Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Award

Centre Number | Candidate Number

Time 3 hours

Paper reference **9811/01**

Advanced Extension Award Mathematics

You must have:
Mathematical Formulae and Statistical Tables
An insert for questions 5, 6 and 7

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may not be used.**
- You must **show all your working.**
- Answers should be given in as simple a form as possible, e.g. $\frac{2\pi}{3}$, $\sqrt{2}$, $3\sqrt{2}$.

Information

- The total mark for this paper is 100 of which **7** marks are for style and clarity of presentation.
- The style and clarity of presentation marks will be indicated as **(+S1) or (+S2)**.
- There are 7 questions in this question paper.
- The marks for each question are shown in brackets.
- The total mark for each question is shown at the end of the question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

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1. $$f(x) = x^{(x^2)} \qquad x > 0$$

Use logarithms to find the x coordinate of the stationary point of the curve with equation $y = f(x)$.

(5)

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Question 1 continued

(Total for Question 1 is 5 marks)

2.

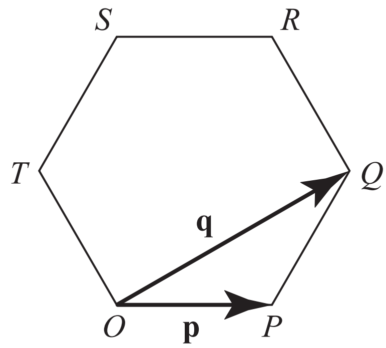

Figure 1

Figure 1 shows a regular hexagon *OPQRST*.

The vectors **p** and **q** are defined by $\mathbf{p} = \overrightarrow{OP}$ and $\mathbf{q} = \overrightarrow{OQ}$

Forces, in Newtons, $\mathbf{F}_P = (\overrightarrow{OP})$, $\mathbf{F}_Q = 2 \times (\overrightarrow{OQ})$, $\mathbf{F}_R = 3 \times (\overrightarrow{OR})$, $\mathbf{F}_S = 4 \times (\overrightarrow{OS})$ and $\mathbf{F}_T = 5 \times (\overrightarrow{OT})$ are applied to a particle.

(a) Find, in terms of **p** and **q**, the resultant force on the particle.

(5)

The magnitude of the acceleration of the particle due to these forces is $13\,\text{m}\,\text{s}^{-2}$

Given that the mass of the particle is 3 kg,

(b) find $|\mathbf{p}|$

(5)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. (a) Use the formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$ to prove that $\tan(90° - \theta) \equiv \cot\theta$

(3)

(b) Solve for $0 < \theta < 360°$

$$2 - \sec^2(\theta + 11°) = 2\tan(\theta + 11°)\tan(\theta - 34°)$$

Give each answer as an integer in degrees.

(8)

(+S1)

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Question 3 continued

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Question 3 continued

Question 3 continued

(Total for Question 3 is 12 marks)

4. Given that $f(x) = e^{x^3 - 2x}$

(a) find $f'(x)$

(2)

The curves C_1 and C_2 are defined by the functions g and h respectively, where

$$g(x) = 8x^3 e^{x^3 - 2x} \qquad x \in \mathbb{R}, x > 0$$

$$h(x) = (3x^5 + 4x)e^{x^3 - 2x} \qquad x \in \mathbb{R}, x > 0$$

(b) Find the x coordinates of the points of intersection of C_1 and C_2

(4)

Given that C_1 lies above C_2 between these points of intersection,

(c) find the area of the region bounded by the curves between these two points. Give your answer in the form $A + Be^C$ where A, B, and C are exact real numbers to be found.

(7)

(+S1)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 14 marks)

5. An aeroplane leaves a runway and moves with a constant speed of V km/h due north along a straight path inclined at an angle $\arctan\left(\frac{3}{4}\right)$ to the horizontal.

A light aircraft is moving due north in a straight horizontal line in the same vertical plane as the aeroplane, at a height of 3 km above the runway.

The light aircraft is travelling with a constant speed of $2V$ km/h.

At the moment the aeroplane leaves the runway, the light aircraft is at a horizontal distance d km behind the aeroplane.

Both aircraft continue to move with the same trajectories due north.

(a) Show that the distance, D km, between the two aircraft t hours after the aeroplane leaves the runway satisfies

$$D^2 = \left(\frac{6}{5}Vt - d\right)^2 + \left(\frac{3}{5}Vt - 3\right)^2$$

(3)

Given that the distance between the two aircraft is never less than 2 km,

(b) find the range of possible values for d.

(7)

(+S1)

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Question 5 continued

Question 5 continued

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Question 5 continued

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 11 marks)

6.

