# Mark Scheme (Results)

## Summer 2016

Pearson Edexcel Advanced Extension Award Mathematics (9801/01)

## Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

## Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2016
Publications Code 9801_01_1606_MS

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL AEA MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 100

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper or ag- answer given
- ⎿ or d… The second mark is dependent on gaining the first mark

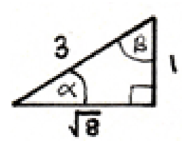
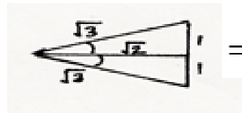
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

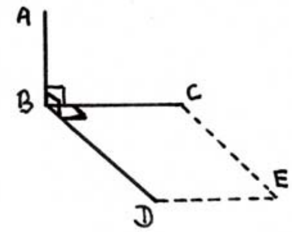
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

| Qu | Scheme | Mark | Notes |
|---|---|---|---|
| **1 (a)** | f(3) = 6 | M1 | Attempt f(3) |
| | [ $f(x)=(x-2)^2+5$ so min (2, 5) therefore] range is **$f \geqslant 6$** | A1 | |
| | | (2) | |
| **(b)** | $gf(x)=\dfrac{10}{x^2-4x+10}$ (o.e.) | B1 | |
| | | (1) | |
| **(c)** | Domain of gf is domain of f i.e. **$x \geqslant 3$** | B1 | |
| | Range of f is $f \geqslant 6$ so smallest $x$ we can put into g is 6 so g(6) = $\frac{10}{7}$ | M1 | Attempt g(6) o.e. |
| | [As $x\to\infty$ then gf and g $\to 0$ so] **gf > 0** | B1 | Not just in words |
| | Therefore range of gf is [ 0 < ] gf $\leqslant \frac{10}{7}$ | A1 | Allow B0A1 for gf $\leqslant \frac{10}{7}$ |
| | | (4) | |
| | | **(7)** | |

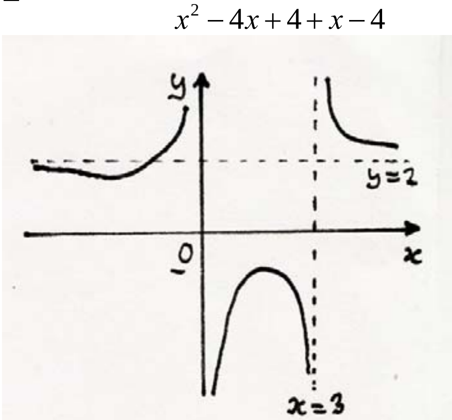
| Qu | Scheme | Mark | Notes |
|---|---|---|---|
| **2** | $\arccos\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$   | B1 | |
| | $\arcsin\left(\frac{1}{3}\right)=\alpha \Rightarrow \sin\alpha=\frac{1}{3}$ | B1 | $\sin\alpha=\ldots$ and $\Delta$ o.e. with $\alpha$ indicated |
| | $2\arctan\left(\frac{1}{\sqrt{2}}\right)=\beta \Rightarrow \tan\left(\frac{\beta}{2}\right)=\frac{1}{\sqrt{2}}$ or use cosine rule or | M1 | Relevant use of $\tan\left(\frac{\beta}{2}\right)$ |
| | $\tan\beta=\dfrac{2\times\frac{1}{\sqrt{2}}}{1-\left(\frac{1}{\sqrt{2}}\right)^2},=2\sqrt{2}$   $\Rightarrow \cos\beta=\frac{1}{3}$ | M1<br>A1 | M1 for attempt to use $\tan(2A)$ formula or get trig $(\beta)$ |
| | Use of triangle or some other link between $\alpha$ and $\beta$ | M1 | |
| | So answer is $\frac{\pi}{4}+\frac{\pi}{2}=\frac{3\pi}{4}$ | A1 | |
| | | **(7)** | |

