Mark Scheme (Results)

October 2020

Pearson Edexcel Advanced Extension Award
In Mathematics (9811/01)

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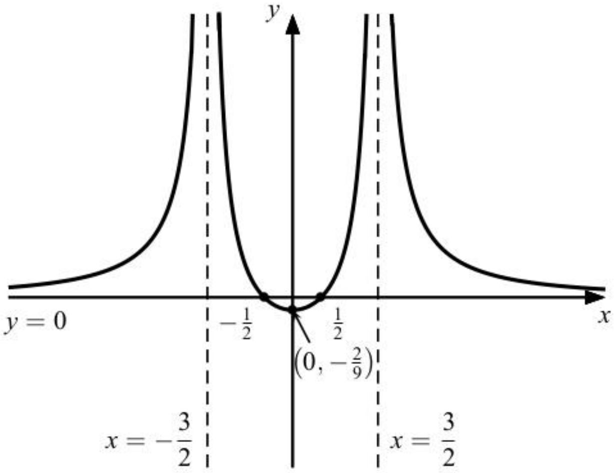
General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

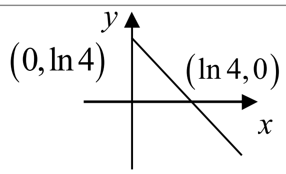
AEA 2006 paper final

Mark scheme

Question	Scheme		Marks	AOs
1(a)	$a = 3$		B1	1
			(1)	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4(2x-3)}{(x^2-3x)^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{3x^2} - \frac{4}{3(x-3)^2}$ oe		B1	1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x = \frac{3}{2}$	Attempts x Correct x coordinate from a correct derivative.	M1 A1	1 1
	$y = 1 - \frac{4}{\frac{3}{2}(\frac{3}{2}-3)} = -\frac{7}{9}$ so P is $\left(\frac{3}{2}, -\frac{7}{9}\right)$	Correct y coordinate from a correct derivative.	A1	1
			(4)	
(c)	Allow marks from a correct P even if from an incorrect derivative. 	Evidence of translation left/ TP Minimum on y-axis	M1	3
		Correct $x = \pm\frac{3}{2}$ asymptotes	A1	3
		Evidence of $\lvert f(\ldots) \rvert$, e.g central $\cup$ shape	M1	2
		Translation down by 1 unit – the $y = 0$ asymptote may be implied (but must be correct if shown).	B1	3
	y intercept is $\left(0, -\frac{2}{9}\right)$ follow through their y coordinate of P		B1ft	3
	Attempts $f(x) = -1$ then subtracts $\frac{3}{2}$ or equivalent work to… …find the x intercepts are $\pm\frac{1}{2}$		M1 A1	2 3
			(7)	
			(12 marks)	

Notes:

(b) Candidates may use symmetry but the method must be justified e.g. by stating the line of symmetry.

