Paper Reference(s)

9801/01

Edexcel

Mathematics

Advanced Extension Award

Friday 30 June 2017 – Morning

Time: 3 hours

Materials required for examination	**Items included with question papers**
Answer book (AB16)	Nil
Graph paper (ASG2)	
Mathematical Formulae (Pink)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.
Check that you have the correct question paper.
Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
There are 8 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The function f is given by

$$f(x) = \sqrt{x+2} \quad \text{for} \quad x \in \mathbb{R}, x \geqslant 0$$

(a) Find $f^{-1}(x)$ and state the domain of f^{-1}

(2)

The function g is given by

$$g(x) = x^2 - 4x + 5 \quad \text{for} \quad x \in \mathbb{R}, x \geqslant 0$$

(b) Find the range of g.

(2)

(c) Solve the equation $fg(x) = x$.

(3)

(Total 7 marks)

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2. (a) Show that the equation

$$\tan x = \frac{\sqrt{3}}{1 + 4\cos x}$$

can be written in the form

$$\sin 2x = \sin(60° - x)$$

(4)

(b) Solve, for $0 < x < 180°$

$$\tan x = \frac{\sqrt{3}}{1 + 4\cos x}$$

(5)

(Total 9 marks)

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3. The line L_1 has equation $\mathbf{r} = \begin{pmatrix} -13 \\ 7 \\ -1 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$.

The line L_2 passes through the point A with position vector $\begin{pmatrix} 1 \\ p \\ 10 \end{pmatrix}$ and is parallel to $\begin{pmatrix} -2 \\ 11 \\ -5 \end{pmatrix}$, where p is a constant.

The lines L_1 and L_2 intersect at the point B.

(a) Find (i) the value of p,

(ii) the position vector of B.

(5)

The point C lies on L_1 and angle ACB is 90°

(b) Find the position vector of C.

(5)

The point D also lies on L_1 and triangle ABD is isosceles with $AB = AD$.

(c) Find the area of triangle ABD.

(3)

(Total 13 marks)

4.

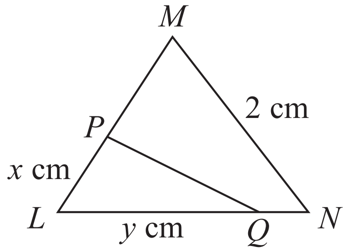

Figure 1

Figure 1 shows the equilateral triangle LMN of side 2 cm. The point P lies on LM such that $LP = x$ cm and the point Q lies on LN such that $LQ = y$ cm. The points P and Q are chosen so that the area of triangle LPQ is half the area of triangle LMN.

(a) Show that $xy = 2$

(2)

(b) Find the shortest possible length of PQ, justifying your answer.

(5)

Mathematicians know that for any closed curve or polygon enclosing a fixed area, the ratio $\dfrac{\text{area enclosed}}{\text{perimeter}}$ is a maximum when the closed curve is a circle.

By considering 6 copies of triangle LMN suitably arranged,

(c) find the length of the shortest line or curve that can be drawn from a point on LM to a point on LN to divide the area of triangle LMN in half. Justify your answer.

(6)

(Total 13 marks)

5.

