvert\mathbf{R}\rvert^2 = (6p)^2 + (21p)^2 - 2(6p)(21p)\cos 60^\circ$ $\left(= (3p)^2\left(2^2 + 7^2 - 2\times 7\right) = (3p)^2 \times 39\right)$	Applies cosine rule to appropriate triangle	**M1**	3
	$\lvert\mathbf{R}\rvert^2 = 39(3p)^2$ or $\lvert\mathbf{R}\rvert = 3p\sqrt{39}$	Correct simplified magnitude or its square	**A1**	2
	$\Rightarrow 3p\sqrt{39} = 3\times 13 \Rightarrow p = \ldots$	Applies $\lvert\mathbf{F}\rvert = m\lvert\mathbf{a}\rvert$ to find *p. Must include attempt at modulus (may assume perpendicular vectors), not solving a linear equation.*	**M1**	3
	$p = \dfrac{13}{\sqrt{39}} = \dfrac{\sqrt{39}}{3}$	Correct answer, allow either form.	**A1**	2
			(5)	
			(Total 10 marks)	

Useful diagram for (b), using $\lvert\mathbf{p}\rvert = \lvert\mathbf{q} - \mathbf{p}\rvert = p$ or for alt on next page:





	Alternatives are possible to (b) and follow the pattern below.			
(b)	$\frac{\lvert\mathbf{q}\rvert}{\sin 120°}=\frac{p}{\sin 30°}\Rightarrow\lvert\mathbf{q}\rvert=p\sqrt{3}$ (oe method)	Formulates a correct strategy – e.g finding length of **q** relative to **p** first.	**M1**	3
	So $\lvert\mathbf{R}\rvert^2=(27p)^2+(21p\sqrt{3})^2-2(27p)(21p\sqrt{3})\cos 60°$ $\left(=3^3p^2(27+49-63)=(3p)^2\times 39\right)$ Or $\lvert\mathbf{R}\rvert^2=\mathbf{R}.\mathbf{R}=21^2\mathbf{q}.\mathbf{q}+27^2\mathbf{p}.\mathbf{p}-2\times 21\times 27\mathbf{q}.\mathbf{p}$ $=(21q)^2+(27p)^2-2\times 21\times 27\times pq\cos 30°$ Etc.	Applies cosine rule to appropriate triangle; Or may use dot product, though not on spec.	**M1**	3
	$\lvert\mathbf{R}\rvert^2=39(3p)^2$ or $\lvert\mathbf{R}\rvert=3p\sqrt{39}$	Correct simplified magnitude or its square	**A1**	2
	$\Rightarrow 3p\sqrt{39}=3\times 13\Rightarrow p=\ldots$	Applies $\lvert\mathbf{F}\rvert=m\lvert\mathbf{a}\rvert$ to find p *(same constraints)*	**M1**	3
	$p=\frac{13}{\sqrt{39}}=\frac{\sqrt{39}}{3}$	Correct answer, allow either form.	**A1**	2
			(5)	
	By resolving:			
	$\frac{\lvert\mathbf{q}\rvert}{\sin 120°}=\frac{p}{\sin 30°}\Rightarrow\lvert\mathbf{q}\rvert=p\sqrt{3}$ (oe method)	Formulates a correct strategy – e.g finding length of **q** relative to **p** first.	**M1**	3
	Let $\mathbf{p}=p\mathbf{i}$ and $\mathbf{q}=p\sqrt{3}\cos 30°\mathbf{i}+p\sqrt{3}\sin 30°\mathbf{j}=\frac{3p}{2}\mathbf{i}+\frac{p\sqrt{3}}{2}\mathbf{j}$ $\mathbf{R}=-27(p\mathbf{i})+21\left(\frac{3p}{2}\mathbf{i}+\frac{p\sqrt{3}}{2}\mathbf{j}\right)=\frac{9p}{2}\mathbf{i}+\frac{21p\sqrt{3}}{2}\mathbf{j}$	Sets up suitable axes and find both vectors in terms of just p and used them to find **R** (need not be simplified, accept any equivalent form).	**M1**	3
	$\lvert\mathbf{R}\rvert^2=\left(\frac{9p}{2}\right)^2+\left(\frac{21p\sqrt{3}}{2}\right)^2=\left(\frac{3p}{2}\right)^2\left(3^2+\left(7\sqrt{3}\right)^2\right)$ $=\frac{(3p)^2}{4}\times 156=39(3p)^2$ (so $\lvert\mathbf{R}\rvert=3p\sqrt{39}$)	Correct simplified magnitude or its square	**A1**	2
	$\Rightarrow 3p\sqrt{39}=3\times 13\Rightarrow p=\ldots$	Applies $\lvert\mathbf{F}\rvert=m\lvert\mathbf{a}\rvert$ to find p *(same constraints)*	**M1**	3
	$p=\frac{13}{\sqrt{39}}=\frac{\sqrt{39}}{3}$	Correct answer, allow either form.	**A1**	2
			(5)	