Question	Scheme		Marks	AOs
2(a)	As the ranges of f and g are $0 \leqslant f(x) < 2$ and $(-\infty <)\ g(x) \leqslant \ln 4$			
	A function fg cannot be formed as the range of g does not lie in the domain of f.	A reason **or** example is acceptable (range for g not needed but if no example is given the range must be given and correct) e.g $g\left(\frac{7}{4}\right) = \ln\left(\frac{15}{16}\right) < 0$ so range of g is not in domain of f.	**B1**	2
	A function gf can be formed as the range of f lies in the domain of g.	Correct range for f must have been found and reason given.	**B1**	2
			(2)	
(b)	$g(f(x)) = \ln\left(4 - \left(2\sqrt{1-e^{-x}}\right)^2\right)$	Attempts the composite.	**M1**	1
	$= \ln\left(4 - \left(2\sqrt{1-e^{-x}}\right)^2\right) = \ln\left(4 - 4\left(1-e^{-x}\right)\right)$ $= \ln\left(4e^{-x}\right) = \ln 4 + \ln\left(e^{-x}\right)$	Correct composite with square evaluated, but need not be simplified.	**A1**	1
	$gf(x) = \ln 4 - x$ or $2\ln 2 - x$	Correct form.	**A1**	1
	Domain is $x \in \mathbb{R}, x \geqslant 0$	Correct domain	**B1**	2
	Range is $(-\infty <)\, gf(x) \leqslant \ln 4$	Correct range	**B1**	2
			(5)	
(c)		A line consistent with their gradient and intercept from (b).	**M1**	1
		Line starting at $(0, \ln 4)$ and passing through $(\ln 4, 0)$	**A1**	1
			(2)	
(d)	(If X is centre, P is $(0, \ln 4)$ and Q is point where circle touches line then triangle XPQ is isosceles right angled, so) $2r^2 = (\ln 4 - (-\ln 9))^2 \Rightarrow r^2 = \ldots$	A complete method to find r or r^2 where r is the radius – longer methods are possible here.	**M1** **(S+)**	3
	$r^2 = \frac{1}{2}(\ln 36)^2 = 2(\ln 6)^2$ oe or $r = \sqrt{2}\ln 6$ etc	Correct r or r^2 award when first seen, need not be simplified.	**A1**	3
	So equation of C is $x^2 + (y + \ln 9)^2 = 2(\ln 6)^2$ oe	Correct equation, need not be simplified but do not isw if eg.	**A1**	3

		$(\ln 6)^2$ is incorrectly simplified to $\ln 36$		
			(3)	
	S1 mark: Award S1 for a clear and concise solution that scores 10+ marks and includes the S+ point – ie must be a well explained solution with all terminology and notation correct (though there may be variations on the notation used).		**S1**	2
				(12+1 marks)

Notes:

(d) S+: For succinct solution

Question	Scheme	Marks	AOs
3(a)(i)	$\left(1+\frac{2}{n}\right)^n = 1+n\left(\frac{2}{n}\right)+\frac{n(n-1)}{2}\left(\frac{2}{n}\right)^2+\frac{n(n-1)(n-2)}{6}\left(\frac{2}{n}\right)^3+\ldots$ No need to simplify but allow if simplified forms are given instead.	**B1**	1
(ii)	$\left(1+\frac{2}{n}\right)^n = 1+2+\frac{2(n-1)}{n}+\frac{4(n^2-3n+2)}{3n^2}+\ldots$(non-negative terms) $=1+2+2-\frac{2}{n}+\frac{4}{3}-\frac{4}{n}+\left(\frac{8}{3n^2}+\right)\ldots$(non-negative terms) Simplifies terms and cancels common factors (+… may be missing).	**M1**	2
	$\therefore\left(1+\frac{2}{n}\right)^n \geqslant \frac{19}{3}-\frac{6}{n}$ (as all other terms are non-negative.) * (Accept all other terms are positive, but S- if no reason is given.)	**A1*** **(S-)**	2
		(3)	
(b)	Expands to e.g. $\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(-\frac{x}{4}\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}\left(-\frac{x}{4}\right)^3+\ldots$ Enough terms to deduce pattern of signs should be given.	**M1** **(S+)**	1
	$\left(1-\frac{x}{4}\right)^{\frac{1}{2}} < 1-\frac{x}{8}$ since we can see all remaining terms are negative. Deduces inequality noting all remaining terms are negative. (Allow if $\leqslant$ used as equality can be rejected as $\sqrt{3}$ is not rational.)	**B1**	3
	(Series converges for $\lvert x\rvert < 4$ so) substituting $x=1$ into the equation gives $\left(\frac{3}{4}\right)^{\frac{1}{2}} < 1-\frac{1}{8}=\frac{7}{8}$ (M0 if attempt with an x with $\lvert x\rvert > 4$)	**(S+)** **M1**	2
	Hence $\frac{\sqrt{3}}{2} < \frac{7}{8} \Rightarrow \sqrt{3} < \frac{7}{4}$ * Simplifies and rearranges, no incorrect working seen. Reason for inequality must have been given.	**A1***	2
		(4)	
(c)	$f(9)=\left(1+\frac{2}{9}\right)^9-3^{9/6} \geqslant \frac{19}{3}-\frac{6}{9}-3\sqrt{3}$ using the result of (a)(ii)	**M1**	3
	$\geqslant \frac{17}{3}-3\times\frac{7}{4}=5\frac{2}{3}-5\frac{1}{4}=\frac{2}{3}-\frac{1}{4}>0$ using the result of (b)	**A1**	3
	$f(10)=\left(1+\frac{2}{10}\right)^{10}-3^{\frac{10}{6}}=\left(\frac{6}{5}\right)^{10}-\left(\sqrt[6]{3}\right)^{10}$	**M1**	1

