Teacher Support Materials

FSMQ

Paper Reference UOM 4/2

Question 1

1 Route planning software that can be used on a computer classifies roads into three different types. It calculates the time taken for journeys by assuming that a motorist will travel at a different average speed on each type of road:

average speed on motorways: 60 miles per hour;
average speed on dual carriageways: 50 miles per hour;
average speed on single carriageway roads: 40 miles per hour.

A motorist plans to travel from home to London. The software gives two different possible routes:

route A: 160 miles on single carriageway roads;

route B: 105 miles on motorways followed by 75 miles on dual carriageways.

(a) Calculate how long the software would calculate that the motorist would take using:

(i) Route A;

(ii) Route B. *(4 marks)*

(b) On the grid on the answer sheet, plot distance travelled from home against time for a motorist who plans to arrive in London at 4 pm, using:

(i) Route A;

(ii) Route B. *(5 marks)*

(c) The motorway section of Route B can be represented by an equation $d_B = a + bt$ where d_B is the distance travelled from home in miles in terms of time t, the number of hours after 12 noon.

(i) Find numerical values for parameters a and b. *(3 marks)*

(ii) Interpret the physical significance of parameter b. *(2 marks)*

Student Response

Leave blank

1 a) i) 160 miles at 40mph $\frac{160}{40}$ = 4 hours ✓ 1

ii) 105 miles at 60mph $\frac{105}{60}$ = 1 hour 45 mins

75 miles at 50mph $\frac{75}{50}$ = 1 hour 30 mins

= 1 hour 45mins + 1 hour 30mins ✓

= 3 hours 15 minutes 3

b) on answer sheet

Leave blank

c) $d_B = a + bt$

$dB = 105$
$t = 1\text{hr } 45$ (1.75 hours)
$105 = a + 1.75b$

5

i) $a = 1.75b - 105$
if $\underline{b = 60}$ (average speed)
$a = 1.75 \times 60 - 105$
$\underline{\underline{a = 0}}$