Question	Scheme		Marks	AOs
3(a)	$\tan(90^\circ-\theta)=\dfrac{\sin(90^\circ-\theta)}{\cos(90^\circ-\theta)}$	Writes in terms of sin and cos	**M1**	1
	$=\dfrac{\sin 90^\circ\cos\theta-\cos 90^\circ\sin\theta}{\cos 90^\circ\cos\theta+\sin 90^\circ\sin\theta}$	Applies both formulae (must be right signs).	**M1**	2
	$=\dfrac{\cos\theta-0}{0+\sin\theta}=\dfrac{\cos\theta}{\sin\theta}=\cot\theta$ *	Correct completion (S+ for anyone who investigates $k180°$)	**A1***	2
			(3)	
(b) Way 1	$2-\sec^2\left(\theta+11^\circ\right)=2\tan\left(\theta+11^\circ\right)\tan\left(\theta-34^\circ\right)$			
	$\Rightarrow\cot\left(\theta-34^\circ\right)=\dfrac{2\tan\left(\theta+11^\circ\right)}{2-\left(1+\tan^2\left(\theta+11^\circ\right)\right)}$	Applies either $\sec^2 x=1+\tan^2 x$ or rearranges to make $\cot\left(\theta-34^\circ\right)$ the subject	**M1**	3
	$\Rightarrow\cot\left(\theta-34^\circ\right)=\dfrac{2\tan\left(\theta+11^\circ\right)}{1-\tan^2\left(\theta+11^\circ\right)}=\tan\left(2(\theta+11^\circ)\right)$	Identifies tan 2t formula and replaces.	**M1**	3
	$\Rightarrow\tan\left(90^\circ-(\theta-34^\circ)\right)=\tan\left(2\theta+22^\circ\right)$	Uses result from (a) appropriately ; correct equation	**M1;** **A1**	3 2
	$124^\circ-\theta=2\theta+22^\circ\left(+k180^\circ\right)\Rightarrow\theta=...$ (S+ for good explanation of additional roots)	Solves for at least one value for θ	**dM1** **(S+)**	3
	$\theta=34^\circ,94^\circ,154^\circ,214^\circ,274^\circ,334^\circ$ Must be from correct work.	At least one correct. At least 3 correct All correct answers and no others.	**A1** **A1** **A1**	3 2 2
			(8)	
	S+ for appropriate substitution to make working easier.			
	Award S1 for: • a fully correct solution that is succinct but does not mention the S+ point • a solution scoring 9+ marks that may be laboured but includes the S+ point		**S1**	2
			(Total 11+1 marks)	

(b) Way 2	$2-\sec^2\left(\theta+11^\circ\right)=2\tan\left(\theta+11^\circ\right)\tan\left(\theta-34^\circ\right)$			
	$2\cos^2\left(\theta+11^\circ\right)-1=2\sin\left(\theta+11^\circ\right)\cos\left(\theta+11^\circ\right)\tan\left(\theta-34^\circ\right)$	Multiplies through by $\cos^2$.	**M1**	3
	$\cos 2\left(\theta+11^\circ\right)=\sin 2\left(\theta+11^\circ\right)\tan\left(\theta-34^\circ\right)$	Replaces with double angle formula	**M1**	3
	$\Rightarrow \tan\left(\theta-34^\circ\right)=\cot\left(2\theta+22^\circ\right)=\tan\left(90^\circ-(2\theta+22^\circ)\right)$	Uses result from (a) appropriately; correct equation	**M1;** **A1**	3 2
	$\theta-34^\circ=68^\circ-2\theta\left(+k180^\circ\right)\Rightarrow\theta=...$ (S+ for good explanation of additional roots)	Solves for at least one value for θ	**dM1** **(S+)**	3
	$\theta=34^\circ,94^\circ,154^\circ,214^\circ,274^\circ,334^\circ$ Must be from correct work.	At least one correct. At least 3 correct All correct answers no no extras	**A1** **A1** **A1**	3 2 2
			(8)	
(b) Way 3	$2-\sec^2\left(\theta+11^\circ\right)=2\tan\left(\theta+11^\circ\right)\tan\left(\theta-34^\circ\right)$			
	$2-\sec^2\left(\theta+11^\circ\right)=2\tan\left(\theta+11^\circ\right)\tan\left(\theta+11^\circ-45^\circ\right)$ $\Rightarrow 2-\left(1+\tan^2\left(\theta+11^\circ\right)\right)=2\tan\left(\theta+11^\circ\right)\dfrac{\tan\left(\theta+11^\circ\right)-\tan 45^\circ}{1+\tan\left(\theta+11^\circ\right)\tan 45^\circ}$	Applies either $\sec^2 x = 1+\tan^2 x$ or compound angle formulae on tan as shown.	**M1**	3
	$\Rightarrow 1-\tan^2\left(\theta+11^\circ\right)=2\tan\left(\theta+11^\circ\right)\dfrac{\tan\left(\theta+11^\circ\right)-1}{1+\tan\left(\theta+11^\circ\right)}$	Attempts both formulae and applies tan45°=1	**M1**	3
	$1-\tan\left(\theta+11^\circ\right)=0\Rightarrow\theta+11^\circ=45^\circ,225^\circ\Rightarrow\theta=...$	Identifies and solves this factor to $\theta=..$	**dM1**	2
	$\theta=34^\circ,214^\circ$ from correct work	Both values and no others from this equation.	**A1**	3
	Or $\left(1+\tan\left(\theta+11^\circ\right)\right)^2=-2\tan\left(\theta+11^\circ\right)\Rightarrow\tan\left(\theta+11^\circ\right)=$	Solves the remaining quadratic in tan	**dM1**	3
	$\tan\left(\theta+11^\circ\right)=-2\pm\sqrt{3}$ oe	Correct values for $\tan\left(\theta+11^\circ\right)$	**A1**	2
	$\theta=94^\circ,154^\circ,274^\circ,334^\circ$ from correct work	At least two correct. All four correct and no extras from this equation. Must be exact degree answers.	**A1** **A1**	3 2
			(8)	