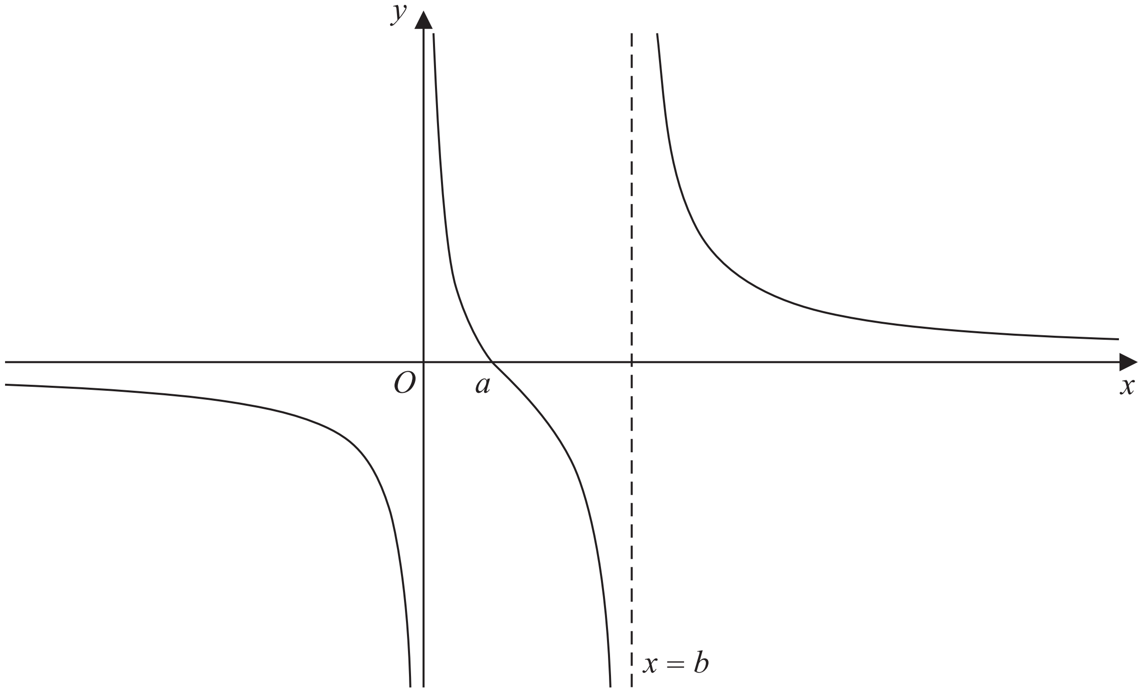

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4(x-1)}{x(x-3)}$$

The curve cuts the x-axis at $(a, 0)$. The lines $y = 0$, $x = 0$ and $x = b$ are asymptotes to the curve.

(a) Write down the value of a and the value of b.

(2)

(b) On separate axes, sketch the curves with the following equations. On your sketches, you should mark the coordinates of any intersections with the coordinate axes and state the equations of any asymptotes.

(i) $y = \mathrm{f}(x + 2) - 4$

(6)

(ii) $y = \mathrm{f}(|x|) - 3$

(6)

(Total 14 marks)

6. (a) Show that

$$\frac{d}{du}\ln\left(u+\sqrt{u^2-1}\right)=\frac{1}{\sqrt{u^2-1}}$$

(2)

(b) Use the result from part (a) and the substitution $x+3=\frac{1}{t}$ to find

$$\int \frac{1}{(x+3)\sqrt{2x+7}}\,dx$$

(6)

(c) Express $\frac{1}{2x^2+13x+21}$ in partial fractions.

(2)

(d) Find

$$\int_1^9 \frac{1}{\left(2x^2+13x+21\right)\sqrt{2x+7}}\,dx$$

giving your answer in the form $\ln r - s$ where r and s are rational numbers.

(6)

(Total 16 marks)

7.

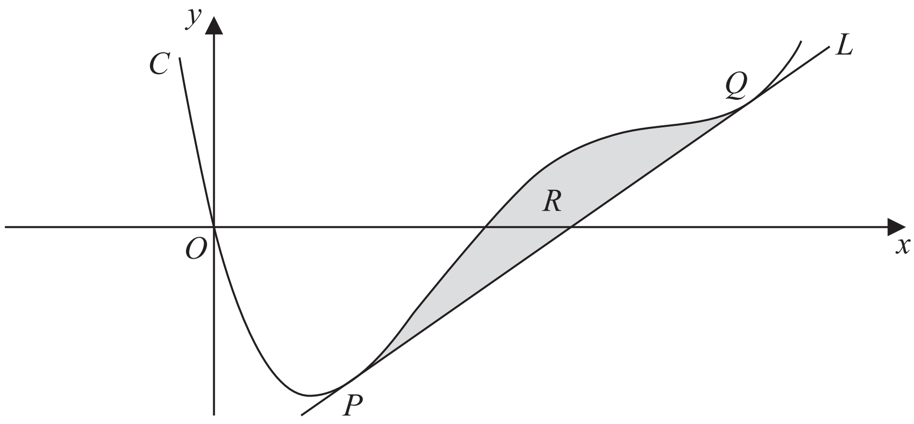

Figure 3

Figure 3 shows part of the curve C with equation $y = x^4 - 10x^3 + 33x^2 - 34x$ and the line L with equation $y = mx + c$.

The line L touches C at the points P and Q with x coordinates p and q respectively.

(a) Explain why

$$x^4 - 10x^3 + 33x^2 - (34 + m)x - c = (x - p)^2(x - q)^2$$

(2)

The finite region R, shown shaded in Figure 3, is bounded by C and L.

(b) Use integration by parts to show that the area of R is $\dfrac{(q-p)^5}{30}$

(6)

(c) Show that

$$(x - p)^2(x - q)^2 = x^4 - 2(p + q)x^3 + Sx^2 - Tx + U$$

where S, T and U are expressions to be found in terms of p and q.

(5)

(d) Using part (a) and part (c) find the value of p, the value of q and the equation of L.

(8)

(Total 21 marks)

FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS
TOTAL FOR PAPER: 100 MARKS

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