	But $\left(\sqrt[6]{3}\right)^{10} > \left(\frac{6}{5}\right)^{10}$ (as $g(x) = x^{10}$ is an increasing function), so $f(10) < 0$ follows. (Correct reason must be given.)	**A1 (S+)**	3
	So $f(x)$ changes sign on [9,10], and as it is a continuous function, thus there is a root in the interval [9,10] Must have scored both M's.	**A1 (S-)**	2
		(5)	
	S1 mark: Award S1 for a clear and concise solution that scores 10+ marks without the S- or scores 9+ and includes an S+ point but no S-.	**S1**	2
		(1)	
			(12 +1 marks)

Notes:

(a) S+ for noting other terms are non-negative rather than positive, or considers cases $n \geqslant 3$ and $n = 1, 2$ separately.

(b) S+ for reasoning for why terms after the second are all negative.
S+ for considering the domain of convergence to give valid expression.
(c) S+ for mentioning increasing function.
S- if continuity not mentioned.

Question	Scheme	Marks	AOs
4(a)	The rectangle must lie under the parabola, so maximum width will occur when the top right vertex also lies on the parabola, ie recognises the symmetry and forms an equation. Allow a suitable sketch as evidence.	M1	1
	By symmetry about the line $x=5$, this occurs at $\left(10-a, \frac{1}{2}a(10-a)\right)$, hence width satisfies $w \leqslant 10-a-a=10-2a$ * Must be convincing reason.	A1*	2
		(2)	
(b)	Maximum area must occur for a full width rectangle, ie when $w=10-2a$	B1	2
	Thus max area occurs for $A=\frac{1}{2}a(10-a)\times(10-2a)$	M1	3
	Attempts $\frac{dA}{da}=\frac{1}{2}(10-2a)\times(10-2a)+\frac{1}{2}(10a-a^2)\times -2\left(=3a^2-30a+50\right)$ and sets $\frac{dA}{da}=0$ and attempts to find a	M1	3
	$\Rightarrow 3(a-5)^2-3\times25+50=0 \Rightarrow a=5\pm\sqrt{\frac{25}{3}}=5\pm\frac{5}{\sqrt{3}}$ — Any correct method to solve the quadratic.	M1	3
	(But need $0<a<5$ to give a valid rectangle and as area is zero at either end of this interval so)	(S+)	
	max area occurs when $a=5-\frac{5\sqrt{3}}{3}$ (oe simplified)	A1	2
		(5)	
(c)	Max square area needs $10-2a=\frac{1}{2}a(10-a) \Rightarrow a=\ldots$ — Sets up correct equation.	M1	3
	$20-4a=10a-a^2 \Rightarrow a^2-14a+20=(a-7)^2-49+20=0$ $\Rightarrow a=7\pm\sqrt{29}$ — Solves the quadratic, any valid means.	dM1	3
	But need $0<a<5$ (and $\sqrt{29}<7$) so $a=7-\sqrt{29}$ — Selects correct root.	(S+) A1*	3
		(3)	
(d)	If area is 36, then width is given by $w=\frac{36}{\frac{1}{2}a(10-a)}=\frac{72}{10a-a^2}$ (oe) Therefore need solutions to $\frac{72}{10a-a^2}\leqslant 10-2a$ OR need solutions to $\frac{1}{2}\left(a+\frac{72}{10a-a^2}\right)\left(10-\left(a+\frac{72}{10a-a^2}\right)\right)\leqslant\frac{1}{2}a(10-a)$ or other valid inequality in a set up e.g. $10a-a^2 \geqslant 10b-b^2 \Rightarrow (b-a)(b+a)+10(a-b)\geqslant 0 \Rightarrow 10-(a+b)\geqslant 0$ (as $a\neq b$) followed by substitution of $b=a+\frac{72}{10a-a^2}$ This mark is for a correct reasoning of the required *inequality*, If no reason is	B1	1