Question	Scheme		Marks	AOs
4(a)	$f'(x)=\left(ax^2+b\right)e^{x^3-2x}$	Valid attempt at chain rule. (Allow b=0)	**M1**	1
	$f'(x)=\left(3x^2-2\right)e^{x^3-2x}$	Correct derivative	**A1**	1
			(2)	
(b)	$g(x)=h(x)\Rightarrow 8x^3e^{x^3-2x}=\left(3x^5+4x\right)e^{x^3-2x}$ $\Rightarrow 8x^3=3x^5+4x$	Equates and cancels or factorise out exponentials	**M1**	1
	$\Rightarrow x\left(3x^4-8x^2+4\right)=0\Rightarrow x\left(3x^2-2\right)\left(x^2-2\right)=0$	Factorises or equivalent. (May cancel the x)	**M1**	2
	($x>0$) (S+ reason for rejecting others) so $x=\sqrt{\frac{2}{3}}=\frac{\sqrt{6}}{3}$ or $x=\sqrt{2}$	A1 for one correct, A2 for both. (Allow if negatives not rejected.)	**A1** **A1** **(S+)**	1 1
			(4)	
(c)	Area $=\int_{\alpha}^{\beta}(8x^3-3x^5-4x)e^{x^3-2x}\,dx$	Allow either way round (limits may not be included yet) **must be combined.**	**M1**	1
	Integral $=(-)\int\left(x^3-2x\right).\left(3x^2-2\right)e^{x^3-2x}\,dx$ $=(-)\left[\left(x^3-2x\right)e^{x^3-2x}-\int\left(3x^2-2\right)e^{x^3-2x}\,dx\right]$	Identifies appropriate parts split and applies. (Limits may or may not be present.) A1 for correct	**M1** **A1**	3 3
	$=\pm\left(-\left(x^3-2x\right)e^{x^3-2x}+e^{x^3-2x}\right)$	Correct result after second integral	**A1**	3
	$\left[e^{x^3-2x}-\left(x^3-2x\right)e^{x^3-2x}\right]_{\frac{\sqrt{6}}{3}}^{\sqrt{2}}=\ldots$	Applies their limits from (b) either way round	**M1**	3
	$=(1)-\left(1-\frac{6\sqrt{6}}{27}+2\frac{\sqrt{6}}{3}\right)e^{\frac{6\sqrt{6}}{27}-2\frac{\sqrt{6}}{3}}$	Simplifies at least the 1 correctly following a correct expression. Must be correct sign by this stage.	**A1**	3
	$=1-\frac{(4\sqrt{6}+9)}{9}e^{-\frac{4\sqrt{6}}{9}}$ oe simplified	Correct in a simplified form. (S+ if any reasoning for correct sign given)	**A1** **(S+)**	3
			(7)	
	Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a fully correct solution that may be laboured but includes an S+ point • A succinct solution that scores 10+ marks that includes at least one S+ point.		**S1**	2

(Total 13+1 marks)			
S+ See notes in scheme.			
Alt for integration marks – may use substitution			
$u = x^3 - 2x \Rightarrow \frac{du}{dx} = 3x^2 - 2$ so integral becomes Integral = $-\int ue^u \, du = -\left[ue^u - \int e^u \, du\right]$	Applies substitution and attempts to integrate to at least the first stage (may be implied by a correct result of the integral)	**M1**	3
$= e^u - ue^u$	Correct result in terms of u	**A1**	3
$= e^{x^3-2x} - (x^3 - 2x)e^{x^3-2x}$ or correct u limits $-\frac{4\sqrt{6}}{9}$ and 0	Correct integral in terms of x OR correct limits for u identified.	**A1**	3

