Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Award

Centre Number | Candidate Number

Wednesday 24 June 2020

Morning (Time: 3 hours) | Paper Reference **9811/01**

Advanced Extension Award Mathematics

You must have:
Mathematical Formulae and Statistical Tables
An insert for questions 3, 4, 5 and 6

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may not be used**.
- You must **show all your working**.
- Answers should be given in as simple a form as possible, e.g. $\frac{2\pi}{3}$, $\sqrt{2}$, $3\sqrt{2}$

Information

- The total mark for this paper is 100 of which **7** marks are for style and clarity of presentation.
- The style and clarity of presentation marks will be indicated as **(+S1) or (+S2)**.
- There are 6 questions in this question paper.
- The marks for each question are shown in brackets.
- The total mark for each question is shown at the end of the question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

1.

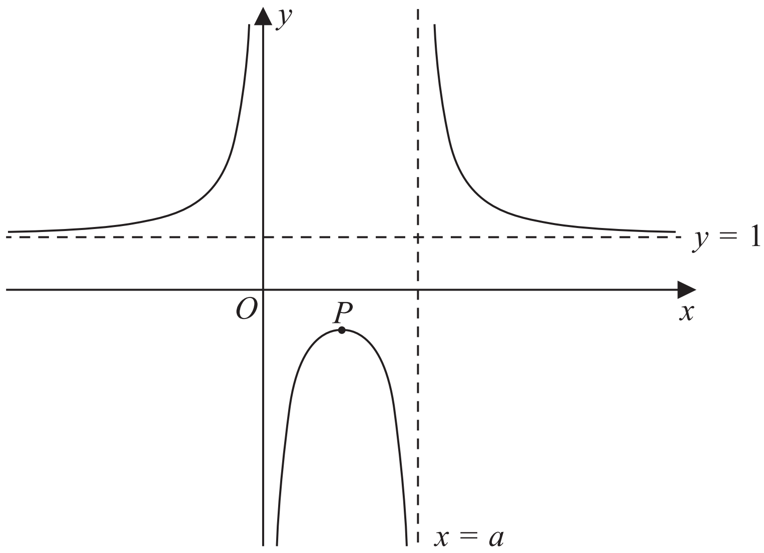

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 1 + \frac{4}{x(x-3)}$$

The curve has a turning point at the point P, and the lines with equations $y = 1$, $x = 0$ and $x = a$ are asymptotes to the curve.

(a) Write down the value of a.

(1)

(b) Find the coordinates of P, justifying your answer.

(4)

(c) Sketch the curve with equation $y = \left|\mathrm{f}\left(x + \frac{3}{2}\right)\right| - 1$

On your sketch, you should show the coordinates of any points of intersection with the coordinate axes, the coordinates of any turning points and the equations of any asymptotes.

(7)

P 6 4 9 1 2 R A 0 2 3 6

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 12 marks)

2. The functions f and g are defined by

$$f(x) = 2\sqrt{1 - e^{-x}} \qquad x \in \mathbb{R}, x \geqslant 0$$
$$g(x) = \ln(4 - x^2) \qquad x \in \mathbb{R}, -2 < x < 2$$

(a) (i) Explain why fg cannot be formed as a composite function.

(ii) Explain why gf can be formed as a composite function.

(2)

(b) (i) Find gf(x), giving the answer in the form gf$(x) = a + bx$, where a and b are constants.

(ii) State the domain and range of gf.

(5)

(c) Sketch the graph of the function gf.

On your sketch, you should show the coordinates of any points where the graph meets or crosses the coordinate axes.

(2)

The circle C with centre $(0, -\ln 9)$ touches the line with equation $y = $ gf(x) at precisely one point.

(d) Find an equation of the circle C.

(3)

(+S1)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 13 marks)

3. (a) (i) Write down the binomial series expansion of

$$\left(1+\frac{2}{n}\right)^n \qquad n \in \mathbb{N}, n > 2$$

in powers of $\left(\frac{2}{n}\right)$ up to and including the term in $\left(\frac{2}{n}\right)^3$

(ii) Hence prove that, for $n \in \mathbb{N}, n \geqslant 3$

$$\left(1+\frac{2}{n}\right)^n \geqslant \frac{19}{3} - \frac{6}{n}$$

(3)

(b) Use the binomial series expansion of $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ to show that $\sqrt{3} < \frac{7}{4}$

(4)

$$\mathrm{f}(x) = \left(1+\frac{2}{x}\right)^x - 3^{\frac{x}{6}} \qquad x \in \mathbb{R}, x > 0$$

Given that the function f(*x*) is continuous and that $\sqrt[6]{3} > \frac{6}{5}$

(c) prove that f(*x*) = 0 has a root in the interval [9, 10]

(5)

(+S1)

Question 3 continued

Question 3 continued

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Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 13 marks)

4.

