Paper Reference(s)

9801

Mathematics

Advanced Extension Award

Materials required for examination
Answer Book (AB16)
Graph Paper (ASG2)
Mathematical Formulae (Lilac)

Items included with question papers
Nil

Candidates may NOT use calculators in answering this paper.

Instructions to Candidates

Full marks may be obtained for answers to ALL questions.
In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title, the paper reference (9801), your surname, other names and signature.

Information for Candidates

A booklet 'Mathematical Formulae including Statistical Formulae and Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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Turn over

1. (a) By considering the series

$$1 + t + t^2 + t^3 + \ldots + t^n ,$$

or otherwise, sum the series

$$1 + 2t + 3t^2 + 4t^3 + \ldots + nt^{n-1}$$

for $t \neq 1$.

(5)

(b) Hence find and simplify an expression for

$$1 + 2\times 3 + 3\times 3^2 + 4\times 3^3 + \ldots + 2001\times 3^{2000} .$$

(1)

(c) Write down an expression for both the sums of the series in part (a) for the case where $t = 1$.

(1)

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2. Given that $S = \int_0^{\frac{\pi}{2}} \mathrm{e}^{2x} \sin x \, \mathrm{d}x$ and $C = \int_0^{\frac{\pi}{2}} \mathrm{e}^{2x} \cos x \, \mathrm{d}x$,

(a) show that $S = 1 + 2C$,

(3)

(b) find the exact value of S.

(6)

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3. Solve for values of θ, in degrees, in the range $0 \leq \theta \leq 360$,

$$\sqrt{2} \times (\sin 2\theta + \cos \theta) + \cos 3\theta = \sin 2\theta + \cos\theta.$$

(12)

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4. A curve C has equation $y = \mathrm{f}(x)$ with $\mathrm{f}'(x) > 0$. The x-coordinate of the point P on the curve is a. The tangent and the normal to C are drawn at P. The tangent cuts the x-axis at the point A and the normal cuts the x-axis at the point B.

(a) Show that the area of ΔAPB is

$$\frac{1}{2}[\mathrm{f}(a)]^2\left(\frac{[\mathrm{f}'(a)]^2+1}{\mathrm{f}'(a)}\right).$$

(8)

(b) Given that $\mathrm{f}(x) = \mathrm{e}^{5x}$ and the area of ΔAPB is e^{5a}, find and simplify the exact value of a.

(4)

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5. The function f is defined on the domain [–2, 2] by:

$$\mathrm{f}(x)=\begin{cases}-kx(2+x) & \text{if } -2\le x<0,\\ kx(2-x) & \text{if } \quad 0\le x\le 2,\end{cases}$$

where k is a positive constant.

The function g is defined on the domain [–2, 2] by $\mathrm{g}(x)=(2.5)^2-x^2$.

(a) Prove that there is a value of k such that the graph of f touches the graph of g.

(8)

(b) For this value of k sketch the graphs of the functions f and g on the same axes, stating clearly where the graphs touch.

(4)

(c) Find the exact area of the region bounded by the two graphs.

(5)

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6. Given that the coefficients of x, x^2 and x^4 in the expansion of $(1+kx)^n$, where $n \geq 4$ and k is a positive constant, are the consecutive terms of a geometric series,

(a) show that $k = \dfrac{6(n-1)}{(n-2)(n-3)}$.

(6)

(b) Given further that both n and k are positive integers, find all possible pairs of values for n and k. You should show clearly how you know that you have found all possible pairs of values.

(5)

(c) For the case where $k = 1.4$, find the value of the positive integer n.

(2)

(d) Given that $k = 1.4$, n is a positive integer and that the first term of the geometric series is the coefficient of x, estimate how many terms are required for the sum of the geometric series to exceed 1.12×10^{12}. [You may assume that $\log_{10} 4 \approx 0.6$ and $\log_{10} 5 \approx 0.7$.]

(5)

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7. The variable y is defined by

$$y = \ln(\sec^2 x + \operatorname{cosec}^2 x) \quad \text{for} \quad 0 < x < \tfrac{\pi}{2}.$$

A student was asked to prove that

$$\frac{dy}{dx} = -4\cot 2x.$$

The attempted proof was as follows:

$$\begin{aligned} y &= \ln(\sec^2 x + \operatorname{cosec}^2 x) \\ &= \ln(\sec^2 x) + \ln(\operatorname{cosec}^2 x) \\ &= 2\ln\sec x + 2\ln\operatorname{cosec} x \\ \frac{dy}{dx} &= 2\tan x - 2\cot x \\ &= \frac{2(\sin^2 x - \cos^2 x)}{\sin x \cos x} \\ &= \frac{-2\cos 2x}{\frac{1}{2}\sin 2x} \\ &= -4\cot 2x \end{aligned}$$

(a) Identify the error in this attempt at a proof.

