General Certificate of Education
Advanced Level Examination
June 2013

# Use of Mathematics (Pilot) USE3/PM

**Mathematical Comprehension**

# Preliminary Material

# Data Sheet

**To be opened and issued to candidates between Friday 3 May 2013 and Friday 17 May 2013**

**REMINDER TO CANDIDATES**

YOU MUST **NOT** BRING THIS DATA SHEET
WITH YOU WHEN YOU SIT THE EXAMINATION.
A CLEAN COPY WILL BE MADE AVAILABLE.

## Measuring inequality

There is inequality in the distribution of wealth throughout the world. **Figure 1** and **Figure 2** contrast the wealth of New York, USA, and its impressive skyscrapers with the poverty of Mumbai, India, and its ramshackle housing.

**Figure 1**
*A wealthy area of the city of New York in the USA*

**Figure 2**
*A poor area of the city of Mumbai in India*

Not only is more of the world's wealth concentrated in some countries than in others, but within all countries some citizens are considerably wealthier than others.

How can we investigate such inequality mathematically?

One way is to use a measure or index that will either allow us to make comparisons between different countries or allow us to track changes in inequality within one particular country over time.

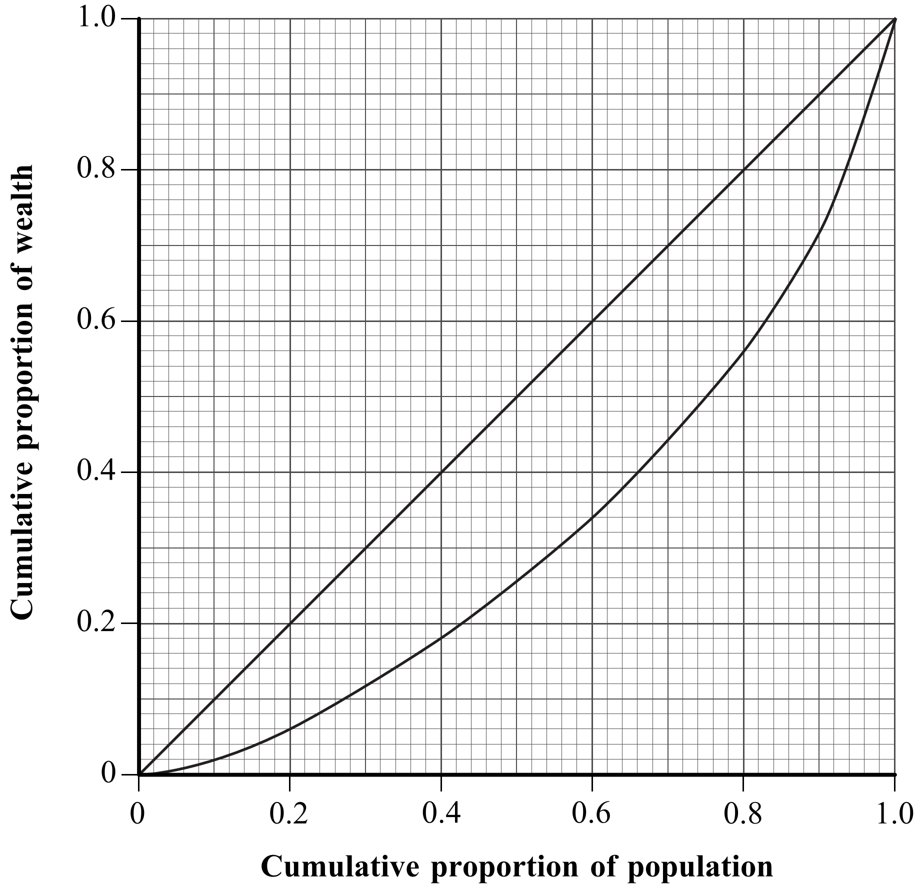
To help explore a commonly used measure, consider the data in **Table 1**. These data show, for the UK, how wealth is distributed in our society by showing the cumulative proportion of wealth that is held by the cumulative proportion of the population. So the least wealthy 10 % of the population have 2 % of the country's wealth, the least wealthy 20 % of the population have 6 % of the country's wealth and so on, until we see that the least wealthy 90 % of the population have 72 % of the country's wealth. In other words, the wealthiest 10 % of the population have 28 % of the wealth.