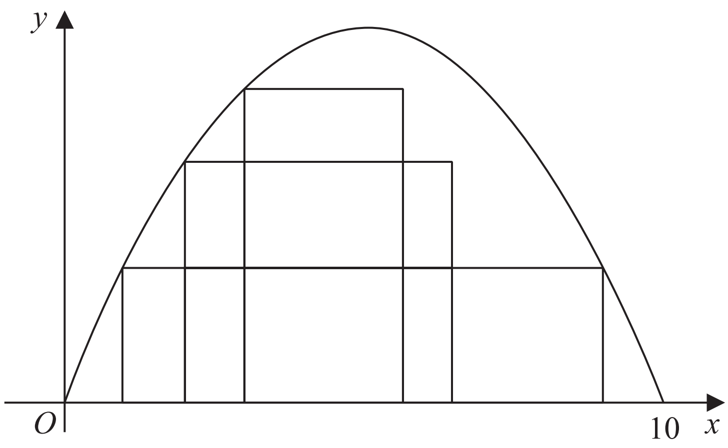

Figure 2

Figure 2 shows a sketch of the parabola with equation $y = \frac{1}{2}x(10 - x)$, $0 \leqslant x \leqslant 10$

This question concerns rectangles that lie under the parabola in the first quadrant. The bottom edge of each rectangle lies along the x-axis and the top left vertex lies on the parabola. Some examples are shown in Figure 2.

Let the x coordinate of the top left vertex be a.

(a) Explain why the width, w, of such a rectangle must satisfy $w \leqslant 10 - 2a$

(2)

(b) Find the value of a that gives the maximum area for such a rectangle.

(5)

Given that the rectangle must be a square,

(c) find the value of a that gives the maximum area for such a square.

(3)

Given that the area of the rectangles is fixed as 36

(d) find the range of possible values for a.

(6)

(+S1)

Question 4 continued

Question 4 continued

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 17 marks)

5. (a) The box below shows a student's attempt to prove the following identity for $a > b > 0$

$$\arctan a - \arctan b \equiv \arctan \frac{a-b}{1+ab}$$

Let $x = \arctan a$ and $y = \arctan b$, so that $a = \tan x$ and $b = \tan y$

So $\tan(\arctan a - \arctan b) \equiv \tan(x - y)$

$$\equiv \frac{\tan x - \tan y}{1 - \tan^2(xy)}$$

$$\equiv \frac{a-b}{1-(ab)^2}$$

$$\equiv \frac{a - ab + ab - b}{(1-ab)(1+ab)}$$

$$\equiv \frac{a\,\cancel{(1-ab)} - b\,\cancel{(1-ab)}}{\cancel{(1-ab)}\,(1+ab)}$$

$$\equiv \frac{a-b}{1+ab}$$

Taking arctan of both sides gives $\arctan a - \arctan b \equiv \arctan \dfrac{a-b}{1+ab}$ as required.

There are three errors in the proof where the working does not follow from the previous line.

(i) Describe these three errors.

(3)

(ii) Write out a correct proof of the identity.

(2)

(b) [*In this question take g to be* $9.8\,\mathrm{m\,s^{-2}}$]

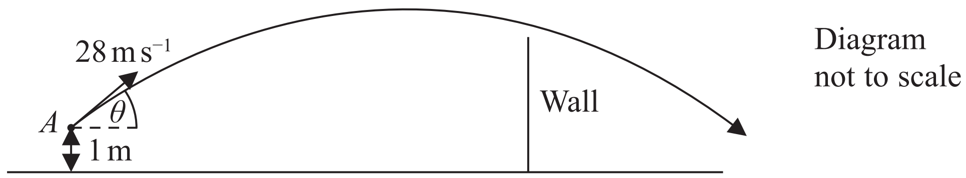

Figure 3

Balls are projected, one after another, from a point, A, one metre above horizontal ground. Each ball travels in a vertical plane towards a 6 metre high vertical wall of negligible thickness, which is a horizontal distance of $10\sqrt{2}$ metres from A.

