Paper Reference(s)

## 9801/01

## Edexcel

## Mathematics

## Advanced Extension Award

Wednesday 25 June 2008 - Afternoon
Time: 3 hours

Materials required for examination Items included with question papers<br>Answer book (AB16)<br>Nil<br>Graph paper (ASG2)<br>Mathematical Formulae (Green)<br>Candidates may NOT use a calculator in answering this paper.

## Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initial(s) and signature.
Check that you have the correct question paper.
Answers should be given in as simple a form as possible, e.g. $\frac{2 \pi}{3}, \sqrt{ } 6,3 \sqrt{ }$.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper.
The total mark for this paper is 100 , of which 7 marks are for style, clarity and presentation.
There are 8 pages in this question paper. Any blank pages are indicated.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. The first and second terms of an arithmetic series are 200 and 197.5 respectively.

The sum to $n$ terms of the series is $S_{n}$.
Find the largest positive value of $S_{n}$.
2. The points $(x, y)$ on the curve $C$ satisfy

$$
(x+1)(x+2) \frac{\mathrm{d} y}{\mathrm{~d} x}=x y .
$$

The line with equation $y=2 x+5$ is the tangent to $C$ at a point $P$.
(a) Find the coordinates of $P$.
(b) Find the equation of $C$, giving your answer in the form $y=\mathrm{f}(x)$.
3. (a) Prove that $\tan 15^{\circ}=2-\sqrt{3}$
(b) Solve, for $0 \leqslant \theta<360^{\circ}$,

$$
\begin{equation*}
\sin \left(\theta+60^{\circ}\right) \sin \left(\theta-60^{\circ}\right)=(1-\sqrt{3}) \cos ^{2} \theta \tag{8}
\end{equation*}
$$

4. 

Figure 1


Figure 1 shows a sketch of the curve $C$ with equation

$$
y=\cos x \ln (\sec x), \quad-\frac{\pi}{2}<x<\frac{\pi}{2} .
$$

The points $A$ and $B$ are maximum points on $C$.
(a) Find the coordinates of $B$ in terms of e .

The finite region $R$ lies between $C$ and the line $A B$.
(b) Show that the area of $R$ is

$$
\frac{2}{\mathrm{e}} \arccos \left(\frac{1}{\mathrm{e}}\right)+2 \ln \left(\mathrm{e}+\sqrt{ }\left(\mathrm{e}^{2}-1\right)\right)-\frac{4}{\mathrm{e}} \sqrt{ }\left(\mathrm{e}^{2}-1\right)
$$

$\left[\arccos x\right.$ is an alternative notation for $\left.\cos ^{-1} x\right]$
5. (i) Anna, who is confused about the rules for logarithms, states that

$$
\begin{gathered}
\quad\left(\log _{3} p\right)^{2}=\log _{3}\left(p^{2}\right) \\
\text { and } \quad \log _{3}(p+q)=\log _{3} p+\log _{3} q .
\end{gathered}
$$

However, there is a value for $p$ and a value for $q$ for which both statements are correct.
Find the value of $p$ and the value of $q$.
(ii) Solve

$$
\begin{equation*}
\frac{\log _{3}\left(3 x^{3}-23 x^{2}+40 x\right)}{\log _{3} 9}=0.5+\log _{3}(3 x-8) . \tag{7}
\end{equation*}
$$

6. 

$$
\mathrm{f}(x)=\frac{a x+b}{x+2} ; \quad x \in \mathbb{R}, x \neq-2,
$$

where $a$ and $b$ are constants and $b>0$.
(a) Find $\mathrm{f}^{-1}(x)$.
(b) Hence, or otherwise, find the value of $a$ so that $\mathrm{ff}(x)=x$.

The curve $C$ has equation $y=\mathrm{f}(x)$ and $\mathrm{f}(x)$ satisfies $\mathrm{f}(x)=x$.
(c) On separate axes sketch
(i) $y=\mathrm{f}(x)$,
(ii) $y=\mathrm{f}(x-2)+2$.

On each sketch you should indicate the equations of any asymptotes and the coordinates, in terms of $b$, of any intersections with the axes.

The normal to $C$ at the point $P$ has equation $y=4 x-39$. The normal to $C$ at the point $Q$ has equation $y=4 x+k$, where $k$ is a constant.
(d) By considering the images of the normals to $C$ on the curve with equation $y=\mathrm{f}(x-2)+2$, or otherwise, find the value of $k$.
7. Relative to a fixed origin $O$, the position vectors of the points $A, B$ and $C$ are

$$
\overrightarrow{O A}=-3 \mathbf{i}+\mathbf{j}-9 \mathbf{k}, \quad \overrightarrow{O B}=\mathbf{i}-\mathbf{k}, \quad \overrightarrow{O C}=5 \mathbf{i}+2 \mathbf{j}-5 \mathbf{k} \text { respectively. }
$$

(a) Find the cosine of angle $A B C$.

The line $L$ is the angle bisector of angle $A B C$.
(b) Show that an equation of $L$ is $\mathbf{r}=\mathbf{i}-\mathbf{k}+t(\mathbf{i}+2 \mathbf{j}-7 \mathbf{k})$.
(c) Show that $|\overrightarrow{A B}|=|\overrightarrow{A C}|$.
(2)

The circle $S$ lies inside triangle $A B C$ and each side of the triangle is a tangent to $S$.
(d) Find the position vector of the centre of $S$.
(e) Find the radius of $S$.

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