| Qu | Scheme | Mark | Notes |
|---|---|---|---|
| **3 (a)** | $\overrightarrow{AB}=\begin{pmatrix}-1\\-5\\-7\end{pmatrix}$ $\overrightarrow{AC}=\begin{pmatrix}3\\-3\\-9\end{pmatrix}$ $\overrightarrow{AD}=\begin{pmatrix}-5\\0\\-10\end{pmatrix}$ $\overrightarrow{BC}=\begin{pmatrix}4\\2\\-2\end{pmatrix}$ $\overrightarrow{BD}=\begin{pmatrix}-4\\5\\-3\end{pmatrix}$ $\overrightarrow{CD}=\begin{pmatrix}-8\\3\\-1\end{pmatrix}$ | M1 | Attempt at least 3 (at least 2 correct) Can be vetors or \|..\| |
| | <u>or</u> $\lvert\overrightarrow{AB}\rvert=\sqrt{75}, \lvert\overrightarrow{AC}\rvert=\sqrt{99}, \lvert\overrightarrow{AD}\rvert=\sqrt{125}, \lvert\overrightarrow{BC}\rvert=\sqrt{24}, \lvert\overrightarrow{BD}\rvert=\sqrt{50}, \lvert\overrightarrow{CD}\rvert=\sqrt{74}$ | | |
| | $\overrightarrow{AB}\bullet\overrightarrow{BC}=-4-10+14=0$<br>$\overrightarrow{AB}\bullet\overrightarrow{BD}=4-25+21=0$<br>$\overrightarrow{BC}\bullet\overrightarrow{BD}=-16+10+6=0$<br><br> | dM1<br><br>A1,A1 | Seek $\perp$ vectors. Check at least one correct pair (Pythag or $\bullet$ )<br><br>2 edges checked, Edges: *AB, BC, BD* |
| | $\overrightarrow{OE}=\overrightarrow{OD}+\overrightarrow{DE}=\overrightarrow{OD}+\overrightarrow{BC}$ (o.e.) | M1 | Correct expression |
| | So $\overrightarrow{OE}=\begin{pmatrix}0\\4\\-13\end{pmatrix}$ | A1<br>(6) | No proof that *AB, BC, BD* (oe) are edges scores max M1M0A0A0M1A1 |
| **(b)** | $\lvert\overrightarrow{AB}\rvert=\sqrt{1+25+49}=\sqrt{75}$ similarly $\lvert\overrightarrow{BC}\rvert=\sqrt{24}$ and $\lvert\overrightarrow{BD}\rvert=\sqrt{50}$ | M1 | Attempt 3 relevant lengths & try area |
| | volume $=\frac{1}{3}\times\left(\frac{1}{2}\times BC\times BD\right)\times AB=\frac{1}{3}\times\frac{1}{2}\times 2\sqrt{6}\times 5\sqrt{2}\times 5\sqrt{3}$ | M1 | Suitable expression could be ***p.q***x***r***/6 |
| | $=$ <u>**50**</u> | A1<br>(3)<br>**(9)** | |