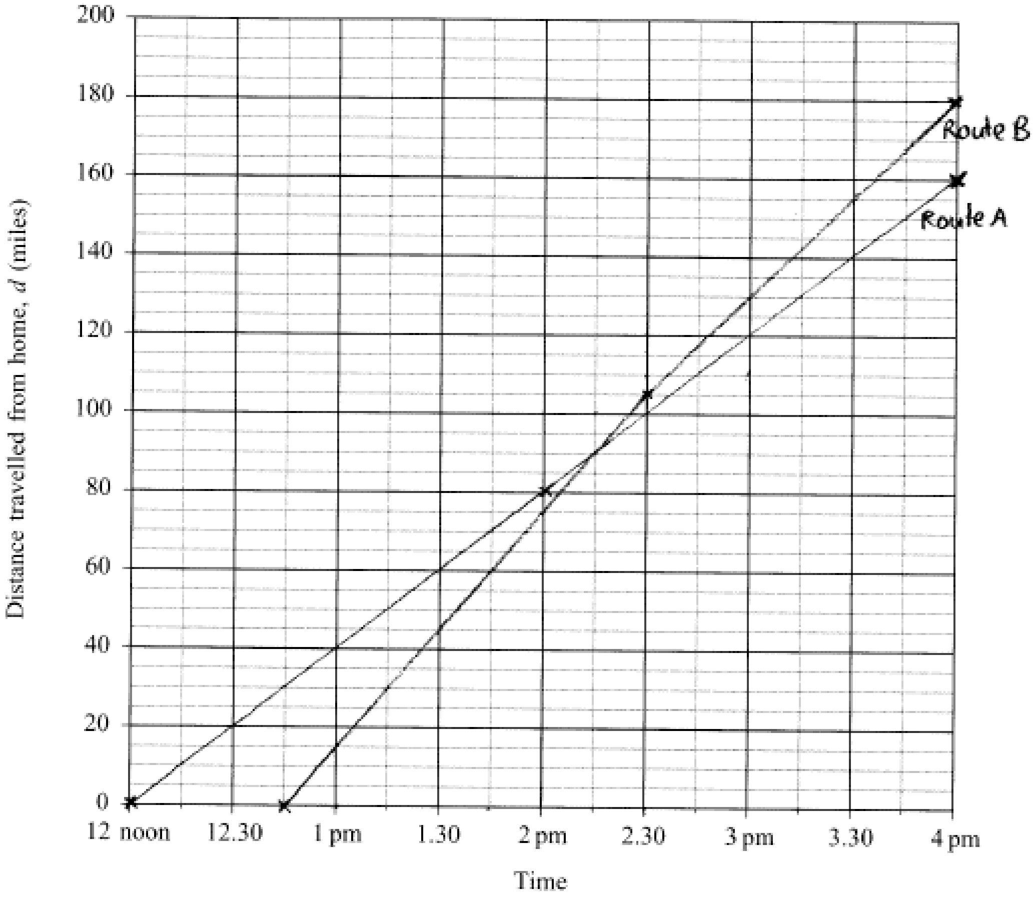
✓1

M0

ii) The parameter b is the average speed.

✓2

Question 1



B1

4

5

Commentary

In this question candidates were expected to work with interrelated recurrence relations in a situation that would hopefully be realisable to many of them. The vast majority of candidates were able to correctly demonstrate the calculations required to show the values repaid in the first year and the loan therefore remaining given the initial values of earnings and loan, as is shown here.

Unfortunately a significant number of candidates misread recurrence relations such as $L_n = L_{n-1} - R_n$ as $L_n = L_n - 1 - R_n$ as illustrated here. This seems to indicate a lack of familiarity with recurrence relation notation. Such misinterpretation did not result in the candidate losing all marks when completing the table and subsequently the graph for part (d).

In answering questions requiring interpretation such as that asked in part (e) candidates should be encouraged to think about the aspects of the situation being modelled: here for example, they are being asked about the variation in loan with time and the factors they should consider are earnings and repayments. The response illustrated here does not refer to these key factors in the model of the situation.

Mark scheme

(a)(i)	$\frac{160}{40} = 4$ hours	**B1**	Accept 4 or 240
(a)(ii)	$\frac{105}{60} + \frac{75}{50} =$ $1\frac{3}{4} + 1\frac{1}{2} =$ $3\frac{1}{4}$ hours	**M1** **A1** **A1**	For either $\frac{105}{60}$ or $\frac{75}{50}$ For either $1\frac{3}{4}$ or $1\frac{1}{2}$ Accept 3.25, 195 **SC2** no working 3 hr 25 min
(b)(i)	Single straight line joining origin to (4, 160)	**B1**	
(b)(ii)	Line starting at (12.45, 0) Line ending at (4[pm], 180) 2 straight lines Intersection of lines at (2.30pm, 105) i.e. gradient changes at (2.30pm, 105)	**M1** **M1** **M1** **A1**	**SC2** two straight lines, one starting at (0,0) other ending at (3.15, 180) **AND SC1** intersection of 2 lines at (1.45, 105) ie **SC3** for all of the above NB SC above only gives possibility of B1 in (c)(i) and B2 in (c)(ii) Dependent on all 3 M1s
(c)(i)	$b = 60$ when $d_B = 0 \quad t = 0.75$ $0 = a + 60 \times 0.75$ $a = -60 \times 0.75 = -45$	**B1** **M1** **A1**	For $b = 60$ Substituting any correct motorway point e.g. (2.5, 105)
(c)(ii)	(motorway) speed	**B2**	Accept 'speed', 'how fast' OE
	TOTAL	**14**	

Question 2

2 The function $m = m_0 e^{kt}$ can be used to model the mass, m grams, of radioactive substance remaining in an organism t years after it died. When it died, the mass of radioactive substance was m_0.

For the radioactive substance carbon-14, $k = -0.000121$.

An organism contained 1 gram of carbon-14 when it died. Therefore, the mass, m grams, of carbon-14, t years after it died, will be given by $m = e^{-0.000121t}$.

(a) Show that it takes approximately 5730 years for the amount of carbon-14 in this organism to decay to half its starting value, that is 0.5 grams. *(2 marks)*

(b) Find the mass of carbon-14 remaining in this organism after a further 5730 years. *(2 marks)*

(c) For this decay, sketch a graph of m against t on the grid given on the answer sheet. *(3 marks)*

(d) Compare the gradient of your graph when $t = 0$ with its gradient when $t = 20000$. Interpret your answer. *(2 marks)*

(e) Find the number of years after which the mass of carbon-14 remaining in the organism will be 0.75 grams by solving $0.75 = e^{-0.000121t}$. *(4 marks)*

(f) For a different radioactive substance with an initial mass of 1 gram, $m = e^{-0.0005t}$.