Question	Scheme		Marks	AOs
5(a)	Taking the point where the plane leaves the runway as the origin and forward as positive *or* setting up other alternative axes… At time t the planes have coordinates $(Vt\cos\alpha, Vt\sin\alpha)$ and $(2Vt-d,3)$ respectively Or $(Vt\cos\alpha+d, Vt\sin\alpha)$ and $(2Vt,3)$ (clear set up for S+)		**(S+)**	
	Vertical distance at time t is $h_v = Vt\sin\alpha - 3$	Establishes correct vertical distance.	**B1**	2
	Horizontal distance between planes is given by $Vt\cos\alpha - (2Vt-d)$	Correct attempt at horizontal distance.	**M1**	1
	$D^2 = \left(\frac{4}{5}Vt - 2Vt + d\right)^2 + \left(\frac{3}{5}Vt - 3\right)^2 = \left(\frac{6}{5}Vt - d\right)^2 + \left(\frac{3}{5}Vt - 3\right)^2$ *	Correct proof including correct trig ratios (terms inside brackets either way)	**A1***	1
			(3)	
(b)	$D \geqslant 2 \Rightarrow D^2 \geqslant 4$ $\Rightarrow \left(d - \frac{6}{5}Vt\right)^2 + \left(\frac{3}{5}Vt - 3\right)^2 \geqslant 4$	Sets up inequality with their distance squared. (If using D then must square and get 4) Accept with $\geqslant$ or $>$	**M1**	3
	$\Rightarrow \frac{9}{5}(Vt)^2 - \left(\frac{12d+18}{5}\right)Vt + d^2 + 5 \geqslant 0$	Expands and collects to a quadratic in Vt or t or $(Vt)/5$ etc. (Inequality may be incorrect)	**M1**	3
	$b^2 - 4ac \leqslant 0 \Rightarrow \frac{36(2d+3)^2}{25} \leqslant 4 \times \frac{9}{5}(d^2+5)$ or $\frac{9}{5}\left(Vt - \frac{2d+3}{3}\right)^2 - \frac{9}{5}\left(\frac{2d+3}{3}\right)^2 + \left(5+d^2\right) \geqslant 0$	Applies $b^2 - 4ac \leqslant 0$ (accept $<$) to their quadratic in Vt or to quadratic in t (oe) and cancels V to achieve inequality in d or completes square.	**M1**	3
	$\Rightarrow (4d^2 + 12d + 9) \leqslant 5(d^2+5) \Rightarrow d^2 - 12d + 16 \geqslant 0$	Correct 3TQ quadratic inequality	**A1**	3
	C.V.s are $\frac{12 \pm \sqrt{144 - 4\times 16}}{2} = \ldots$	Attempts critical values for their quadratic.	**dM1**	1
	Depends on second M and having attempted $b^2 - 4ac \ldots 0$ with = or any inequality, or completion of square, to produce a quadratic in d only.			
	Need $d \leqslant 6 - 2\sqrt{5}$ and $d \geqslant 6 + 2\sqrt{5}$	Chooses "outsides"	**M1**	2
	(Second aircraft is behind first at start so $d > 0$) Hence $0 < d \leqslant 6 - 2\sqrt{5}$ or $d \geqslant 6 + 2\sqrt{5}$	(Explains $d > 0$) Correct solution but allow $<$ or $\leqslant$ at 0	**(S+)** **A1**	3
			(7)	
	Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a fully correct solution that may be laboured but includes an S+ point • A succinct solution that scores 8+ marks that includes at least one S+ point.		**S1**	2
			(Total 10+1 marks)	

S+ for clear set up of vector coordinates (oe) ; Explanation of why d cannot be negative.

