General Certificate of Education
Advanced Subsidiary Examination
June 2014

Use of Mathematics UOM4/1PM

Applying Mathematics Paper 1

Preliminary Material

Data Sheet

To be opened and issued to candidates between
Friday 23 May 2014 and Friday 30 May 2014

REMINDER TO CANDIDATES

YOU MUST **NOT** BRING THIS DATA SHEET
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Bounciness

Figure 1 *A range of balls used in different sports*

To ensure fairness in ball games, it is important that there are regulations governing how bouncy the ball is. For example, if you drop a number of different tennis balls from the same height onto the same piece of ground, you expect each ball to bounce to approximately the same height.

For a tennis ball, the International Tennis Federation states that:

'Each ball shall have a bounce of more than 134.62 cm and less than 147.32 cm when dropped 254.00 cm upon a flat, rigid surface, eg concrete.'

This statement allows for some variation: the bounce height depends not only on the manufacture of the ball but also on other factors such as the surface on which it bounces, temperature and humidity.

Table 1 shows the results of a number of experiments where different sports balls were dropped onto a hard surface and the average bounce height, h_{rebound}, was found for a drop height, h_{drop} metres, when $h_{\text{drop}} = 1$. For each ball, the mean of five bounce heights was found to give h_{rebound}.

Table 1 *Results of experiments conducted dropping different balls onto the same hard surface from a height of 1 metre and measuring the bounce height*

Ball	**Bounce heights**					
	Drop 1	Drop 2	Drop 3	Drop 4	Drop 5	h_{rebound}
Soccer	0.38 m	0.41 m	0.42 m	0.38 m	0.41 m	0.40 m
Tennis	0.51 m	0.51 m	0.51 m	0.48 m	0.49 m	0.50 m
Golf	0.62 m	0.64 m	0.65 m	0.64 m	0.65 m	0.64 m
Baseball	0.33 m	0.34 m	0.31 m	0.31 m	0.31 m	0.32 m
Basketball	0.36 m	0.36 m	0.37 m	0.36 m	0.35 m	0.36 m

If a ball is dropped, it will continue to bounce before eventually coming to rest. Assuming that the height of the bounce is always the same fraction, k, of the drop height, then $h_1 = kh_0$, $h_2 = kh_1$, $h_3 = kh_2$, where h_0 is the initial drop height and h_1, h_2, h_3 and so on are successive bounce heights. Because a drop height of 1 metre was used for the experiments in **Table 1**, k is given by the numerical value of the mean bounce height, as given in the final column.

We can use the recurrence relation $h_{n+1} = kh_n$ to model how each successive bounce height can be found from the previous height. For the case of the tennis ball investigated in the experiment above, $k = 0.50$ and the initial drop height $h_0 = 1.00$. **Table 2** gives the heights of successive bounces of this tennis ball using this recurrence relation.

Table 2 *Bounce height for successive bounces of a tennis ball using* $h_{n+1} = kh_n$

Bounce number	**Height (metres)**
0	1.00
1	0.50
2	0.25
3	0.13
4	0.06
5	0.03
6	0.02