Write a brief description of how the decay of this substance would be different from that of carbon-14. *(2 marks)*

Student response

3 a)

i) $R_n = 0.09\,(E_n - 15000)$

$= 0.09\,($[redacted] $25000 - 15000)$

$= 900$

$R_1 = 900$ ✓ 2

ii) $L_n = L_{n-1} - R_n$

$L_n =$ ~~8100~~ $10\,000 - 900$

$= 9100$

$L_1 = 9100$ ✓ 2

c) 86 years ✗ 0

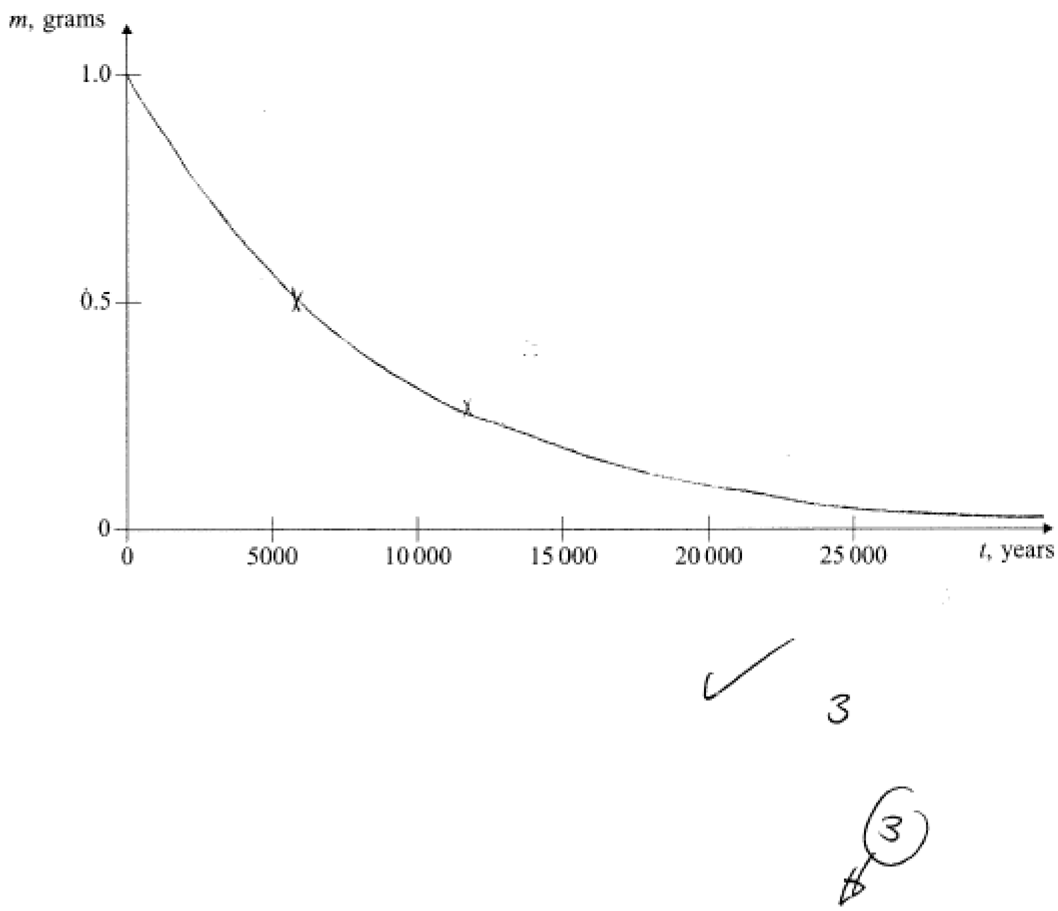
e) When the loan is repaid the pattern of the graph changes ✗ 0

4

8

3

Question 2



Commentary

The response to this question illustrates much good practice: overall, candidates often produced good answers and showed that they were able to work well with exponential functions and had a good understanding of natural logarithms.

This candidate substitutes a value of 5730 for *t* into the exponential function and shows that this gives a mass of 0.5 grams and that the mass therefore decays to half its starting value in this time. A number of candidates reversed the process which required the use of natural logarithms: it was pleasing to see many successful such solutions although candidates should always attempt to take the most direct, and often easiest route.