Alt (b) By calculus	$\frac{\mathrm{d}}{\mathrm{d}t}\left(D^2\right)=2\left(\frac{6}{5}Vt-d\right)\times\frac{6V}{5}+2\left(\frac{3}{5}Vt-d\right)\times\frac{3V}{5}$ or $D^2=\frac{9V^2t^2}{5}-\frac{(12d+18)Vt}{5}+d^2+9\rightarrow$ $\frac{\mathrm{d}}{\mathrm{d}t}\left(D^2\right)=\frac{18V^2t}{5}-\frac{(12d+18)V}{5}$	May use other notation. Attempts to differentiate D^2 wrt t	**M1**	3
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(D^2\right)=0\Rightarrow t=\frac{2d+3}{3V}$	Sets derivative to zero and solves for t or kVt (need not be simplified)	**M1**	3
	Min $D^2=\left(\frac{6V}{5}\left(\frac{2d+3}{3V}\right)-d\right)^2+\left(\frac{3V}{5}\left(\frac{2d+3}{3V}\right)-3\right)^2$ $=\left(\frac{6-d}{5}\right)^2+\left(\frac{2(d-6)}{5}\right)^2=\frac{1}{5}(d-6)^2$	Applies t value to the formula for D^2 (may use their expanded form) need not be simplified.	**M1**	3
	Hence $\frac{1}{5}(d_{2\text{km}}-6)^2=2^2$	Correct equation or inequation using the given condition.	**A1**	3
	$\Rightarrow (d_{2\text{km}}-6)^2=20\Rightarrow d_{2\text{km}}=6\pm\sqrt{20}$	Attempts critical values for their quadratic (allow from inequality)	**dM1**	1
	Need $d\leqslant 6-2\sqrt{5}$ and $d\geqslant 6+2\sqrt{5}$	Chooses "outsides"	**M1**	2
	(Second aircraft is behind first at start so $d>0$) Hence $0<d\leqslant 6-2\sqrt{5}$ or $d\geqslant 6+2\sqrt{5}$	(Explains $d>0$) Correct solution but allow < or $\leqslant$ at 0	**(S+) A1**	3
			(7)	

Question	Scheme		Marks	AOs
6(a)	(Let length of L_n be S_n then $S_0 = 4$ and $S_1 = 5\times\frac{4}{4} = 5$ and has 3 horizontal and 2 sloped segments) so	(For a good explanation of their formula)	(S+)	
	$S_2 = 3\times 5\times\frac{1}{4}+2$ or $2+9\times\frac{1}{4}+6\times\frac{1}{4}$ oe	A correct expression/ identifies correct terms	**M1**	1
	$=\frac{23}{4}$ *	Correctly shown	**A1***	1
			(2)	
(b)(i)	There are 3^n horizontal line segments sides in the L_n	Correct answer only	**B1**	2
(ii)	6	cao	**B1**	1
	(Each horizontal line splits to 3 new horizontal lines and 2 new sloped lines)	(S+ for explanation)	**(S+)**	
(iii)	There are 2×3^n new sloped lines.	for 2×their (b)(i)	**B1ft**	2
			(3)	
(c)	$(S_{n+1}) = (0+)2\times 1+6\times\frac{1}{4}+18\times\left(\frac{1}{4}\right)^2+\ldots$	Starts by working out the length of sloped sides – may use S_n or S_{n+1}	**M1**	3
	$\ldots+2\times 3^n\times\left(\frac{1}{4}\right)^n+3^{n+1}\times\left(\frac{1}{4}\right)^n$ or $\ldots+5\times 3^n\times\left(\frac{1}{4}\right)^n$	Identifies the general term for sloped sides and considers the horizontals ; Correct expression (for their sum) either added considered separately.	**M1;** **A1**	3 2
	$=2\times\sum_{r=0}^{n}\left(\frac{3}{4}\right)^r+3\times\left(\frac{3}{4}\right)^n$ (oe)	Recognises a G.S. excluding the final term (accept equivalents)	**M1**	2
	$\to 2\times\frac{1}{1-\frac{3}{4}}(+3\times 0)$ (as $\left(\frac{3}{4}\right)^n \to 0$ as $n\to\infty$)	Applies sum of GS (S+ for explanation of last term disappearing)	**M1** **(S+)**	3
	So $S_\infty = 8$	Correct answer	**A1**	3
			(6)	
(d)	Area = $\frac{(a+b)h}{2} = \frac{(2+1)\times 1\sin(60°)}{2} = \ldots$ Or height of trapezium is $\sqrt{1^2-\left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ so Area $\frac{(a+b)h}{2} = \frac{(2+1)\times 'h'}{2} = \ldots$	Correct method to find the height of the trapezium (trig or Pythagoras) and applies correct area formula	**M1**	1
	$=\frac{3\sqrt{3}}{4}$	Correct answer	**A1**	1
			(2)	