	given and/or it is an equation it is B0, but all other marks are possible.		
	Forms a suitable cubic using the maximum width and height (may be equation or inequation).	**M1**	3
	$\Rightarrow a^3 - 15a^2 + 50a - 36 \geqslant 0$ — Correct cubic achieved as equation or inequation.	**A1**	3
	Identifies $(a-1)$ as factor (factor theorem) and attempts to factorise $\Rightarrow (a-1)\left(a^2 - 14a + 36\right) \geqslant 0$	**M1**	3
	$a^2 - 14a + 36 = (a-7)^2 - 49 + 36 \Rightarrow$ CVs are $a = 1, 7 \pm \sqrt{13}$ — Finds CVs	**M1**	3
	(positive cubic with roots $1 < 7 - \sqrt{13}\ (< 5) < 7 + \sqrt{13}$ (as $3 < \sqrt{13} < 4$)) So possible values of a are $1 \leqslant a \leqslant 7 - \sqrt{13}$	**(S+)** **A1**	2
		(6)	
S1	S1 mark: Award S1 for a clear and concise solution that is - fully correct with no S- point or - that scores 13+ and includes an S+ point and no S-.	**(1)**	2
			(16 + 1 marks)

Notes:

(b) S+ for explaining clearly why the root outside $0 < a < 5$ is rejected.
(c) S+ for justifying the root lies in acceptable domain for *a*.
(c) S- for a cumbersome strategy. **S+** for justification of roots/which are in valid domain.

Question	Scheme		Marks	AOs
5(a)(i)	The expansion of $\tan(x-y)$ is incorrect between lines 2 and 3, the denominator should be $1+\tan x\tan y$		**B1**	2
	Between lines 3 and 4 when replacing tangents $\tan^2(xy)$ is not $(ab)^2$ - the student has incorrectly assumed $(\tan xy)^2=(\tan x\tan y)^2$		**B1**	2
	The factorisation between lines 5 and 6 is incorrect, eg $a-ab=a(1-b)$ not $a(1-ab)$		**B1**	2
			(3)	
(a)(ii)	Let $x=\arctan a$ and $y=\arctan b$. We have $\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}=\frac{a-b}{1+ab}$		**M1**	1
	Hence taking arctan of both sides gives $x-y=\arctan a-\arctan b=\arctan\left(\frac{a-b}{1+ab}\right)$ as required.		**A1* (S+)**	2
			(2)	
(b)(i)	Horizontal motion is given by $s_x=28\cos\theta\times t$ so ball is in motion for $t=\frac{10\sqrt{2}}{28\cos\theta}$ seconds to reach wall (oe equation in *t*).		**B1**	1
	Vertical motion relative to *A* is given by $s_y=28\sin\theta\times t-\frac{1}{2}gt^2$ OR vertical motion relative to ground given by $s_y=28\sin\theta\times t-\frac{1}{2}gt^2+1$ (Any correct set up for vertical motion acceptable.)		**B1 (S+)**	3
	Ball clears wall if $s_y>5$ (for motion relative to *A*) OR $s_y>6$ (if relative to ground. Must be consistent with their equation for vertical motion.		**M1**	3
	$\Rightarrow 28\sin\theta\times\frac{10\sqrt{2}}{28\cos\theta}-\frac{1}{2}g\left(\frac{10\sqrt{2}}{28\cos\theta}\right)^2>5$ oe	Substitutes for *t*	**M1**	3
	$\Rightarrow \tan\theta\times10\sqrt{2}-\frac{98}{10}\frac{100}{28\times28}\sec^2\theta>5$ $\Rightarrow 2\sqrt{2}\tan\theta-\frac{49}{14\times14}\left(1+\tan^2\theta\right)>1$	Attempts simplification and applies $\frac{\sin\alpha}{\cos\alpha}=\tan\theta$ and $\sec^2\alpha=1+\tan^2\theta$	**M1**	3
	$\Rightarrow \tan^2\theta-8\sqrt{2}\tan\theta+5>0$	Correct quadratic in $\tan\theta$	**A1**	3
	$\Rightarrow \tan\theta=\frac{8\sqrt{2}\pm\sqrt{\left(8\sqrt{2}\right)^2-4\times1\times5}}{2\times1}$	Attempts to solve quadratic – any acceptable method.	**M1**	3
	$\tan\theta=4\sqrt{2}\pm\sqrt{27}=4\sqrt{2}\pm3\sqrt{3}$ oe	Correct roots	**A1**	3

