Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Award

Centre Number | Candidate Number

Wednesday 26 June 2019

Morning (Time: 3 hours) | Paper Reference **9811/01**

Advanced Extension Award Mathematics

You must have:
Mathematical Formulae and Statistical Tables

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may not be used**.
- You must **show all your working**.
- Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{2}$, $3\sqrt{2}$.

Information

- The total mark for this paper is 100 of which **7** marks are for style and clarity of presentation.
- The style and clarity of presentation marks will be indicated as **(+S1) or (+S2)**.
- There are 7 questions in this question paper.
- The marks for each question are shown in brackets.
- The total mark for each question is shown at the end of the question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. (a) By writing $u = \log_4 r$, where $r > 0$, show that

$$\log_4 r = \frac{1}{2}\log_2 r$$

(2)

(b) Solve the equation

$$\log_4\left(5x^2 - 11\right) = \log_2\left(3x - 5\right)$$

(5)

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Question 1 continued

(Total for Question 1 is 7 marks)

2. The discrete random variable X follows the binomial distribution

$$X \sim \mathrm{B}(n, p)$$

where $0 < p < 1$. The mode of X is m.

(a) Write down, in terms of m, n and p, an expression for $\mathrm{P}(X = m)$

(1)

(b) Determine, in terms of n and p, an interval of width 1, in which m lies.

(5)

(c) Find a value of n where $n > 100$, and a value of p where $p < 0.2$, for which X has **two** modes. For your chosen values of n and p, state these two modes.

(2)

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Question 2 continued

(Total for Question 2 is 8 marks)

3. Given that $\phi = \frac{1}{2}\left(\sqrt{5} + 1\right)$,

(a) show that

(i) $\phi^2 = \phi + 1$

(ii) $\frac{1}{\phi} = \phi - 1$

(4)

(b) The equations of two curves are

$$y = \frac{1}{x} \qquad x > 0$$

and $$y = \ln x - x + k \qquad x > 0$$

where k is a positive constant.
The curves touch at the point P.
Find in terms of ϕ

(i) the coordinates of P,

(ii) the value of k.

(6)

(+S1)

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Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

4. (a) Prove the identity

$$(\sin x + \cos y)\cos(x - y) \equiv (1 + \sin(x - y))(\cos x + \sin y)$$

(5)

(b) Hence, or otherwise, show that

$$\frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} \equiv \frac{1 + \tan\theta}{1 - \tan\theta}$$

(6)

(c) Given that $k > 1$, show that the equation $\frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} = k$ has a unique solution in the interval $0 < \theta < \frac{\pi}{4}$

(4)

(+S2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 17 marks)

5. Points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$, respectively, relative to an origin O, and are such that OAB is a triangle with $OA = a$ and $OB = b$.

The point C, with position vector $\mathbf{c}$, lies on the line through O that bisects the angle AOB.

(a) Prove that the vector $b\mathbf{a} - a\mathbf{b}$ is perpendicular to $\mathbf{c}$.

(4)

The point D, with position vector $\mathbf{d}$, lies on the line AB between A and B.

(b) Explain why $\mathbf{d}$ can be expressed in the form $\mathbf{d} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ for some scalar λ with $0 < \lambda < 1$

(2)

(c) Given that D is also on the line OC, find an expression for λ in terms of a and b only and hence show that

$$DA : DB = OA : OB$$

(8)

(+S2)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 16 marks)