(e)	E.g. from first to second we have 3 new trapezia $\frac{1}{16}^{\text{th}}$ the area of the one in the first iteration.	Identifies the scaling factor of trapezia between iterations OR number of new trapezia. Accept from iteration 1 to 2, or between any two.	**M1**	1
	In the $(n+1)^{\text{th}}$ iteration the new trapezia have area $\left(\frac{1}{16}\right)^n$ and there are 3^n such, so area scales by $\left(\frac{3}{16}\right)^n$ (Allow if indices out by 1)	Brings both facts together in relation to the $(n+1)$th iteration. (S+ good explanation)	**M1 (S+)**	3
	So increase in area is $\left(\frac{1}{16}\right)^n \times \frac{3\sqrt{3}}{4} \times 3^n$	Correct expression	**A1**	3
			(3)	
(f)	$A_\infty = \frac{3\sqrt{3}}{4} + 3\times\frac{3\sqrt{3}}{4\times16} + 3^2\times\frac{3\sqrt{3}}{4\times16^2} + \ldots + \left(\frac{3}{16}\right)^n \times \frac{3\sqrt{3}}{4} + \ldots$ Correct consideration of the area – sum of their term from (e) attempted		**M1**	3
	$= \frac{3\sqrt{3}}{4} \times \frac{1}{1-\frac{3}{16}}$	Correct unsimplified expression follow through their (e) as long as it is a G.S.	**A1ft**	3
	$= \frac{12\sqrt{3}}{13}$		**A1**	3
			(3)	
(g)	Area of triangle is $\frac{1}{2}a^2\sin(60^\circ) = \frac{a^2\sqrt{3}}{4}$ so limiting area of shape is $\frac{a^2\sqrt{3}}{4} + 3\times\left(\frac{a}{4}\right)^2 \times \frac{12\sqrt{3}}{13}$	Finds area of triangle and attempts to add (3×) a scaled area from (f). Accept e.g. with $\frac{a}{4}$ or a^2 for this mark.	**M1**	1
	$\frac{a^2\sqrt{3}}{4} + 3\times\left(\frac{a}{4}\right)^2 \times \frac{12\sqrt{3}}{13} = 26\sqrt{3} \Rightarrow a^2 = \ldots$	Applies **correct** 3× scaling to area from (f) and solves as far as a^2	**dM1**	3
	$\Rightarrow a = \frac{26}{\sqrt{11}} = \frac{26\sqrt{11}}{11}$ (Accept either) Alt : may find whole area for triangle of side length 4 before scaling.		**A1**	3
			(3)	
	Award S2 for a solution scoring 20+ marks that is succinct and includes some S+ points (see notes below). Award S1 for:		**S2**	2 2

	• a fully correct solution that is succinct but does not mention any S+ points • a fully correct solution that may be laboured but includes an S+ point • A succinct solution that scores 18+ marks that includes at least one S+ point.			
				(Total 22+2 marks)

Note: S+ marks for good explanations at the point indicated.

(c) Alt 1 (Further maths)	$L_{n+1} = \frac{1}{4} \times L_n \times 3 + 2,\ L_0 = 4$	Identifies a recurrence relation for the total length.	**M1**	3
	$\Rightarrow L_n = \alpha + \beta \left(\frac{3}{4}\right)^n$	Identifies general form for the solution; correct general form. May consider an complementary and particular part separately.	**M1;** **A1**	3 2
	E.g. $L_0 = 4 \Rightarrow \alpha + \beta = 4, L_1 = 5 \Rightarrow \alpha + \frac{3}{4}\beta = 5 \Rightarrow \alpha = ..., \beta = ...$ or $PS: \alpha = 2 + \frac{3}{4}\alpha \Rightarrow \alpha = .., L_0 = 4 \Rightarrow \beta = ...$	Full method to find the constants. $\alpha = 8, \beta = -4$	**M1**	2
	$L_n = 8 - 4\left(\frac{3}{4}\right)^n \to 8$ (as $\left(\frac{3}{4}\right)^n \to 0$ as $n \to \infty$)	Evaluates limit (S+ for explanation of term disappearing)	**M1** **(S+)**	3
	So $L_\infty = 8$	Correct answer	**A1**	3
			(6)	
(c) Alt 1	$L_0 = 4, L_1 = 5, L_2 = \frac{23}{4}, L_3 = \frac{101}{6}\left(= 2 + 6 \times \frac{1}{4} + 5 \times 9 \times \frac{1}{16}\right)$	Investigates total lengths for first first terms up to L_3	**M1**	3
	$\Rightarrow L_n = 4 + 1 + \frac{3}{4} + ... + \left(\frac{3}{4}\right)^{n-1}$	Attempts to form the general expression for total length; correct expression (so an expression for total length of a general iteration)	**M1;** **A1**	3 2
	Note inaccuracy with the first term loses the A, e.g. $5 + \sum_{r=1}^{n} \left(\frac{3}{4}\right)^{r-1}$ is A0			
	$\Rightarrow L_n = "4" + \sum_{r=1}^{n} \left(\frac{3}{4}\right)^{r-1}$	Identifies Geometric series identified within the total length	**M1**	2
	$L_\infty = 4 + \frac{1}{1 - \frac{3}{4}}$	Applies sum of GS (S+ for dealing well with first term)	**M1** **(S+)**	3
	So $L_\infty = 8$	Correct answer	**A1**	3
			(6)	