In part (b) it was very pleasing to see that some candidates had a very clear understanding of exponential functions and were able to immediately deduce that the mass of the radioactive substance will have again halved. However, the response shown here was a more common approach.

As illustrated here, in response to part (c), although a sketch was required a reasonable degree of accuracy is expected: here the candidate has taken care to ensure that account is taken of the half life of the substance.

In response to part (d) the candidate here drops a mark, as although he has correctly described the key features of the gradient at different part so the graph he has failed to interpret them in terms of the situation they model.

The response to part (e) was invariably tackled well as is illustrated here. Although this is a relatively straight forward case of solving an equation using logarithms it was pleasing to see many correct solutions such as that illustrated here.

Finally, in part (f) candidates were expected to consider the effect of varying a parameter in the model for exponential decay: for a range of functions and situations this is often required in this paper and in preparation candidates should be encouraged to explore such situations. The candidate here responds by explaining in terms of both variation in rate of decay and half life although only a clear explanation in terms of one of these was required.

Mark Scheme

(a)	$m = e^{-0.000121 \times 5730} = 0.4999 = 0.500$	**M1** **A1**	M1 0.000121×5730 or $0.693\ldots$
(b)	$m = e^{-0.000121 \times 2 \times 5730} = 0.2499 = 0.250$	**M1** **A1**	M1 $0.000121 \times 2 \times 5730$ or $1.386\ldots$ or 1.39
(c)	General shape of exponential decay passing through (0,1) half-life approximately 5700 years	**B1** **B1** **B1**	Can touch but not cut axis
(d)	Answer in terms of gradient only: steeper when $t = 0$ than when t = 20000 Interpreting physical significance: rate of decay greater when $t = 0$	**B1** **B1**	Accept decaying faster when $t = 0$
(e)	$0.75 = e^{-0.000121\, t}$ $\ln(0.75) = -0.000121t$ $-0.28768 = -0.000121t$ $t = 2377.5 = 2380$	**M1** **M1** **A1** **A1**	Use of logs eg $\ln 0.75 = \ln e^{-0.000121\, t}$ Eliminating t Use of $\ln 0.75 = -0.287...$ or -0.288. Dependent on first M1 Full marks for answer only **SC3** no working 2370
(f)	More rapid decay OR shorter half life	**B2**	Anything which sensibly indicates this
	TOTAL	**15**	

Question 3

3 Maria has completed a university course and has a student loan of £10 000 to repay. Each year, Maria has to pay back 9% of anything she earns over £15 000 during that year.

Throughout this question, you may assume that interest is not charged on a student loan.

Maria earns £25 000 in the first year and gets an increase in her earnings of £2000 per year in each subsequent year.

The following recurrence relations are used to model this situation until the loan is repaid:

- $E_n = E_{n-1} + 2000$ gives Maria's earnings each year $(E_1 = 25\,000)$;
- $R_n = 0.09(E_n - 15\,000)$ gives the amount Maria repays each year;
- $L_n = L_{n-1} - R_n$ gives the amount of Maria's loan that remains at the end of each year $(L_0 = 10\,000)$.

(a) Given that $E_1 = 25\,000$ and $L_0 = 10\,000$, show that:

(i) $R_1 = 900$;

(ii) $L_1 = 9100$. *(4 marks)*

(b) Use the recurrence relations to complete the table on the answer sheet. *(6 marks)*

(c) How long will it take Maria to pay back her student loan? *(1 mark)*

(d) Complete the graph of loan remaining, L_n, plotted against year, n, on the grid on the answer sheet. *(2 marks)*

(e) Explain why the points on your graph do not lie on a straight line. *(2 marks)*

Student Response

3 a)