Figure 2 *Graph of bounce height for successive bounces of a tennis ball*

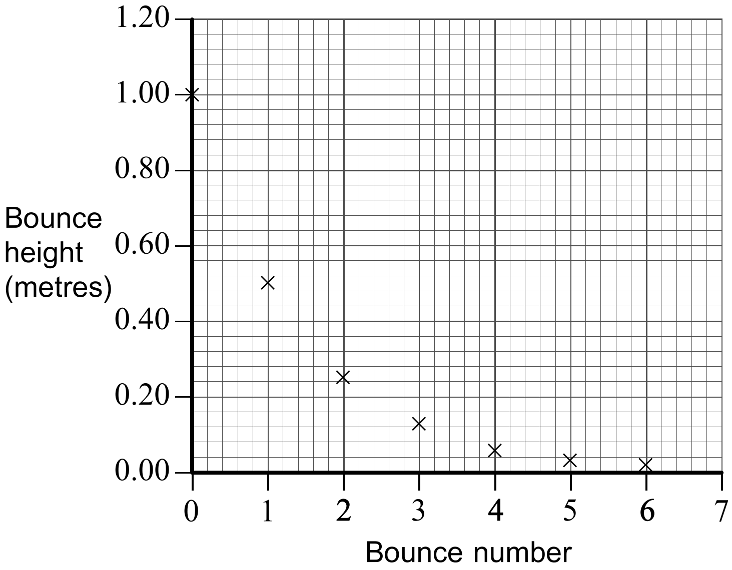

Figure 2 shows a graph of bounce height plotted against bounce number for this ball. This shows clearly how successive bounce heights decrease. The expression $h_n = k^n h_0$ gives the height of a particular bounce number when the drop height is h_0.

The bouncing of a ball against a stationary surface is just a particular case of a collision. When investigating collisions, it is more usual to look at what is known as the coefficient of restitution which is a measure of the impact. This coefficient is a constant that represents the ratio of the speed of the ball just after impact, v_1, to the speed of the ball just before impact, v_0. The coefficient of restitution is defined as $\frac{v_1}{v_0}$.

If we neglect the effects of any air resistance, these speeds, just before and just after impact of the ball with the surface, are related quite simply to the drop and bounce heights. In the case of a ball dropped from a height, h metres, the speed just before impact with the hard surface is given by the equation $v^2 = 2gh$ where g is the acceleration due to gravity, which is often taken to have a numerical value of 9.8.

If we consider a bounce of the ball when it is dropped from a height, h_0, then just before the impact with the hard surface, the speed of the ball, v_0, is given by $v_0 = \sqrt{2gh_0}$. The speed of the ball just after the impact with the hard surface, v_1, is related to the maximum height of the ball after the bounce, h_1, by the equation $v_1 = \sqrt{2gh_1}$.

In this case, therefore, the coefficient of restitution $= \sqrt{k}$.

This allows us to find values of the coefficients of restitution for the different balls in **Table 1**. For example, the coefficient of restitution of the tennis ball is 0.707.

There are some other equations of motion under gravity that can be used to model the bounce of a ball. For example, when the ball is first dropped, the time taken, t_0 seconds, for the ball to reach the ground is related to the height, h_0, by the equation $h_0 = \frac{g}{2}t_0^2$.

So, $t_0 = \sqrt{\frac{2h_0}{g}}$.

Turn over ▶

Similarly, the time taken, t_1 seconds, for the ball to travel from the hard surface to its second bounce height is given by the equation $t_1 = \sqrt{\dfrac{2h_1}{g}}$.

This means that, after the initial drop, the time between a bounce and the next bounce will be given by $t_n = 2\sqrt{\dfrac{2h_n}{g}}$ where n is the bounce number. If you drop a ball, the height reached by it between successive bounces gets less and less and hence there is less time between bounces. The total time for a ball bouncing, T seconds, is therefore given by

$$T = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + 2\sqrt{\frac{2h_3}{g}} + ...$$

On page 2, it was established that

$$h_1 = kh_0$$
$$h_2 = kh_1 = k^2h_0$$
$$h_3 = kh_2 = k^3h_0$$

and so on. Therefore

$$T = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2kh_0}{g}} + 2\sqrt{\frac{2k^2h_0}{g}} + 2\sqrt{\frac{2k^3h_0}{g}} + ...$$
$$= \sqrt{\frac{2h_0}{g}}\,(1 + 2\sqrt{k} + 2\sqrt{k^2} + 2\sqrt{k^3} + ...)$$

The ball eventually stops bouncing and the time this takes is given by

$$T = \sqrt{\frac{2h_0}{g}}\left(\frac{1+\sqrt{k}}{1-\sqrt{k}}\right)$$

Therefore, for the tennis ball in the experiment, dropped from a height of 1 metre

$$T = \sqrt{\frac{2}{9.8}}\left(\frac{1+0.707}{1-0.707}\right) = 2.63$$

So, from being released, the ball takes 2.63 seconds to come to rest.

As you can see, we can use mathematics to model the motion of a ball as it bounces. Using mathematics in this way can be helpful to manufacturers as they experiment with new materials when designing balls for use in different sports.

END OF DATA SHEET

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Bounciness: © International Tennis Federation – rules of tennis