Question	Scheme		Marks	AOs
7(a)	$C:\left(x-a\right)^2+\left(y-b\right)^2=r^2$; $l:\ y=mx+c$ Meet when $\left(x-a\right)^2+\left(mx+c-b\right)^2=r^2 \Rightarrow x^2+\ldots+m^2x^2+\ldots=r^2$	Attempts to substitute $y=mx+c$ into $(x\pm a)^2+(y\pm b)^2=r^2$ and expands	**M1**	1
	$\Rightarrow x^2-2ax+a^2+m^2x^2+2mx(c-b)+(c-b)^2=r^2$ $\Rightarrow (m^2+1)x^2-2\left(a-m(c-b)\right)x+a^2+(c-b)^2-r^2=0$ *	Achieves correct result with intermediate step.	**(S+) A1***	2
			(2)	
(b)	(Tangent if equation in (a) has one solution, so need) $4\left(a-m(c-b)\right)^2-4(m^2+1)\left(a^2+(c-b)^2-r^2\right)=0$	(Explanation) Attempts $b^2-4ac=0$ on equation from (a) (allow if slips e.g. sign error copying terms).	**(S+) M1**	3
	$\Rightarrow 0=4a^2-8am(c-b)+4m^2(c-b)^2$ $-4m^2a^2-4m^2(c-b)^2+4m^2r^2-4(a^2+(c-b)^2-r^2)$	Expands both brackets ; Any correct expansion	**M1** **A1**	1 2
	$\Rightarrow (c-b)^2+2am(c-b)+m^2a^2-m^2r^2-r^2=0$ or $c^2+2(am-b)c+b^2-2amb+m^2a^2-m^2r^2-r^2=0$ or $b^2-2(am+c)b+c^2+2amc+m^2a^2-m^2r^2-r^2=0$	Cancels terms and forms a quadratic in *c* or *b* or (*c-b*) (oe)	**M1**	3
	$\Rightarrow\left[(c-b)+am\right]^2-a^2m^2+a^2m^2-r^2(m^2+1)=0\Rightarrow c=\ldots$ or e.g. (or equivalent for *b*) $c=\dfrac{-2(am-b)\pm\sqrt{4(am-b)^2-4\left(b^2-2amb+m^2a^2-m^2r^2-r^2\right)}}{2}$	Solves via completing the square or formula	**dM1**	3
	$\Rightarrow c=b-am\pm r\sqrt{m^2+1}$ *	Correct result with no errors seen.	**A1***	2
			(6)	
(c)	All normals to a circle must pass through the centre, so there is only one common normal, the line through both centres. Alt. Use differentiation and obtain unique solution.	Explains normals pass through centres, so only one - ie justifies only one.	**B1**	2
	C_1 has centre *O* and C_2 has centre (10,5) so equation is $(y-0)=\frac{5}{10}(x-0)$	Extracts centres and attempts equation	**M1**	1
	so $y=\frac{1}{2}x$ or any equivalent (eg $2y-x=0$)	Correct equation	**A1**	1
			(3)	
(d)	C_1 has centre (0,0) and radius 4, and C_2 has centre	Attempts to find	**M1**	2

	(10,5) and radius $\sqrt{100+25-89}=\sqrt{36}=6$	centre **and** radius for each circle (may be seen in (c)		
	(Horizontal distance between the centres is 10 and sum of radii is 10 so there is a common vertical tangent) One common tangent is $x=4$	(Explains and) identifies the vertical common tangent	**(S+) B1**	2
	From (b) other common tangents $y=mx+c$ satisfy $0-0m\pm4\sqrt{m^2+1}=c=5-10m\pm6\sqrt{m^2+1}$	Uses the result of (b) with both their centres and radii (these equations imply first M)	**M1**	3
	$\pm10\sqrt{m^2+1}=5-10m\Rightarrow4(m^2+1)=1-4m+4m^2\Rightarrow m=...$	Attempts and solves this combination of signs	**M1**	3
	$\Rightarrow m=-\frac{3}{4}$	Correct m from first equation above (and not rejected)	**A1**	2
	Other possibility is $\pm2\sqrt{m^2+1}=5-10m\Rightarrow4(m^2+1)=25-100m+100m^2$	Attempts the other possibility of signs	**M1**	3
	or $96m^2-100m+21=0\Rightarrow(24m-7)(4m-3)=0$ $\Rightarrow m=\frac{3}{4},\frac{7}{24}$	Attempts to solve a quadratic in m. Depends on previous M	**dM1**	3
	For $m=\pm\frac{3}{4}:c=\pm4\sqrt{\frac{9}{16}+1}=\pm4\sqrt{\frac{25}{16}}=\pm5$ (or $c=5-10\left(\pm\frac{3}{4}\right)\pm6\sqrt{\frac{9}{16}+1}=5\pm\frac{15}{2}\pm6\sqrt{\frac{25}{16}}$ $=5$ or 5 ± 15) or for $m=\frac{7}{24}:c=\pm4\sqrt{\frac{7^2}{24^2}+1}=\pm4\sqrt{\frac{25^2}{24^2}}=\pm\frac{25}{6}$ (or $c=5-10\left(\frac{7}{24}\right)\pm6\sqrt{\frac{7^2}{24^2}+1}=5-\frac{35}{12}\pm\frac{25}{4}$)	Attempts to find c for at least one value of m ; Obtains one correct equation (box below) ; Attempts to find c for all values of m	**M1** **A1** **M1**	3 3 2
	The only possibilities satisfying both equations for c are $y=-\frac{3}{4}x+5$; $y=\frac{3}{4}x+5$ and $y=\frac{7}{24}x-\frac{25}{6}$	All three equations found and no others.	**A1**	3
			(11)	
	Award S2 for a solution scoring 20+ marks that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a fully correct solution that may be laboured but includes an S+ point • A succinct solution that scores 18+ marks that includes at least one S+ point.		**S2**	2 2
				(Total 22+2 marks)

