Paper Reference(s)

9801

Edexcel

Mathematics

Advanced Extension Award

Thursday 26 June 2003 – Afternoon

Time: 3 hours

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, other names and signature.

Answers should be given in as simple a form as possible e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
This paper has seven questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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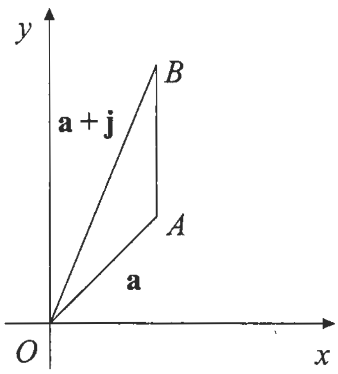
1.

Figure 1



The point A is a distance 1 unit from the fixed origin O. Its position vector is $\mathbf{a} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$.

The point B has position vector $\mathbf{a} + \mathbf{j}$, as shown in Figure 1.

By considering ΔOAB, prove that $\tan \frac{3\pi}{8} = 1 + \sqrt{2}$.

(5)

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2. Find the values of $\tan \theta$ such that

$$2 \sin^2\theta - \sin \theta \sec \theta = 2 \sin 2\theta - 2.$$

(8)

3.

Figure 2

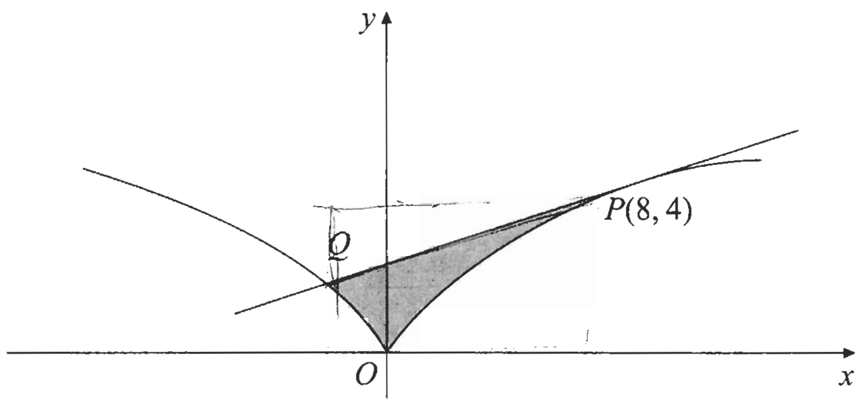

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t^3, \quad y = t^2.$$

The tangent at the point $P(8, 4)$ cuts C at the point Q.

Find the area of the shaded region between PQ and C.

(11)

4. $$f(x) = \frac{1-3x}{(1+3x^2)(1-x)^2}, \quad x \neq 1.$$

(a) Find the constants A, B, C and D such that

$$f(x) \equiv \frac{Ax+B}{1+3x^2} + \frac{C}{1-x} + \frac{D}{(1-x)^2}.$$

(5)

(b) Find a series expansion for f(x) in ascending powers of x, up to and including the term in x^4.
(4)

(c) Find an equation of the tangent to the curve with equation $y = f(x)$ at the point where $x = 0$.
(2)

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5. The function f is given by

$$f(x) = \frac{1}{\lambda}(x^2 - 4)(x^2 - 25),$$

where x is real and λ is a positive integer.

(a) Sketch the graph of $y = f(x)$ showing clearly where the graph crosses the coordinate axes.

(3)

(b) Find, in terms of λ, the range of f.

(5)

(c) Find the sets of values of the positive integers k and λ such that the equation

$$k = |f(x)|$$

has exactly k distinct real roots.

(9)

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6. (a) Show that

$$\sqrt{2+\sqrt{3}} - \sqrt{2-\sqrt{3}} = \sqrt{2}.$$

(3)

(b) Hence prove that

$$\log_{\frac{1}{8}}\left(\sqrt{2+\sqrt{3}} - \sqrt{2-\sqrt{3}}\right) = -\tfrac{1}{6}.$$

(3)

(c) Find all possible pairs of integers a and n such that

$$\log_{\frac{1}{n}}\left(\sqrt{a+\sqrt{15}} - \sqrt{a-\sqrt{15}}\right) = -\tfrac{1}{2}.$$

(13)

7.

Figure 3

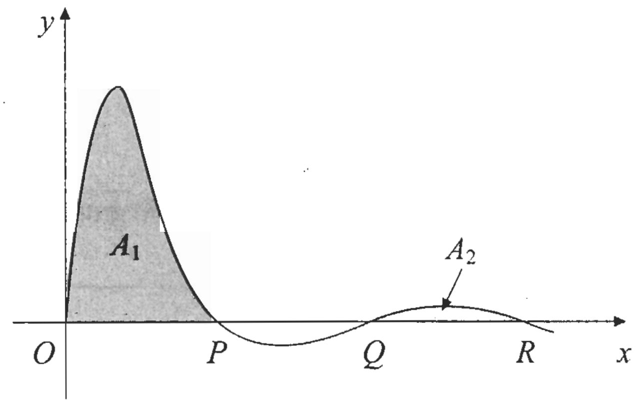

Figure 3 shows a sketch of part of the curve C with equation

$$y = e^{-x}\sin x, \quad x \geqslant 0.$$

(a) Find the coordinates of the points P, Q and R where C cuts the positive x-axis.

(2)

(b) Use integration by parts to show that

$$\int e^{-x}\sin x \, dx = -\tfrac{1}{2}e^{-x}(\sin x + \cos x) + \text{constant}.$$

(5)

The terms of the sequence $A_1, A_2, \ldots, A_n, \ldots$ represent areas between C and the x-axis for successive portions of C where y is positive. The areas represented by A_1 and A_2 are shown in Figure 3.

(c) Find an expression for A_n in terms of n and π.

(6)

(d) Show that $A_1 + A_2 + \ldots + A_n + \ldots$ is a geometric series with sum to infinity

$$\frac{e^{\pi}}{2(e^{\pi} - 1)}.$$

(5)

(e) Given that

$$\int_0^{\infty} e^{-x} \sin x \, dx = \tfrac{1}{2},$$

find the exact value of

$$\int_0^{\infty} |e^{-x} \sin x| \, dx$$

and simplify your answer.

(4)

Marks for style, clarity and presentation: 7

TOTAL FOR PAPER: 100 MARKS

END