i) $Rn = 0{\cdot}09\,(E_n - 15000)$

$= 0{\cdot}09\,($ ~~[illegible]~~ $25000 - 15000)$

$= 900$

$R1 = 900$ ✓ 2

ii) $Ln = Ln-1 - Rn$

$Ln =$ ~~8100~~ $10{\cdot}000 - 900$

$= 9100$

$L_1 = 9100$ ✓ 2

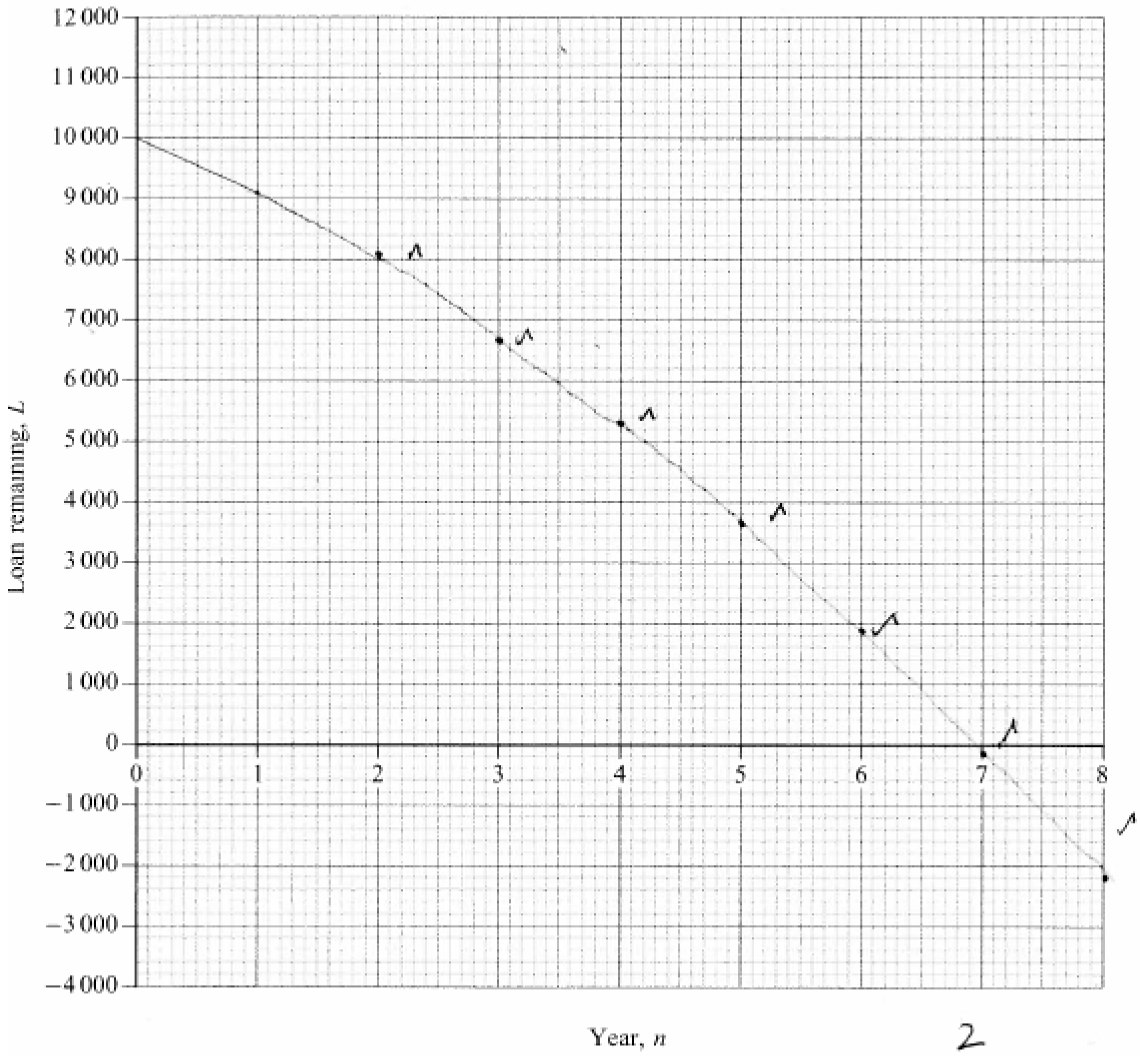
c) ~~8~~ 6 years ✗ 0

e) When the loan is repaid the pattern of the graph changes ✗ 0

4

8

Question 3



(4)

Question 3

Year, n	Earnings, E	Repaid, R	Loan remaining, L
0	0	0	10 000
1	25 000	900	9 100
2	26 999	1079.91	8019.09
3	28 998	1259.82	6758.27
4	30 997	1439.73	5317.54
5	32 996	1619.64	3696.9
6	34 995	1799.55	[illegible]
7	36 994	1979.46	[illegible]
8	38 993	2159.37	[illegible]

x0

B2

Commentary

All too often this question, as is often the case with questions based on straight lines, was badly done in general.