	$\beta-\alpha=\arctan\left(\frac{\left(4\sqrt{2}+3\sqrt{3}\right)-\left(4\sqrt{2}-3\sqrt{3}\right)}{1+\left(4\sqrt{2}+3\sqrt{3}\right)\left(4\sqrt{2}-3\sqrt{3}\right)}\right)=\arctan\left(\frac{6\sqrt{3}}{1+(32-27)}\right)$ $=\arctan\left(\sqrt{3}\right)=60°$		M1 A1 (S+)	3 3
			(10)	
(ii)	Probability that any given ball successfully clears the wall is given by $p=\frac{60°}{90°}=\frac{2}{3}$ *		B1*	2
			(1)	
(iii)	If X is the random variable "number of balls passing over wall out of 10 attempts" then $X \sim \mathrm{B}\left(10,\frac{2}{3}\right)$ (Realises binomial distribution needed)		B1 (S+)	3
	$P(X=2)={}^{10}C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^8$	Applies binomial formula	M1	3
	$\left(=\frac{10\times 9}{2}\times\frac{2^2}{3^{10}}=5\times\frac{4}{3^8}\right)=\frac{20}{3^8}$	Correct answer.	A1	3
			(3)	
(b)(iv)	Taking air resistance into account would decrease range, so the <u>probability of success would be reduced</u> and hence <u>the probability for 2 successes would **increase**.</u>		B1	3
			(1)	
S2	Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a fully correct solution that is slightly laboured but includes an S+ point • A succinct solution that scores 17 or more that includes at least one S+ point.		(2)	2 2

(20 + 2 marks)

Notes:

(a) S+ a clearly set out proof, with all notation etc correct.
S+ for indicating $a > b > 0$ implies $90° > \arctan a - \arctan b > 0$ so will give principle value for $\arctan\left(\tan\left(\arctan a - \arctan b\right)\right)$

(b) S+ for clear set out of the vertical motion equation and condition needed.
S+ for clear, concise solution
S+ for clear definition of random variable.

Alt : for first four marks in (b), if trajectory formula is used award as follows:

B1: Correct trajectory formula $y = x\tan\theta - \dfrac{gx^2}{2v_0^2\cos^2\theta}$ used (quoted and used or implied).

B1: Correct inequality formed.
M1: Attempt to use the trajectory formula – allow one slip if not quoted (but if quoted must be correct)
M1: Attempts to set up the inequality with correct height used.