| Qu | Scheme | Mark | Notes |
|---|---|---|---|
| **4(a)** | $\log_x y = k \Rightarrow x^k = y \Rightarrow y^n = ...$ <u>or</u> $\log_x y^n = nk \Rightarrow y^n = ..$ <u>or</u> base change | M1 | Out of logs and $y^n$ attempt or suitable first step. |
| | $y^n = \left(x^k\right)^n = x^{nk} = \left(x^n\right)^k$ therefore $\log_{x^n} y^n = k = \log_x y$ **(*)** | A1cso | |
| | | (2) | |
| **(b)(i)** | LHS = $4\log_2 u$ | M1 | All same base and + |
| | $\therefore \log_2 u = \frac{5}{4}$ so $\underline{u = 2^{\frac{5}{4}}}$ | A1 | Allow $p$ = 1.25 oe |
| | | (2) | |
| **(ii)** | $\log_{16} v^4 + \log_{16} v^2 + \log_{16} v^{\frac{4}{3}} + \log_{16} v$ <u>or</u> $\log_2 v + \log_2 v^{\frac{1}{2}} + \log_2 v^{\frac{1}{3}} + \log_2 v^{\frac{1}{4}}$ | M1 | All same base |
| | $= \log_{16} v^{\frac{25}{3}}$ $= \log_2 v^{\frac{25}{12}}$ | M1 | Single log |
| | so $v = 2^{\frac{60}{25}} = \underline{2^{\frac{12}{5}}}$ | A1 | Allow $p$ = 2.4 oe |
| | | (3) | |
| **(iii)** | LHS = $\log_2 w + \dfrac{3\times 2}{\log_2 w}$ | M1 | Evaluate $\log_8 64$ and logs to same base |
| | Sub $t = \log_2 w$ gives $t^2 - 5t + 6 = 0$ or $(t-3)(t-2) = 0$ | M1 | Reduce to 3TQ |
| | $\log_2 w = 2 \Rightarrow w = \underline{2^2}$ and $\log_2 w = 3 \Rightarrow w = \underline{2^3}$ (accept 4 and 8 ) | A1,A1 | 1st A1 for 2 and 3<br>2nd A1 for 4 and 8 |
| | | (4)<br>**(11)** | |

| Qu | Scheme | Mark | Notes |
|---|---|---|---|
| 5(a) | GP with $a=1, r=\frac{1}{x}$ so $S_{n+1}=\frac{1\left(1-x^{-(n+1)}\right)}{1-x^{-1}}$ | M1 | Identify corr. GP and attempt sum. Allow $n$ not $(n+1)$ |
| | Multiply T and B by $x$ and open bracket $=\underline{\frac{x}{x-1}-\frac{x^{-n}}{x-1}}$ | A1cso | No incorrect working seen |
| | | (2) | |
| (b) | $\frac{d}{dx}\left(\sum_{r=0}^{n} x^{-r}\right)=-\sum_{r=0}^{n} rx^{-(r+1)}$ so differentiate | M1 | Identify need to differentiate & $\underline{+}$ LHS |
| | $\frac{d}{dx}(\text{RHS})=\left[\frac{(x-1).1-x.1}{(x-1)^2}\right]-\left[\frac{(x-1).\left(-nx^{-(n+1)}\right)-x^{-n}.1}{(x-1)^2}\right]$ | M1 | Some correct use of quotient rule |
| | So $\sum_{r=0}^{n} rx^{-(r+1)}=\left[\frac{1}{(x-1)^2}+\left\{\frac{-nx^{-n}+nx^{-(n+1)}-x^{-n}}{(x-1)^2}\right\}\right]$ | A1 | Correct diff'n RHS |
| | $=\underline{\frac{1+nx^{-(n+1)}-(n+1)x^{-n}}{(x-1)^2}}$ (o.e.) | A1 | A correct expre'n. Common denom' Like terms collected. |
| | | (4) | |
| (c) | Sum = $\sum_{r=0}^{n} 3\times 2^{-r}+\sum_{r=0}^{n} 5r\times 2^{-r}$ | M1 | Split sum |
| | $\sum_{r=0}^{n} 3\times 2^{-r}=3(2-2^{-n})$ | M1<br>A1 | Use of $x$ = 2 in (a) |
| | $\sum_{r=0}^{n} 5r\times 2^{-r}=10\sum_{r=0}^{n} r\times 2^{-(r+1)}=,10\left[1+\frac{n\times 2^{-(n+1)}-(n+1)\times 2^{-n}}{1}\right]$ | M1, | Into form for (b) ($x$ = 2 & 10 needed) |
| | | dM1 | Use $x$ = 2 in their(b) |
| | Sum $=16-\frac{(13+5n)}{2^n}$ or ***a* = 16, *b* = 13** and ***c* = 5** | A2<br>(7) | −1 eeoo |
| | | **(13)** | |