**Table 1**
*UK data of wealth distribution*

| Cumulative proportion of population | Cumulative proportion of wealth |
|---|---|
| 0 | 0 |
| 0.1 | 0.02 |
| 0.2 | 0.06 |
| 0.4 | 0.18 |
| 0.6 | 0.34 |
| 0.8 | 0.56 |
| 0.9 | 0.72 |
| 1.0 | 1.0 |

A graph of these data, known as a Lorenz curve, is shown in **Figure 3**. The figure also shows a straight line joining the origin to the point (1, 1). This straight line would be obtained for a society in which there was total equality. This would mean that every member of society would have the same wealth regardless of factors such as their age.

**Figure 3**
*Lorenz curve for the UK*



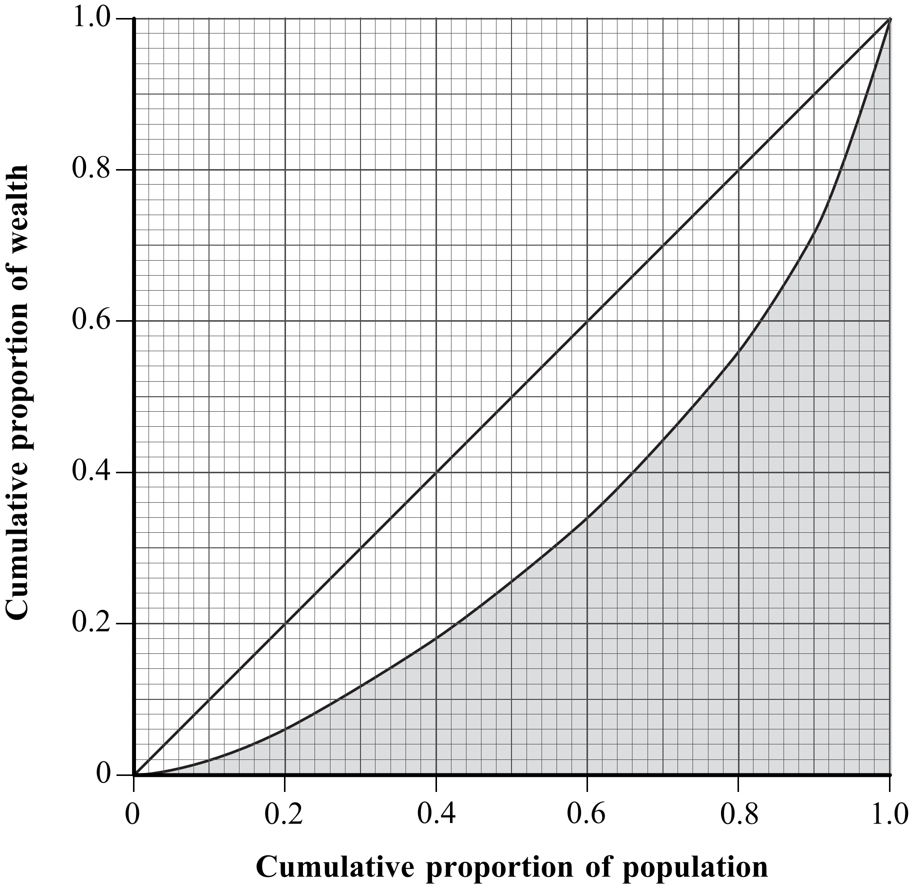

As is often the case, a single numerical value that is a measure of inequality might be preferred to a graph: this would make it easier to make comparisons. The Gini coefficient is often used in conjunction with the Lorenz curve. It gives an overall measure of how far away the data are from the line of equality. It is based on the area that is enclosed by the curve representing the data and the line of equality. The area is calculated and doubled to give the Gini coefficient. This means that the coefficient will lie between 0 (representing equal distribution of wealth) and 1 (representing the most unequal distribution of wealth possible).

**Turn over**

In the case of the UK, therefore, to calculate the Gini coefficient we first need to determine the area that is shaded in **Figure 4**. One method would be to find the area using a numerical method of integration. Using the trapezium rule with five intervals of 0.2 gives a value of 0.328 for the shaded area in **Figure 4**. This leads to an approximate value of $G = (0.5 - 0.328) \times 2 = 0.344$ for the Gini coefficient.