Part (a) of the question was often answered well as it is here, although not all candidates were as careful to work in hours and minutes with many expressing times in decimal hours (ie 1.75 and 1.5 hours): in this case credit was given for such responses although in general candidates should take care as they are often asked to work with times expressed in hours and minutes.

In response to part (b) this candidate correctly draws graphs to illustrate the two different routes so that each ends at 4pm. A significant number of candidates drew a graph for route B that started at 12 noon and consequently was completed at quarter past three: partial credit was given in such cases.

As is illustrated here part (c) was often less successfully answered with only a small number of candidates recognising that the parameter *b* was the speed of the vehicle and only a small proportion of these going on to make any progress in finding a value for *a*. It appeared that some candidates were confused about which parameter represented gradient and which the intercept with the vertical axis.

Mark Scheme

(a)(i)	$R_1 = 0.09(E_1 - 15000) = 0.09(25000 - 15000)$	**M1**	
	$= 0.09 \times 10000$ $= 900$	**A1**	
(a)(ii)	$L_1 = L_0 - R_1 = 10000 - 900 = 9100$	**M1A1**	
(b)			

Year, *n*	Earnings, *E*	Repaid, *R*	Loan remaining, *L*				
0	0	0	10000				
1	25000	900	9100				
2	27000	1080	8020				
3	29000	1260	6760				
4	31000	1440	5320				
5	33000	1620	3700				
6	35000	1800	1900				
7	37000	1980	–80	or	37000	1900	0
8	39000	2160	–2240		39000	0	0

Earnings column: **B1** for $n = 2,3,4$ + **B1ft** for $n = 5,6,7,8$	Condone 8 missing
Repaid column: **B1** for $n = 2,3,4$ + **B1ft** for $n = 5,6,7,8$	Condone 8 missing ft from *E* column and from previous value
Loan column: **B1** for $n = 2,3,4$ + **B1ft** for $n = 5,6,7,8$	Condone 8 missing ft from *R* column and from previous value

(c)	Just less than 7 years	**B1**	Accept 7 or less than 7 Do not accept 6
(d)	**B1** for their points plotted correctly for $n = 2,3,4$ + **B1** for (their) $n = 5,6,7$ (within 1 square)		
(e)	Maria pays back a different amount/more each year Because her earnings increase each year	**B1** **B1**	
	TOTAL	**15**	

Question 4

4 A bank manager carries out a simulation to find out whether a single queue should replace the multiple queues that customers currently form in his branch when waiting to see one of three cashiers.

The bank manager carries out a survey and uses this to set up the simulation. The table below shows how integers generated randomly between 0 and 9 inclusive are assigned by the manager to simulate how long customers take to complete their transactions.

Time to complete transaction in minutes	**Random integer assigned**
3	0, 1
4	2, 3, 4
5	5, 6, 7, 8
6	9

(a) (i) Write down the probability that a customer takes 5 minutes to complete their transaction. *(1 mark)*

(ii) Explain how you deduced your answer. *(1 mark)*

(iii) Complete **Table 1** on the answer sheet, using the given random numbers, to show how long each customer takes to complete their transaction at the bank. *(2 marks)*

Question 4 continues on the next page

(b) First of all, the manager simulates the multiple queuing system in which customers join one of three queues as they arrive at the bank.

To run the simulation, the manager assumes that:

- a single customer arrives at the bank every minute;
- customers immediately join the shortest queue available;
- if there is more than one queue of the same shortest length, the customer joins the queue to see the lowest numbered cashier;
- no time is used between a customer finishing a transaction and the next starting;
- no customer changes queue after having joined one.

Table 2 on the answer sheet has been completed for the first six customers, A–F, showing where each of them is from when they arrive at the bank until they have completed their transactions and leave.

(i) Customer D finds this system of queuing annoying. Explain why this is the case. *(2 marks)*

(ii) Use information from **Table 1** to complete **Table 2** for the next eight customers that arrive at the bank. *(4 marks)*

(c) For the alternative simulation, there is a single queue which each customer joins when they enter the bank if one of the cashiers is not immediately available.

Table 3 on the answer sheet has been completed for the first six customers, A–F, showing where each of them is from when they arrive at the bank until they have completed their transactions using the alternative simulation.