| Qu | Scheme | Mark | Notes |
| --- | --- | --- | --- |
| **6 (a)** | $y' = -\sin(\cos x)\times(-\sin x)\times\sin x + \cos(\cos x)\times\cos x$ | M1 | Use of prod. Rule 2 terms - allow slips |
| | $= \sin(\cos x)\times\sin^2 x + \cos(\cos x)\times\cos x$ | A1+A1<br>(3) | |
| **(b)** | Sub $x = \frac{\pi}{2} \Rightarrow y' = \sin(0).\sin^2(\frac{\pi}{2}) + \cos(\frac{\pi}{2}).\cos(0) = 0.1 + 0.1 = 0$ | B1ft | Shows TP correctly<br>(OK for their $y'$) |
| | $y$ coordinate is $\cos(\cos\frac{\pi}{2}).\sin\frac{\pi}{2} = \cos(0).1 = 1.1 = 1$ | B1<br>(2) | |
| **(c)** | $\int \cos(\cos x).\sin x \, dx = -\sin(\cos x)$ | M1A1 | Suitable method<br>A1 needs – |
| | Area = $\left[-\sin(\cos x)\right]_0^{\pi} = \left(-\sin(-1)\right) - \left(-\sin(1)\right)$ | dM1 | Correct use of limits |
| | = **<u>2sin1</u>** | A1<br>(4) | <u>Not</u> sin(1) – sin(–1) |
| **(d)**<br>S+ for $\sin x \neq 0$ comment | $\sin(\cos x).\sin x = \cos(\cos x).\sin x \Rightarrow, \tan(\cos x) = 1$ (with $\sin x$ struck out on both sides) | M1,M1 | Form eq' and cancel<br>Get to tan(cos$x$) o.e. |
| | So $\cos x = \frac{\pi}{4}$ i.e. $a$ ( or $x$) = <u>$\arccos(\frac{\pi}{4})$</u> | A1 | |
| | Triangle or method for sin$x$ ,so $b = \frac{1}{\sqrt{2}}\times\frac{\sqrt{16-\pi^2}}{4}$ or $\frac{\sqrt{32-2\pi^2}}{8}$ | M1,<br>A1<br>(5) | Dep on non-trivial $x$<br>(Must see $\pi$)<br>A1 for correct surd |
| **(e)** | $\int_0^a \sin(\cos x)\sin x \, dx = \left[\cos(\cos x)\right]_0^a$ (outer curve) | M1<br>A1 | Suitable method |
| | $= \cos(\frac{\pi}{4}) - \cos 1$ ,= $\frac{1}{\sqrt{2}} - \cos 1$ | M1,A1 | M1 ft use of their limits |
| | $\left\{\int_0^a \cos(\cos x)\sin x \, dx = \left[-\sin(\cos x)\right]_0^a\right\} = -\sin(\frac{\pi}{4}) + \sin 1$ (inner curve) | M1 | Use of their limits. Follow through their integ' from (c) |
| | $= -\frac{1}{\sqrt{2}} + \sin 1$ | A1 | |
| | Shaded area = $\frac{1}{\sqrt{2}} - \cos 1 - \left[-\frac{1}{\sqrt{2}} + \sin 1\right]$ | M1 | Ft their outer - inner |
| | = <u>$\sqrt{2} - \left[\cos 1 + \sin 1\right]$</u> | A1<br>(8)<br>**(22)** | |