The balls are modelled as particles and it is assumed that there is no air resistance.

Each ball is projected with an initial speed of $28\,\mathrm{m\,s^{-1}}$ and at a random angle θ to the horizontal, where $0 < \theta < 90°$

Question 5 continued

Given that a ball will pass over the wall precisely when $\alpha \leqslant \theta \leqslant \beta$

(i) find, in degrees, the angle $\beta - \alpha$

(10)

(ii) Deduce that the probability that a particular ball will pass over the wall is $\frac{2}{3}$

(1)

(iii) Hence find the probability that exactly 2 of the first 10 balls projected pass over the wall.

You should give your answer in the form $\frac{P}{Q^k}$ where P, Q and k are integers and P is not a multiple of Q.

(3)

(iv) Explain whether taking air resistance into account would increase or decrease the probability in (b)(iii).

(1)

(+S2)

Question 5 continued

Question 5 continued

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Question 5 continued

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 22 marks)

6. (a) Given that f is a function such that the integrals exist,

(i) use the substitution $u = a - x$ to show that

$$\int_0^a \mathrm{f}(x)\,\mathrm{d}x = \int_0^a \mathrm{f}(a-x)\,\mathrm{d}x$$

(2)

(ii) Hence use symmetry of $\mathrm{f}(\sin x)$ on the interval $[0, \pi]$ to show that

$$\int_0^{\pi} x\mathrm{f}(\sin x)\,\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} \mathrm{f}(\sin x)\,\mathrm{d}x$$

(4)

(b) Use the result of (a)(i) to show that

$$\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x}\,\mathrm{d}x$$

is independent of n, and find the value of this integral.

(4)

(c) (i) Prove that

$$\frac{\cos x}{1+\cos x} \equiv 1 - \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

(ii) Hence use the results from (a) to find

$$\int_0^{\pi} \frac{x\sin x}{1+\sin x}\,\mathrm{d}x$$

(7)

(d) Find

$$\int_0^{\pi} \frac{x\sin^4 x}{\sin^4 x + \cos^4 x}\,\mathrm{d}x$$

(4)

(+S2)

Question 6 continued

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Question 6 continued

Question 6 continued

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Question 6 continued

Question 6 continued

(Total for Question 6 is 23 marks)

TOTAL FOR PAPER: 100 MARKS

Pearson Edexcel Award

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Advanced Extension Award Mathematics

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Pearson

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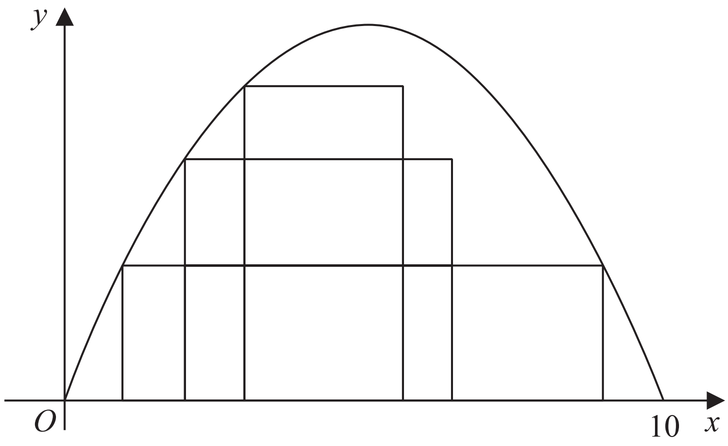
(5)

(+S1)

(Total for Question 3 is 13 marks)

4.



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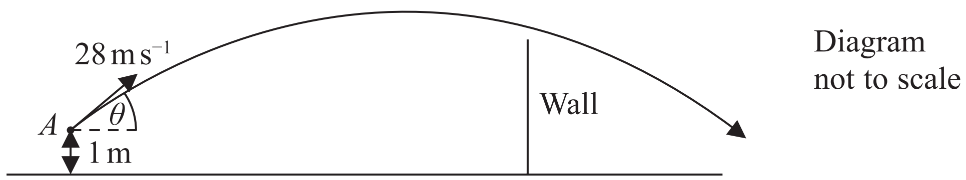

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(+S2)

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(7)

(d) Find

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(4)

(+S2)

(Total for Question 6 is 23 marks)

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