(i) Does this queuing system improve the situation for customer D? Explain your answer. *(2 marks)*

(ii) Use information from **Table 1** to complete **Table 3** for the next eight customers that arrive at the bank. *(4 marks)*

(d) Would you advise the bank manager to introduce the single queuing system? Use evidence from the simulations to support your answer. *(2 marks)*

(e) Give **two** ways in which the simulations could be improved. *(2 marks)*

Student Response

On next page

7

4b. Customer D finds this annoying because they now have to share a cashier whereas others get their own → and they will have to go faster than the others as there as two people. ✗0

4ci NO, because they are still seen 2 minutes late. ✓2

Leave blank

4d no, because the queuing can only increase and you will have more people standing around ✗0

4e • Cannot assume ~~[illegible]~~ a single customer arrives → more than one mostlikely ✓1
• Cannot assume it is every minute → needs to be changed to longer periods aswell ✓1

13

Table 1

Customer	Time arrives at bank	Random number	Time to complete transaction
A	10:00	8	5
B	10:01	2	4
C	10:02	7	5
D	10:03	6	5
E	10:04	1	3
F	10:05	9	6
G	10:06	0	3
H	10:07	4	4
I	10:08	3	4
J	10:09	1	3
K	10:10	8	5
L	10:11	7	5
M	10:12	9	6
N	10:13	3	4

✓2

Table 2

Time	Cashier 1	Cashier 2	Cashier 3
10:00	A		
10:01	A	B	
10:02	A	B	C
10:03	A D	B	C
10:04	A D	B E	C
10:05	D F	E	C
10:06	D F	E	C G K
10:07	D F	E H	G
10:08	D F	H	G I K
10:09	D F	H J	G I
10:10	F	H J	I ~~K~~ K
10:11	F ~~L~~ L	H J	I ~~K~~ K
10:12	F ~~L~~ L	J ~~M~~ M	I ~~K~~ K
10:13	F ~~L~~ L	J ~~M~~ M	I ~~K~~ K n
10:14	F ~~L~~ L	J ~~M~~ M	~~K~~ K n
10:15	F ~~L~~ L	~~M~~ M	~~K~~ K n
10:16	~~L~~ L	~~M~~ M	~~K~~ K n
10:17	~~L~~ L	~~M~~ M	~~K~~ K n
10:18	~~L~~ L	~~M~~ M	~~K~~ K n
10:19	~~L~~ L	~~M~~ M	n
10:20	~~L~~ L	~~M~~ M	n
10:21			n
10:22			n
10:23			
10:24			
10:25			

B1

B0

B0

B0.

Table 3

Time	Queue	Cashier 1	Cashier 2	Cashier 3
10:00		A		
10:01		A	B	
10:02		A	B	C
10:03	D	A	B	C
10:04	D E	A	B	C
10:05	F	D	E	C
10:06	F G	D	E	C
10:07	G H	D	E	F
10:08	H I	D	G	F
10:09	H I J	D	G	F
10:10	I J K	H	G	F
10:11	J K L	H	I	F
10:12	J K L M	H	I	F
10:13	K L M N	H	I	J
10:14	L M N	K	I	J
10:15	M N	K	L	J
10:16	N	K	L	M
10:17	N	K	L	M
10:18	N	K	L	M
10:19		N	L	M
10:20		N		M
10:21		N		M
10:22		N		
10:23				
10:24				
10:25				

✓

✓

4.

✓

7

END OF ANSWER SHEET

Commentary

Although simulation questions such as this often require candidates to engage with a relatively substantial amount of reading to make sense of the situation invariably this appears to present few problems as often this is one of the most successfully answered questions on the paper.

A few, but significant, number of candidates in part (a) of the question thought that as the highest integer was 9 the probability fractions should be expressed in ninths. However, in general part (a) was answered well by many candidates.

Part (b) of the of the question focuses on the simulation of a multiple queuing system. Many candidates worked well with this, however, as is illustrated by this candidate care was not always taken to follow the rules of the simulation carefully. Here a customer should join the lowest numbered cashier when there is more than one queue having equal lengths: this is illustrated by careful inspection of the completed part of the simulation table. Candidates should be encouraged to carefully examine parts of tables that have been completed in the answer sheet. The response here shows how customers have been allocated by the candidate to the highest numbered cashier.

The single queuing system allowed this candidate to complete Table 3 more successfully as their misinterpretation of the situation was confined to the multiple queuing system.