6. Figure 1 shows a sketch of part of the curve with equation $y = x\sin(\ln x)$, $x \geqslant 1$

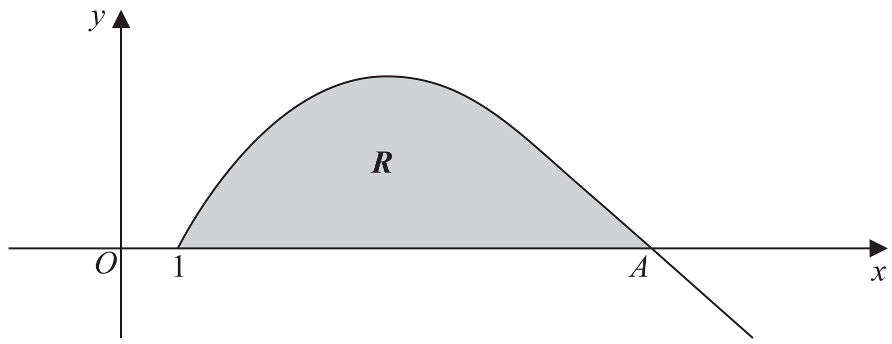

Figure 1

For $x > 1$, the curve first crosses the x-axis at the point A.

(a) Find the x coordinate of A.

(3)

(b) Differentiate $x\sin(\ln x)$ and $x\cos(\ln x)$ with respect to x and hence find

$$\int \sin(\ln x)\,\mathrm{d}x \text{ and } \int \cos(\ln x)\,\mathrm{d}x$$

(7)

(c) (i) Find $\int x\sin(\ln x)\,\mathrm{d}x$.

(ii) Hence show that the area of the shaded region ***R***, bounded by the curve and the x-axis between the points (1, 0) and A, is

$$\frac{1}{5}\left(\mathrm{e}^{2\pi} + 1\right)$$

(9)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 19 marks)

7. Figure 2 shows a rectangular section of marshland, $OABC$, which is a metres long by b metres wide, where $a > b$.

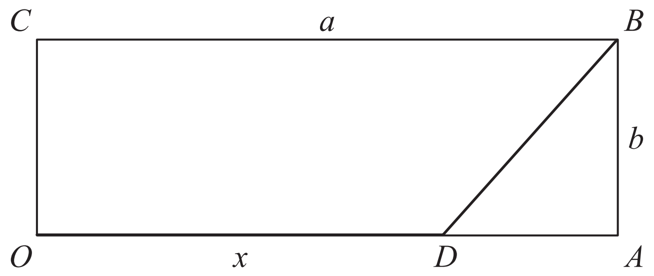

Figure 2

Edgar intends to get from O to B in the shortest possible time. In order to do this, he runs along edge OA for a distance x metres ($0 \leqslant x < a$) to the point D before wading through the marsh directly from D to B.

Edgar can wade through the marsh at a constant speed of $1\,\text{m s}^{-1}$, and he can run along the edge of the marsh at a constant speed of $\lambda\,\text{m s}^{-1}$, where $\lambda > 1$

(a) By finding an expression in terms of x for the time taken, t seconds, for Edgar to reach B from O, show that

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{\lambda} - \frac{a - x}{\sqrt{(a-x)^2 + b^2}}$$

(5)

(b) (i) Find, in terms of a, b and λ, the value of x for which $\frac{\mathrm{d}t}{\mathrm{d}x} = 0$

(ii) Show that this value of x lies in the interval $0 \leqslant x < a$ provided $\lambda \geqslant \sqrt{1 + \frac{b^2}{a^2}}$

(iii) For λ in this range, show that the value of x found in (b)(i) gives a minimum value of t.

(8)

(c) Find the minimum time taken for Edgar to get from O to B if

(i) $\lambda \geqslant \sqrt{1 + \frac{b^2}{a^2}}$

(ii) $1 < \lambda < \sqrt{1 + \frac{b^2}{a^2}}$

(4)

Edgar's friend, Frankie, also runs at a constant speed of $\lambda\,\text{m s}^{-1}$. Frankie runs along the edges OA and AB. Given that $\lambda \geqslant \sqrt{1 + \frac{b^2}{a^2}}$

(d) find the range of values of λ for which Frankie gets to B from O in a shorter time than Edgar's minimum time.

(3)

(+S2)

P 6 0 7 0 2 A 0 2 2 2 8

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 22 marks)

FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS
TOTAL FOR PAPER: 100 MARKS

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