Note: S+ marks for good explanations at the points indicated (o appropriate other places).

S+ mark for a good diagram				
Alternatives for 7(b)				
(b)	$2\left(x-a\right)+2\left(y-b\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	Differentiates circle equation implicitly.	**(S+) M1**	2
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=m\Rightarrow m\left(y-b\right)=a-x\Rightarrow x=\ldots \text{ or } y=\ldots$	Sets $\dfrac{\mathrm{d}y}{\mathrm{d}x}=m$ and solves with $y=mx+c$ leading to x or y	**M1**	1
	$x=\dfrac{a+m(b-c)}{m^2+1},\ y=\dfrac{ma+m^2b+c}{m^2+1}$	Obtains x and y coordinates	**M1**	3
	$\Rightarrow\left(\dfrac{a+m(b-c)}{m^2+1}-a\right)^2+\left(\dfrac{ma+m^2b+c}{m^2+1}-b\right)^2=r^2$	Substitutes coordinates into circle equation.	**dM1**	3
	$\Rightarrow\left(\dfrac{a+m(b-c)-(m^2+1)a}{m^2+1}\right)^2+\left(\dfrac{ma+m^2b+c-(m^2+1)b}{m^2+1}\right)^2=r^2$ $\Rightarrow\left(\dfrac{m\left(b-c-ma\right)}{m^2+1}\right)^2+\left(\dfrac{ma+c-b}{m^2+1}\right)^2=r^2\Rightarrow\left(m^2+1\right)\left(\dfrac{ma+c-b}{m^2+1}\right)^2=r^2$ $\Rightarrow\left(ma+c-b\right)^2=(m^2+1)r^2$	Uses algebra (e.g. expanding) to identify common factors and reduce to equation shown or equivalent.	**dM1;**	3
	$\Rightarrow c=b-am\pm r\sqrt{m^2+1}$ *	Correct result with no errors seen.	**A1***	2
			(6)	
(b)	Let A be where tangent meets y-axis and B be $(0,b)$. (Both triangle XPA and XAB are right-angled so…) $r^2+PA^2=XA^2=a^2+(b-c)^2$ and $PA^2=p^2+(q-c)^2$ from triangle PAC where C is $(0,q)$	(Good explanation) Sets up triangles and applies Pythagoras to $\triangle XPA$ & $\triangle XAP$	**(S+) M1**	2
	So $r^2+p^2+(q-c)^2=a^2+(b-c)^2$	Combines equations	**M1**	1
	$\Rightarrow r^2+p^2+\left(q-c-(b-c)\right)\left(q-c+(b-c)\right)=a^2$ $\Rightarrow(q-b)(q+b-2c)=a^2-r^2-p^2$ $\Rightarrow c=\dfrac{r^2+p^2+q^2-a^2-b^2}{2(q-b)}$	Simplifies to eliminate c^2 terms and makes c the subject	**M1**	3
	(But circle has equation $p^2-2ap+a^2+q^2-2bq+b^2-r^2=0$ so...) $=\dfrac{2r^2+2ap+2bq-2(a^2+b^2)}{2(q-b)}$	Applies circle equation to eliminate p^2 and q^2	**dM1**	3

	$= \dfrac{r^2 + a(p-a) + b(q-b)}{(q-b)}$	Rearranges and factorises appropriately	**dM1;**	3
	$m = -\dfrac{p-a}{q-b} \Rightarrow m^2 + 1 = \dfrac{r^2}{(q-b)^2} \Rightarrow$ $c = \dfrac{r^2}{q-b} + -ma + b = b - ma \pm r\sqrt{m^2+1}$ *	Uses gradient to achieve correct result.	**A1***	2
			(6)	
	Alternatives to (d) are possible, the first two marks will be the same in each. If relevant further progress is made, schemes will be issued when responses seen. Look out for e.g. - Use of symmetry about $y = 5$ for two tangents which meet at on the y-axis. - Use of ratio of radii to find the point of intersection of internal and external tangents.			

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