| Qu | Scheme | Mark | Notes |
|---|---|---|---|
| **7 (a)** | $x^2+3x+8=kx^2+kx-2k \;\Rightarrow\; 0=(k-1)x^2+(k-3)x-(2k+8)$ | M1 | 3TQ in $x$ (2 correct coeff'n $\pm$ 1 & $\pm$ 3) |
| | No real roots so "$b^2-4ac<0$" $\Rightarrow (k-3)^2+4(k-1)(2k+8)\;[<0]$ | M1 | Attempt discrim' |
| | So $9k^2+18k-23[<0]$ | M1A1 | Form 3TQ in $k$ |
| | $(k+1)^2-1-\frac{23}{9}[<0]$ | M1 | Attempt cvs. At least 1 |
| | $k=-1\pm\frac{4\sqrt{2}}{3}$ so $\underline{-1-\frac{4\sqrt{2}}{3}<k<-1+\frac{4\sqrt{2}}{3}}$ (o.e.) | A1cso | Correct limits<br>[S+ clearly use <0] |
| | | (6) | |
| **(b)** | $x^2+x-2=(x+2)(x-1)$ | M1 | Factorize denom |
| | $x=-2,\; x=1$ or $a=-2$ and $b=1$ | A1A1 | |
| | Division or limits of $x$ $y=1$ or $c=1$ | B1 | |
| | | (4) | |
| **(c)** | f($x$) = 2 $\Rightarrow$ $x^2-x-12=0$ | M1 | Form suitable eqn |
| | i.e. $(x-4)(x+3)=0$ so $x=4$ or $-3$ | M1 | Solve as far as $x$ = |
| | Coordinates are **(−3, 2)** and **(4, 2)** | A1 | $x$ = −2 , $y$ = 2 etc is OK |
| | | (3) | |
| **(d)** | Check $r$ = 0 and −1 gives pairs **(0, −4)** and **(−1, −3)** | B1, B1 | |
| | Check $s$ = 1 gives **(−5, 1)** | M1 | M1 for $y$ = 1 check |
| | From (c) or listing $s$ = 2 gives **(−3, 2) and (4, 2)** | A1 | A1 for all 3 pairs<br>Extra points A0<br>[S+ for full arg.] |
| | Only then need to check $r$ = 2 and $r$ = 3 and neither yield a solution | | |
| | | (4) | |
| **(e)** | Consider denominator and see $m=-2$ | B1 | |
| | Consider coefficients of $x^2$giving $n=1$ | B1 | |
| | $\text{f}(x-2)+1=\frac{(x-2)^2+3(x-2)+8}{(x-2)^2+(x-2)-2}+1$ | M1 | Attempt to show by suitable substitution |
| | May use these for M1<br>$\text{f}(x)=1+\frac{-2}{x+2}+\frac{4}{x-1}$<br>$\text{g}(x)=2+\frac{-2}{x}+\frac{4}{x-3}$ | | |
| | $=\frac{x^2-4x+4+3x-6+8+x^2-4x+4+x-4}{x^2-4x+4+x-4}=\frac{2x^2-4x+6}{x^2-3x}$ | A1cso | |
| | | (4) | |
| **(f)** |  | | |
| | Horizontal translation to right <u>or</u> middle part (No crossing with $x$- axis) | B1ft | Follow through their $m$ |
| | Vertical translation up <u>or</u> LH & RH parts (LH must cross their $y$ = 2) | B1ft | Follow through their $n$ |
| | Correct asymptotes and no int' with axes | B1 | $x$ =3 and $y$ = 2 (Must be equations condone no $x$ = 0) |
| | | (3) | |
| | | **(24)** | |

| Awarding of S and T marks | | |
|---|---|---|
| **Questions** | **Marks** | |
| 2, 3, 4 | S1 | For a fully correct solution that is succinct or includes an S+ point |
| | | |
| 5, 6, 7 | S2 | For a fully correct solution that is succinct and includes some S+ points |
| 5, 6, 7 | S1 | For a fully correct solution that is succinct but does not mention any S+ points |
| 5, 6, 7 | S1 | For a fully correct solution that is slightly laboured but includes an S+ point |
| 5, 6, 7 | S1 | For a score of $n$ -1 but solution is otherwise succinct or contains an S+ point |
| **Maximum S score is 6** | | |
| ALL | T1 | For at least half marks on all questions |

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London WC2R 0RL