Question	Scheme		Marks	AOs
6(a)(i)	$u = a - x \Rightarrow \dfrac{du}{dx} = -1 \Rightarrow \int_0^a f(x)\,dx = \int_a^0 f(a-u)(-du)$	Finds derivative and applies substitution.	**M1**	1
	$= \int_0^a f(a-u)\,du = \int_0^a f(a-x)\,dx$ *	Correct work, with limits treated correctly, to achieve given result.	**A1***	2
			(2)	
(a)(ii)	$\int_0^\pi x f(\sin x)\,dx = \int_0^\pi (\pi - x) f(\sin(\pi - x))\,dx$		**M1**	1
	$= \int_0^\pi \pi f(\sin x)\,dx - \int_0^\pi x f(\sin x)\,dx$ (since $\sin(\pi - x) = \sin x$ for all x)		**A1** **(S+)**	3
	Hence $2\int_0^\pi x f(\sin x)\,dx = \pi \int_0^\pi f(\sin x)\,dx = 2\pi \int_0^{\frac{\pi}{2}} f(\sin x)\,dx$ as f(sin x) is symmetric about the line with equation $x = \dfrac{\pi}{2}$		**M1** **(S+)**	2
	so $\int_0^\pi x f(\sin x)\,dx = \pi \int_0^{\frac{\pi}{2}} f(\sin x)\,dx$ *		**A1***	3
			(4)	
(b)	$\int_0^{\frac{\pi}{2}} \dfrac{\sin^n x}{\sin^n x + \cos^n x}\,dx = \int_0^{\frac{\pi}{2}} \dfrac{\sin^n\left(\frac{\pi}{2} - x\right)}{\sin^n\left(\frac{\pi}{2} - x\right) + \cos^n\left(\frac{\pi}{2} - x\right)}\,dx$ $= \int_0^{\frac{\pi}{2}} \dfrac{\cos^n x}{\cos^n x + \sin^n x}\,dx \left(= \int_0^{\frac{\pi}{2}} \dfrac{\cos^n x}{\sin^n x + \cos^n x}\,dx \right)$		**M1** **(S+)**	3
	Applies result from (a)(i) and uses symmetry of cos and sin.			
	So $\int_0^{\frac{\pi}{2}} \dfrac{\sin^n x}{\sin^n x + \cos^n x}\,dx + \int_0^{\frac{\pi}{2}} \dfrac{\cos^n x}{\sin^n x + \cos^n x}\,dx = \int_0^{\frac{\pi}{2}} 1\,dx$	Realises the need to add the two integrals.	**M1**	2
	Thus	Uses the equality of integrals to achieve expression in just initial integral.	**M1**	3

	$2\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x+\cos^n x}\,\mathrm{d}x=[x]_0^{\frac{\pi}{2}}=\frac{\pi}{2}$			
	Thus $\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x+\cos^n x}\,\mathrm{d}x=\frac{\pi}{4}$ for any n, hence the integral is independent of n and its value is $\frac{\pi}{4}$	Correct values and explanation of independence.	**A1**	2
			(4)	
(c)(i)	$\frac{\cos x}{1+\cos x}=\frac{1+\cos x-1}{1+\cos x}=1-\frac{1}{1+\cos x}$	Splits the fraction, or if in reverse, combines the separate fractions.	**M1**	2
	$=1-\frac{1}{1+\left(2\cos^2\left(\frac{x}{2}\right)-1\right)}$	Use of correct double angle formula to get expression in terms of $\cos\left(\frac{x}{2}\right)$ (at any stage)	**B1**	1
	$=1-\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$ *	Correct result, no errors seen.	**A1***	2
Alt for (i)	$\frac{\cos x}{1+\cos x}=\frac{\left(2\cos^2\left(\frac{x}{2}\right)-1\right)}{1+\left(2\cos^2\left(\frac{x}{2}\right)-1\right)}$ $=\frac{2\cos^2\left(\frac{x}{2}\right)-1}{2\cos^2\left(\frac{x}{2}\right)}=\ldots$	Attempts double angle formula substitution and proceeds to simplify	**M1**	2
	$\cos(x)=2\cos^2\left(\frac{x}{2}\right)-1$ used to eliminate $\cos(x)$	Use of correct double angle formula to get expression in terms of $\cos\left(\frac{x}{2}\right)$ (at any stage)	**B1**	1
	$\frac{\cos x}{1+\cos x}=1-\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$ *	Correct result, no errors seen.	**A1***	2
(c)(ii)	$\int_0^{\pi} \frac{x\sin x}{1+\sin x}\,\mathrm{d}x=\pi\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\sin x}\,\mathrm{d}x$	Applies result of (a)(ii)	**M1**	3