**Figure 4**

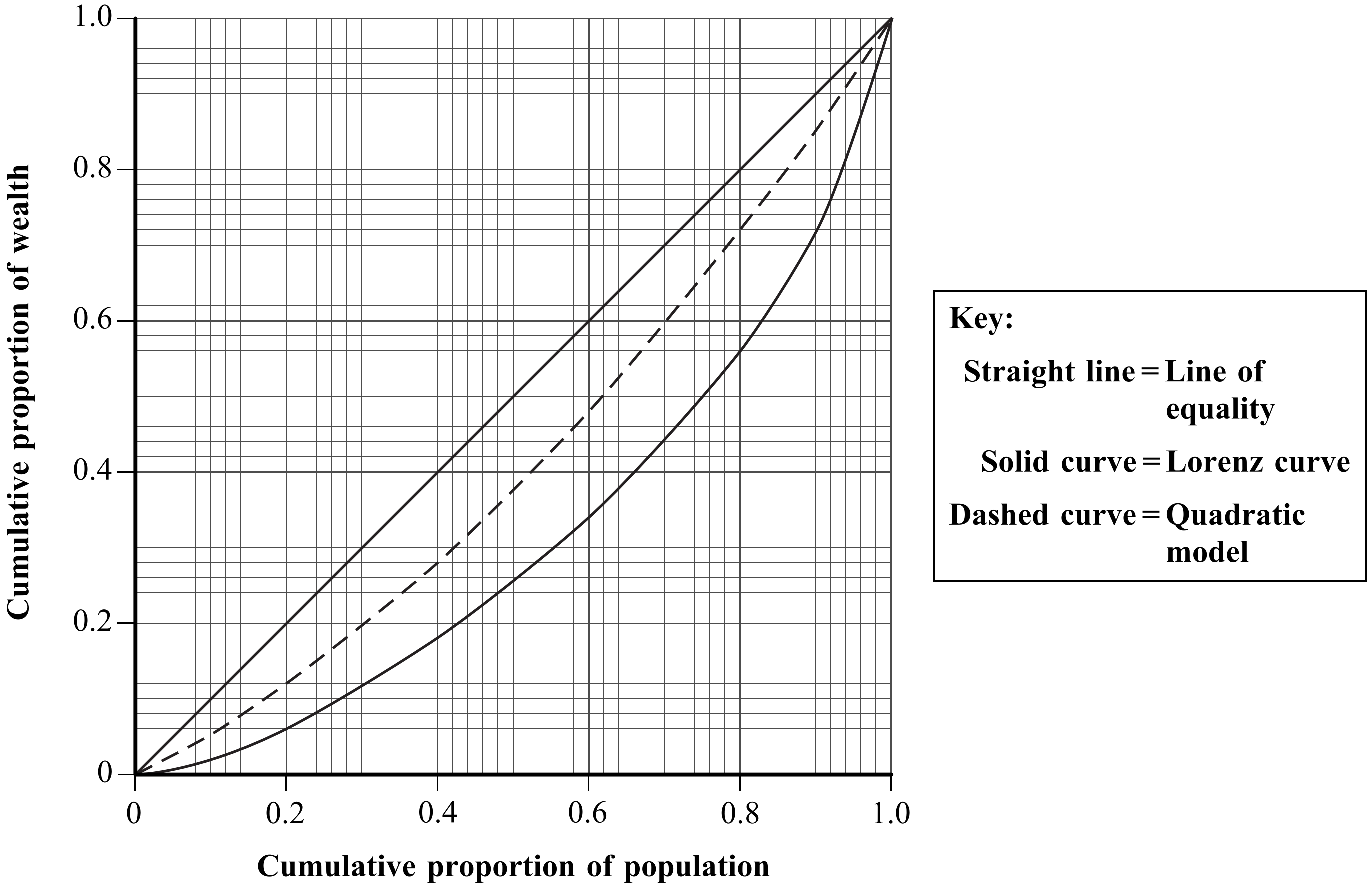
*Lorenz curve for the UK, with the area under the curve shaded*




If we know a function that models the data effectively and passes through (0, 0) and (1, 1), we can use integration to find the area under the curve and then calculate a value for the Gini coefficient. **Figure 5** shows the data for the UK together with the quadratic function $f(x) = 0.5x^2 + 0.5x$, where $x$ is the cumulative proportion of the population.

**Figure 5**

*Lorenz curve for the UK modelled by the quadratic function* $f(x) = 0.5x^2 + 0.5x$




It can be seen from the graph that this is not a very accurate model of the data. In this case, the area under the curve, $A$, is given by $\int_0^1 (0.5x^2 + 0.5x)\,dx$. This gives a value for the Gini coefficient of 0.17. This shows that this quadratic model was not effective.