(1)

(b) Give a correct version of the proof.

(5)

(c) Find and simplify a general relationship between p and q, where p and q are variables that depend on x, such that the student would obtain the correct result when differentiating $\ln(p + q)$ with respect to x by the above incorrect method.

(8)

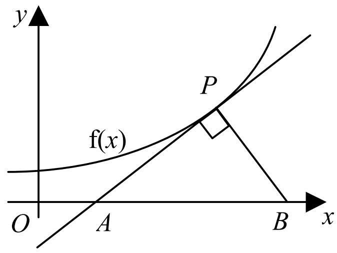
(d) Given that $p(x) = k \sec rx$ and $q(x) = \operatorname{cosec}^2 x$, where k and r are positive integers, find the values of k and r such that p and q satisfy the relationship found in part (c).

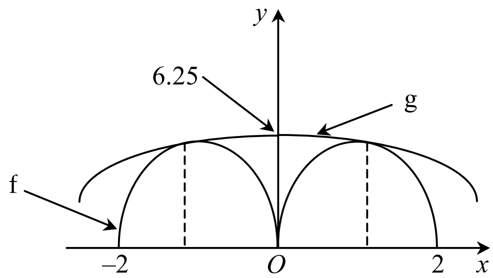
(4)

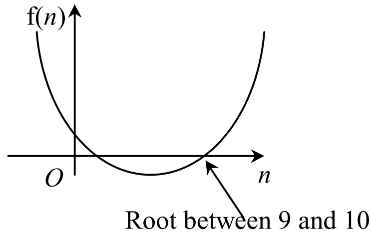
END

Marks for presentation: 7
TOTAL MARKS: 100

No		Working	Marks
1	(a)	G.P. $a = 1, r = t, \quad S_n = \dfrac{t^{n+1} - 1}{t - 1}$	M1 A1
		$S = 1 + 2t + \ldots + nt^{n-1} = \dfrac{\mathrm{d}}{\mathrm{d}t}$ (1st series)	M1
		$\dfrac{\mathrm{d}}{\mathrm{d}t}(S_n) = \dfrac{(t-1)(n+1)t^n - (t^{n+1} - 1).1}{(t-1)^2}$ Quotient rule	M1 A1
		$= \dfrac{nt^{n+1} - (n+1)t^n + 1}{(t-1)^2}$	(5)
	(b)	$t = 3, n = 2001$ $\Rightarrow \text{Sum} = \dfrac{2001.3^{2002} - 2002.3^{2001} + 1}{2^2} = \dfrac{4001.3^{2001} + 1}{4}$	B1 (1)
	(c)	$1 + t + \ldots + t^{n+1} \to n + 1$; $1 + 2t + \ldots + nt^{n-1} \to \dfrac{n(n+1)}{2}$	B1 (both) (1) **(7)**
2	(a)	$S = \int_0^{\frac{\pi}{2}} e^{2x} \ \mathrm{d}(-\cos x)\mathrm{d}x = \left[-\cos x \, e^{2x}\right]_0^{\frac{\pi}{2}} + 2\int_0^{\frac{\pi}{2}} e^{2x} \cos x \ \mathrm{d}x$	M1 A1 (ignore limits)
		$S = (-0) - (-1) + 2C$ or $1 + 2C$	A1 cso (3)
	(b)	$C = \int_0^{\frac{\pi}{2}} e^{2x}\mathrm{d}(\sin x) = \left[\sin x e^{2x}\right]_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} e^{2x} \sin x \mathrm{d}x$	M1 A1 (ignore limits)
		$C = (e^{\pi}) - (0) - 2S$ i.e. $C = e^{\pi} - 2S$	A1
		Solving $S = 1 + 2(e^{\pi} - 2S)$ $= 1 + 2e^{\pi} - 4S$	M1
		$5S = 1 + 2e^{\pi}$ and $S = \dfrac{1}{5}(1 + 2e^{\pi})$	M1, A1 (6) **(9)**