As this candidate, in response to parts (b) (i) and (c) (i), illustrates there is often a need for candidates to interpret carefully in terms of the situation what is happening. Candidates should be encouraged to think very carefully about what is happening. Again this is the case in part (d) of the question. It is important that candidates refer to evidence from *their* simulation: credit will be given if they have made an error in an earlier part of the question provided that they make a careful considered response.

Finally in part (e), as is illustrated here, to improve a simulation candidates should look to the simulations on which it is based. often candidates give responses that suggest the simulation should be run for longer or more times: this effectively might lead to more accurate results but not improve the simulation.

Mark Scheme

TABLE 1

(a)(i)	$\frac{4}{10}$, $\left(\frac{2}{5}\right)$ or decimal equivalents	**B1**	
(a)(ii)	4 integers are assigned out of a possible 10	**B1**	
(a)(iii)			

Customer	Time arrives at bank	Random number	Time to complete transaction
A	10:00	8	5
B	10:01	2	4
C	10:02	7	5
D	10:03	6	5
E	10:04	1	3
F	10:05	9	6
G	10:06	0	3
H	10:07	4	4
I	10:08	3	4
J	10:09	1	3
K	10:10	8	5
L	10:11	7	5
M	10:12	9	6
N	10:13	3	4

	Customers G,H,I,J	**B1**	
	Customers K,L,M,N	**B1**	
(b)(i)	Customer D queues and waits and customer E arrives after him/her and is seen at the same time. E does not have to wait as long as D	**B2**	Do not allow customer E finishes before D (D takes longer). Accept general comment on multiple queue systems

TABLE 2

(b)(ii)

Time	Cashier 1	Cashier 2	Cashier 3
10:00	A		
10:01	A	B	
10:02	A	B	C
10:03	A D	B	C
10:04	A D	B E	C
10:05	D F	E	C
10:06	D F	E G	C
10:07	D F	E G	H
10:08	D F	G I	H
10:09	D F	G I	H J
10:10	F K	G I	H J
10:11	F K	I L	J
10:12	F K	I L	J M
10:13	F K N	I L	J M
10:14	F K N	I L	M
10:15	F K N	L	M
10:16	K N	L	M
10:17	K N	L	M
10:18	K N	L	M
10:19	K N	L	M
10:20	K N		
10:21	N		
10:22	N		
10:23	N		
10:24	N		
10:25			

Arrival time	**B1**	Each at 1 minute interval correct
Customers G, H	**B1**	
Customers I, J	**B1ft**	**FT** (arrive at correct time)
All table correct	**B1**	

TABLE 3

<table>
<tr><td>(c)(i)</td><td>Yes, each customer is now dealt with in order of arrival

Or No, D still waits the same length of time</td><td>B2</td><td>Comment on its own B2</td></tr>
<tr><td>(c)(ii)</td><td colspan="3">

Time	Queue	Cashier 1	Cashier 2	Cashier 3
10:00		A		
10:01		A	B	
10:02		A	B	C
10:03	D	A	B	C
10:04	D E	A	B	C
10:05	F	D	E	C
10:06	F G	D	E	C
10:07	G H	D	E	F
10:08	H I	D	G	F
10:09	H I J	D	G	F
10:10	I J K	H	G	F
10:11	J K L	H	I	F
10:12	J K L M	H	I	F
10:13	K L M N	H	I	J
10:14	L M N	K	I	J
10:15	M N	K	L	J
10:16	N	K	L	M
10:17	N	K	L	M
10:18	N	K	L	M
10:19		N	L	M
10:20		N		M
10:21		N		M
10:22		N		
10:23				
10:24				
10:25				

</td></tr>
</table>

	Customers G, H	**B1**	
	Customers I, J	**B1ft**	**FT** arrive at correct time
	All table correct	**B1**	
	All customers arrive at correct 1 minute interval	**B1**	
(d)	No – not worth the effort as it makes little difference in efficiency of customers being seen	**B1**	Allow alternative argument based on fairness for customers e.g. yes, customers perceive a fairer system
	Specific result from table	**B1**	e.g. F, K or N is quicker or H, J or M takes longer
(e)	Any sensible that will improve realism for example < allow for customers to arrive at different times customers take a greater range of time to complete transactions customers to swap queues etc.	**B1** **B1**	
	TOTAL	**20**	

+ up to 3 marks for ability to present information accurately using correct notation.
+ up to 3 marks for mathematical arguments presented clearly and logically.

	TOTAL MARK FOR PAPER	**70**	