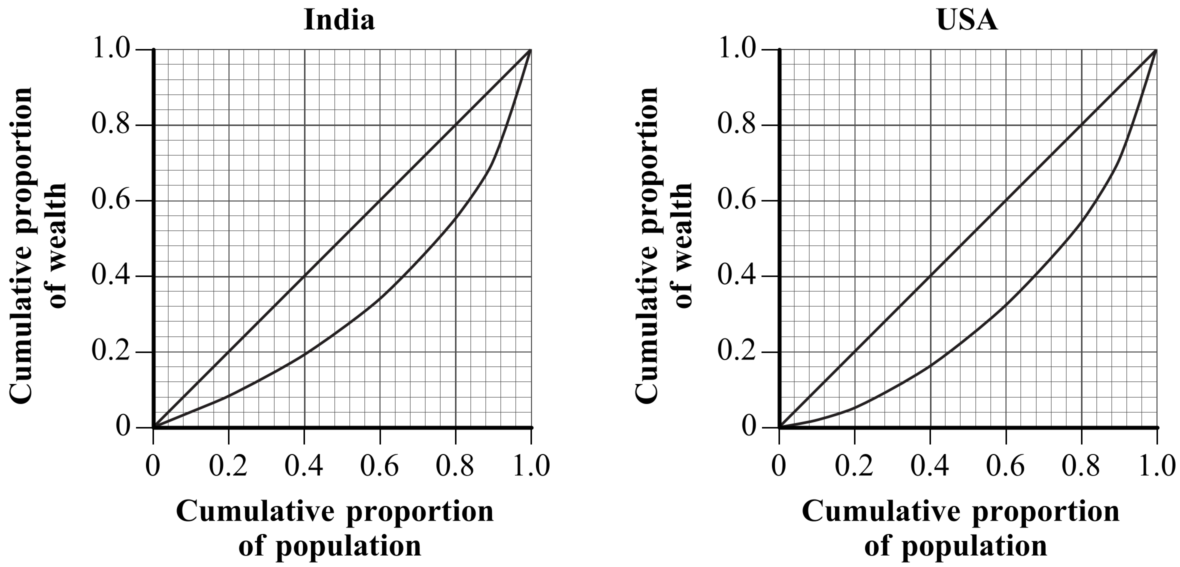
Of course, the more accurate the function is as a model of the data, the more accurate the value that we obtain for the Gini coefficient will be. For example, using the function $f(x) = 1.04x^3 - 0.38x^2 + 0.34x$ gives a value of 0.393 for the Gini coefficient. The numerical value on its own gives you little information about inequality: it is only of use when compared with other values for different countries or for different times within the same country.

**Table 2** again shows the distribution of wealth in the UK and, additionally, in India and the USA, giving the cumulative proportion of wealth that is held by the cumulative proportion of the population in each country. Although in general the Lorenz curves plotted using these data in **Figure 6** look quite similar, you can see that, for example, more of the wealth is owned by the least wealthy 20 % of the population in India than in the USA.

**Table 2**
*Data of wealth distribution in the UK, India and the USA*

| | **Cumulative proportion of wealth** | | |
|---|---|---|---|
| **Cumulative proportion of population** | **UK** | **India** | **USA** |
| 0 | 0 | 0 | 0 |
| 0.1 | 0.02 | 0.04 | 0.02 |
| 0.2 | 0.06 | 0.08 | 0.05 |
| 0.4 | 0.18 | 0.19 | 0.16 |
| 0.6 | 0.34 | 0.34 | 0.32 |
| 0.8 | 0.56 | 0.55 | 0.54 |
| 0.9 | 0.72 | 0.69 | 0.70 |
| 1.0 | 1.0 | 1.0 | 1.0 |

**Figure 6**
*Lorenz curves for India and the USA*



The function $g(x) = 1.04x^3 - 0.58x^2 + 0.54x$ can be used to model the data for India which gives a value of 0.327 for the Gini coefficient. Similarly, the function $h(x) = 1.15x^3 - 0.44x^2 + 0.29x$ can be used to model the data for the USA which gives a value of 0.428 for the Gini coefficient. These results suggest that for the three countries considered here, wealth is distributed least equally in the USA and most equally in India, with the UK lying somewhere between the two.

Another potential way of considering inequality is to find the rate of change of the model with respect to $x$, at the origin. The closer the value of the gradient is to 1, the more equal the distribution of wealth amongst the least wealthy. For example, using the cubic functions to model the data for each of the UK, India and the USA, you can find that when $x = 0$, the rate of increase of these functions with respect to $x$, is 0.34, 0.54 and 0.29 respectively. This again suggests that the distribution of wealth is most unequal in the USA and most equal in India, with the UK somewhere in between.

As you can see, the two mathematical measures of inequality explored here suggest that although the USA is a wealthy country, its wealth is distributed unequally amongst its citizens. In India, although there is a great deal of poverty, the wealth of the country is distributed more equitably. The mathematical measures explored here can give us insights into issues of equality. However, there are many factors that we should consider in relation to employment, education and health before we can start to get a comprehensive understanding of the situation.

**END OF DATA SHEET**

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