No	Working	Marks
3	$(\sqrt{2}-1)(\sin 2\theta+\cos\theta)+\cos 3\theta=0$	M1
	$(\sqrt{2}-1)\cos\theta(1+2\sin\theta)+\cos 3\theta=0$	
	$\cos 3\theta \equiv \cos 2\theta\cos\theta-\sin 2\theta\sin\theta$	M1
	$\equiv \cos\theta[\cos 2\theta-2\sin^2\theta]$	M1
	$\equiv \cos\theta[1-4\sin^2\theta]$	A1
	So: $\cos\theta((\sqrt{2}-1)(1+2\sin\theta)+(1-2\sin\theta)(1+2\sin\theta))=0$	M1
	i.e. $\cos\theta(1+2\sin\theta)(\sqrt{2}-1+1-2\sin\theta)=0$	M1
	i.e. $\sqrt{2}\cos\theta(1+2\sin\theta)(1-\sqrt{2}\sin\theta)=0$	M1
	$\cos\theta=0 \Rightarrow \theta=90°,\ 270°$	A1 (both)
	$1+2\sin\theta=0 \Rightarrow \sin\theta=-\frac{1}{2} \Rightarrow \theta=210°,\ 330°$	A1, A1
	$1-\sqrt{2}\sin\theta=0 \Rightarrow \sin\theta=\frac{1}{\sqrt{2}} \Rightarrow \theta=45°,\ 135°$	A1, A1 **(12)**
4 (a)	 Equation of tangent: $y - f(a) = f'(a)(x-a)$	M1
	$\Rightarrow A$ is $\left(a-\dfrac{f(a)}{f'(a)}, 0\right)$	A1
	Gradient of normal is $-\dfrac{1}{f'(a)}$;	M1
	Equation of normal: $y-f(a)=-\dfrac{1}{f'(a)}(x-a)$	M1
	$\Rightarrow B$ is $(a+f(a)f'(a), 0)$	A1
	Area ΔAPB is $\frac{1}{2}AB.f(a)=\frac{1}{2}f(a)\left[f(a)f'(a)+\dfrac{f(a)}{f'(a)}\right]$	M1, A1
	$=\frac{1}{2}[f(a)]^2\left(\dfrac{[f'(a)]^2+1}{f'(a)}\right)$	A1 c.s.o. (8)
(b)	$f'(a)=5e^{5a}$ $\quad\therefore e^{5a}=\frac{1}{2}(e^{5a})^2\left(\dfrac{25(e^{5a})^2+1}{5e^{5a}}\right)$	M1, A1
	$10(e^{5a})^2=(e^{5a})^2(25e^{10a}+1)$	
	$e^{10a}=\frac{9}{25} \Rightarrow a=\frac{1}{10}\ln\left(\frac{9}{25}\right)=\frac{1}{5}\ln\frac{3}{5}$	M1
		A1 c.s.o. (4) **(12)**

No		Working	Marks
5	(a)	By symmetry consider RHS: $\frac{25}{4} - x^2 = 2kx - kx^2$	M1
		i.e. $(k-1)x^2 - 2kx + \frac{25}{4} = 0$	M1
		Equal roots $\Rightarrow$ $4k^2 = 4\times\frac{25}{4}\times(k-1)$	M1
		i.e. $4k^2 - 25k + 25 = 0$	A1
		i.e. $(4k-5)(k-5) = 0$	M1
		i.e. $k = 5$ or $\frac{5}{4}$	A1
		$x = \dfrac{-b}{2a} = \dfrac{2k}{2(k-1)} = \dfrac{k}{k-1}$ and $\because x < 2$, we need $k = 5$	M1, A1
			(8)
	(b)		
		Symmetric	B1
		touch × 2	B1
		touch at $\left(\pm\frac{5}{4}, \frac{75}{16}\right)$	B1, B1
			(4)
	(c)	Area $= 2\times\int_0^{\frac{5}{4}}\left(6.25 - x^2 - \left[10x - 5x^2\right]\right)dx$	M1
		$= 2\left[6.25x - 5x^2 + \frac{4}{3}x^3\right]_0^{\frac{5}{4}}$	M1 A1
		$= 2\left[\left(\frac{25}{4}\times\frac{5}{4} - 5\times\frac{25}{16} + \frac{4}{3}\times\frac{125}{64}\right) - (0)\right]$	M1
		$= 2\times\frac{125}{48} = \frac{125}{24} = 5\frac{5}{24}$	A1
			(5)
			(17)