	$=\pi\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)}\,\mathrm{d}x=\pi\int_0^{\frac{\pi}{2}}\frac{\cos x}{1+\cos x}\,\mathrm{d}x$	Uses the results to achieve integral in terms of cos x only.	**A1 (S+)**	2
	$=\pi\int_0^{\frac{\pi}{2}}1-\frac{1}{2}\sec^2\left(\frac{x}{2}\right)\mathrm{d}x=\pi\left[x-\tan\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}$	Uses result from (i) and integrates correctly.	**M1**	3
	$=\pi\left(\frac{\pi}{2}-1\right)=\frac{\pi^2}{2}-\pi$ oe		**A1**	2
			(7)	
(d)	$\int_0^{\pi}\frac{x\sin^4 x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x=\int_0^{\pi}\frac{x\sin^4 x}{\sin^4 x+\left(1-\sin^2 x\right)^2}\,\mathrm{d}x$	Correct integral in terms of sin only, or clear explanation that $\cos^4 x=(1-\sin^2 x)^2$ so integral is really an integral in sin$x$	**B1 (S+)**	3
	$=\pi\int_0^{\frac{\pi}{2}}\frac{\sin^4 x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x$	Applies the result of (a)(ii) to their adapted integral.	**M1**	3
	$=\pi\times\frac{\pi}{4}$	Applies the result of (b)	**M1**	3
	$=\frac{\pi^2}{4}$	Correct result obtained.	**A1**	3
Alt for(d)	$\int_0^{\pi}\frac{x\sin^4 x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x=\int_0^{\pi}\frac{(\pi-x)\sin^4(\pi-x)}{\sin^4(\pi-x)+\cos^4(\pi-x)}\,\mathrm{d}x$ $=\int_0^{\pi}\frac{(\pi-x)\sin^4 x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x$		**B1 (S+)**	3
	Deduces correct integral, using $\sin(\pi-x)=\sin x$ and $\cos^4(\pi-x)=(-\cos x)^4=\cos^4 x$			
	$\Rightarrow 2\int_0^{\pi}\frac{x\sin^4 x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x=\pi\int_0^{\pi}\frac{\sin^4 x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x$		**M1**	3

	$\int_0^{\pi} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}\,dx = \int_0^{\pi} \frac{\cos^4 x}{\sin^4 x+\cos^4 x}\,dx$ (as both have period π so equal by symmetry) hence $2\int_0^{\pi} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}\,dx = [x]_0^{\pi} = \pi$	**M1** **(S+)**	3
	$\Rightarrow \int_0^{\pi} \frac{x\sin^4 x}{\sin^4 x+\cos^4 x}\,dx = \frac{\pi}{2}\times\frac{\pi}{2} = \frac{\pi^2}{4}$	**A1**	3
		(4)	
S2	Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a fully correct solution that is slightly laboured but includes and S+ point • a score of >18 but solution is otherwise succinct or contains and S+ point	**(2)**	2 2
		(21 + 2 marks)	

Notes:

(a),(b),(c),(d) S+ for a clear explanation of the symmetries of sin *x* and cos x used.
(a) S+ for clear demonstration of the change of limits.
(d) S+ for concise reasoning, developing the theme.

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