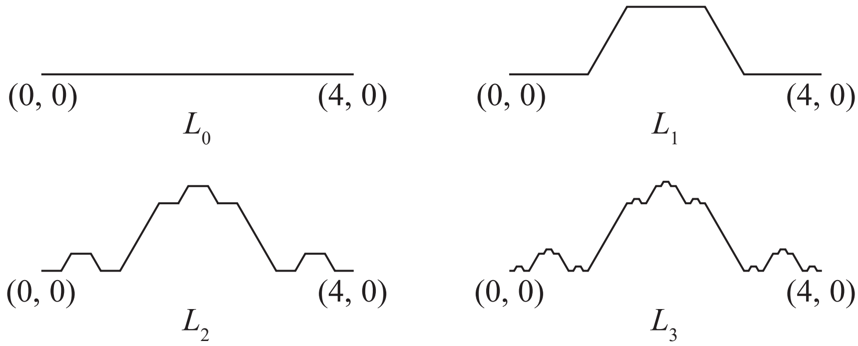

Figure 2

Figure 2 shows the first few iterations in the construction of a curve, L.

Starting with a straight line L_0 of length 4, the middle half of this line is replaced by three sides of a trapezium above L_0 as shown, such that the length of each of these sides is $\frac{1}{4}$ of the length of L_0

After the first iteration each line segment has length one.

In subsequent iterations, each line segment parallel to L_0 similarly has its middle half replaced by three sides of a trapezium above that line segment, with each side $\frac{1}{4}$ the length of that line segment.

Line segments in L_n are either parallel to L_0 or are sloped.

(a) Show that the length of L_2 is $\frac{23}{4}$ **(2)**

(b) Write down the number of

(i) line segments in L_n that are parallel to L_0

(ii) sloped line segments in L_2 that are not in L_1

(iii) **new** sloped line segments that are created by the $(n + 1)$th iteration. **(3)**

(c) Hence find the length of L_n as $n \to \infty$ **(6)**

The area enclosed between L_0 and L_n is A_n

(d) Find the value of A_1 **(2)**

(e) Find, in terms of n, an expression for $A_{n+1} - A_n$ **(3)**

(f) Hence find the value of A_n as $n \to \infty$ **(3)**

P 6 7 0 5 7 A 0 2 2 3 6

Question 6 continued

The same construction as described above is applied externally to the three sides of an equilateral triangle of side length a.

Given that the limit of the area of the resulting shape is $26\sqrt{3}$

(g) find the value of a.

(3)

(+S2)

Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 24 marks)

7. A circle C has centre $X(a, b)$ and radius r.

A line l has equation $y = mx + c$

(a) Show that the x coordinates of the points where C and l intersect satisfy

$$(m^2 + 1)x^2 - 2(a - m(c - b))x + a^2 + (c - b)^2 - r^2 = 0$$

(2)

Given that l is a tangent to C,

(b) show that

$$c = b - ma \pm r\sqrt{m^2 + 1}$$

(6)

The circle C_1 has equation

$$x^2 + y^2 - 16 = 0$$

and the circle C_2 has equation

$$x^2 + y^2 - 20x - 10y + 89 = 0$$

(c) Find the equations of any lines that are normal to both C_1 and C_2, justifying your answer.

(3)

(d) Find the equations of all lines that are a tangent to both C_1 and C_2

[*You may find the following Pythagorean triple helpful in this part*: $7^2 + 24^2 = 25^2$]

(11)

(+S2)

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Question 7 continued

Question 7 continued

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Question 7 continued

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FOR STYLE AND CLARITY OF PRESENTATION: 7 MARKS
TOTAL FOR PAPER IS 100 MARKS

Pearson Edexcel Award

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(7)

(+S1)

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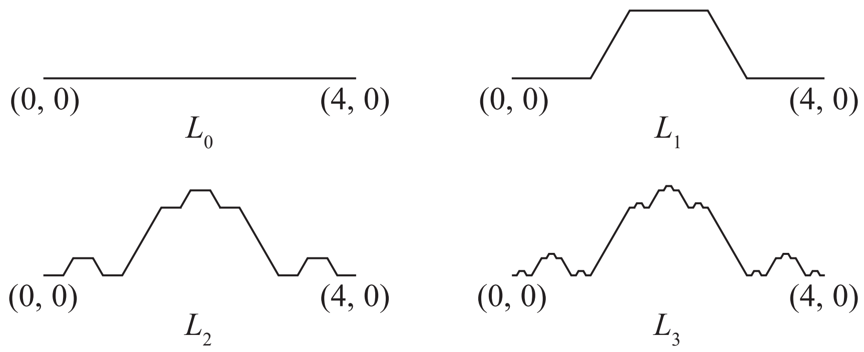

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(3)

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