No	Working	Marks
6 (a)	$(1+kx)^n = 1 + nkx + \frac{n(n-1)}{2}k^2x^2 + \ldots + \frac{n(n-1)(n-2)(n-3)}{4!}k^4x^4 + \ldots$	M1 A2/1/0
	Let r = ratio of geometric series	
	$r = \frac{n(n-1)}{2} \times \frac{k^2}{nk} = \frac{n(n-1)(n-2)(n-3)k^4 \times 2}{4!\ n(n-1)k^2}$	M1
	i.e. $\frac{(n-1)k}{2} = \frac{(n-2)(n-3)k^2}{12}$ i.e. $k = \frac{6(n-1)}{(n-2)(n-3)}$	M1, A1 cso (6)
(b)	$n = 4 \Rightarrow k = \frac{6.3}{2.1} = 9$; $n = 5 \Rightarrow k = \frac{6.4}{3.2} = 4$	B1, B1
	$n = 6 \Rightarrow k = 2.5$, $n = 7 \Rightarrow k = 1.8$, $n = 8 \Rightarrow k = 1.4$, $n = 9 \Rightarrow k = \frac{8}{7}$	M1
	$k < 1 \Rightarrow 6n - 6 < n^2 - 5n + 6$	M1
	i.e. $0 < n^2 - 11n + 12$	
	So $k < 1\ \forall n \geq 10$ 	A1 (5)
(c)	$k = 1.4 \Rightarrow 7(n^2 - 5n + 6) = 30n - 30$	
	$7n^2 - 65n + 72 = 0$	M1
	$(7n-9)(n-8) = 0$ $\therefore n = 8$ is the integer solution	A1 (2)
(d)	$k = 1.4$, $n = 8$, $a = nk = 11.2$, $r = \frac{(n-1)k}{2} = \frac{7}{2} \times \frac{7}{5} = 4.9$	B1, B1
	$S_m > 1.12 \times 10^{12} \Rightarrow \frac{11.2(4.9^m - 1)}{3.9} > 1.12 \times 10^{12}$	M1
	$4.9^m - 1 > 3.9 \times 10^{11}$	
	$4.9^m > 3.9 \times 10^{11} + 1 \approx 3.9 \times 10^{11}$	A1
	$m > \frac{\log 3.9 + 11}{\log 4.9} \approx \frac{11.6}{0.7} = 16.\ldots$	
	So need $m = 17$	A1 (5) **(18)**

No		Working	Marks
7	(a)	In line 1 the student assumes that ln $(a + b)$ = ln a + ln b [Although it can be shown that for these functions this is true, it is not a general statement.]	B1 (1)
	(b)	$y = \ln\left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right)$	
		$= \ln\left(\frac{1}{\cos^2 x \sin^2 x}\right)$	M1
		$= -2\ln(\sin x \cos x)$	M1
		$= -2\ln\frac{1}{2} - 2\ln(\sin 2x)$	M1
		$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{\sin 2x} \times 2\cos 2x = -4\cot 2x$	M1,A1 (5)
	(c)	Require	
		$\frac{p'+q'}{p+q} = \frac{p'}{p} + \frac{q'}{q}$	M1, M1
		$= \frac{p'q + pq'}{pq}$	A1
		i.e.	
		$\frac{\mathrm{d}}{\mathrm{d}x}[\ln(p+q)] = \frac{\mathrm{d}}{\mathrm{d}x}[\ln(pq)]$	M1, A1, A1
		$\ln(p+q) = \ln(pq) + \ln A$	M1
		$(p+q) = A(pq)$	A1 (8)
	(d)	$p(1 - Aq) = -q$	
		$p = \frac{q}{Aq - 1}$	M1
		$q = \operatorname{cosec}^2 x \quad \Rightarrow \quad p = \frac{\operatorname{cosec}^2 x}{A\operatorname{cosec}^2 x - 1} \times \left(\frac{\sin^2 x}{\sin^2 x}\right)$	M1
		$p = k\sec rx \quad \Rightarrow \quad k\sec rx = \frac{1}{A - \sin^2 x}$	M1
		$A = \frac{1}{2} \quad \Rightarrow k\sec rx = \frac{2}{\cos 2x}$	
		$\therefore k = r = 2$	A1 (4) **(18)**

ADVANCED EXTENSION AWARD in MATHEMATICS
SPECIMEN PAPER – MARK SCHEME

Presentation

1 (a) 1 mark may be awarded for a good and largely accurate attempt at the whole paper.

1 (b) For a novel or neat solution to any question award, in each of up to 3 questions. 2 marks if the solution is fully correct and 1 mark if the solution is sound in principle but a minor algebraic or numerical slip occurs.