Paper Reference(s)

9801/01

Edexcel

Mathematics

Advanced Extension Award

Wednesday 24 June 2009 – Afternoon

Time: 3 hours

Materials required for examination	Items included with question papers
Answer book (AB16) Graph paper (ASG2) Mathematical Formulae (Orange or Green)	Nil

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.
Check that you have the correct question paper.
Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
There are 8 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) On the same diagram, sketch

$$y = (x + 1)(2 - x) \quad \text{and} \quad y = x^2 - 2|x|.$$

Mark clearly the coordinates of the points where these curves cross the coordinate axes.

(3)

(b) Find the x-coordinates of the points of intersection of these two curves.

(5)

(Total 8 marks)

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2. The curve C has equation $y = x^{\sin x}$, $\quad x > 0$.

(a) Find the equation of the tangent to C at the point where $x = \frac{\pi}{2}$.

(6)

(b) Prove that this tangent touches C at infinitely many points.

(3)

(Total 9 marks)

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3. (a) Solve, for $0 \leqslant \theta < 2\pi$,

$$\sin\left(\frac{\pi}{3} - \theta\right) = \frac{1}{\sqrt{3}}\cos\theta .$$

(5)

(b) Find the value of x for which

$$\arcsin(1 - 2x) = \frac{\pi}{3} - \arcsin x, \qquad 0 < x < 0.5$$

[$\arcsin x$ is an alternative notation for $\sin^{-1} x$]

(7)

(Total 12 marks)

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4. (a) The function $f(x)$ has $f'(x) = \frac{u(x)}{v(x)}$. Given that $f'(k) = 0$,

show that $f''(k) = \frac{u'(k)}{v(k)}$.

(3)

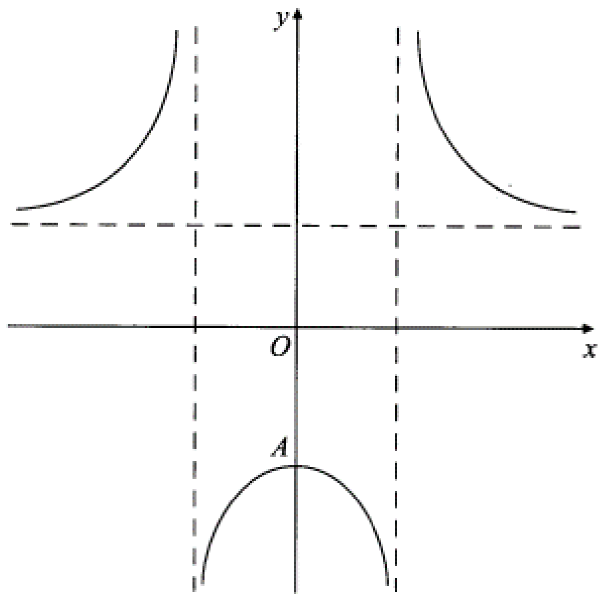

Figure 1

(b) The curve C with equation

$$y = \frac{2x^2 + 3}{x^2 - 1}$$

crosses the y-axis at the point A. Figure 1 shows a sketch of C together with its 3 asymptotes.

(i) Find the coordinates of the point A.

(1)

(ii) Find the equations of the asymptotes of C.

(2)

The point $P(a, b)$, $a > 0$ and $b > 0$, lies on C. The point Q also lies on C with PQ parallel to the x-axis and $AP = AQ$.

(iii) Show that the area of triangle PAQ is given by $\frac{5a^3}{a^2 - 1}$.

(2)

(iv) Find, as a varies, the minimum area of triangle PAQ, giving your answer in its simplest form.

(6)

(Total 14 marks)

5. (a) The sides of the triangle ABC have lengths $BC = a$, $AC = b$ and $AB = c$, where $a < b < c$. The sizes of the angles A, B and C form an arithmetic sequence.

(i) Show that the area of triangle ABC is $ac\frac{\sqrt{3}}{4}$.

(4)

Given that $a = 2$ and $\sin A = \frac{\sqrt{15}}{5}$, find

(ii) the value of b,

(2)

(iii) the value of c.

(4)

(b) The internal angles of an n-sided polygon form an arithmetic sequence with first term 143° and common difference 2°.

Given that all of the internal angles are less than 180°, find the value of n.

(5)

(Total 15 marks)

6.

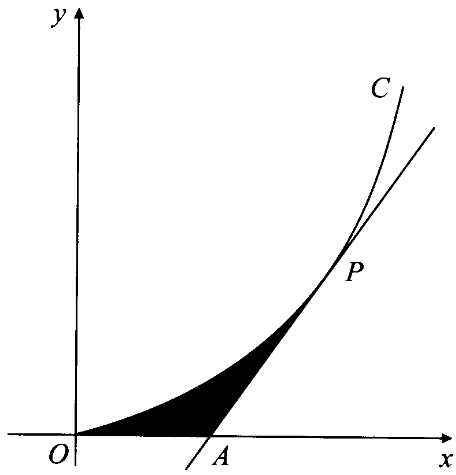

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\sin t, \qquad y = \ln(\sec t), \qquad 0 \leqslant t < \frac{\pi}{2}.$$

The tangent to C at the point P, where $t = \frac{\pi}{3}$, cuts the x-axis at A.

(a) Show that the x-coordinate of A is $\frac{\sqrt{3}}{3}(3 - \ln 2)$.

(6)

The shaded region R lies between C, the positive x-axis and the tangent AP as shown in Figure 2.

(b) Show that the area of R is $\sqrt{3}(1 + \ln 2) - 2\ln(2 + \sqrt{3}) - \frac{\sqrt{3}}{6}(\ln 2)^2$.

(11)

(Total 17 marks)

7. Relative to a fixed origin O the points A, B and C have position vectors

$$\mathbf{a} = -\mathbf{i} + \tfrac{4}{3}\mathbf{j} + 7\mathbf{k},\ \ \mathbf{b} = 4\mathbf{i} + \tfrac{4}{3}\mathbf{j} + 2\mathbf{k} \text{ and } \ \mathbf{c} = 6\mathbf{i} + \tfrac{16}{3}\mathbf{j} + 2\mathbf{k} \text{ respectively.}$$

(a) Find the cosine of angle ABC.

(3)

The quadrilateral $ABCD$ is a kite K.

(b) Find the area of K.

(3)

A circle is drawn inside K so that it touches each of the 4 sides of K.

(c) Find the radius of the circle, giving your answer in the form $p\sqrt{(q)} - q\sqrt{(p)}$, where p and q are positive integers.

(5)

(d) Find the position vector of the point D.

(7)

(Total 18 marks)

(For Style, Clarity and Presentation: 7 marks)

TOTAL FOR PAPER: 100 MARKS

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