General Certificate of Education
Advanced Subsidiary Examination
June 2013

Use of Mathematics UOM4/1PM

Applying Mathematics Paper 1

Preliminary Material

Data Sheet

To be opened and issued to candidates between Friday 10 May 2013 and Friday 17 May 2013

REMINDER TO CANDIDATES

YOU MUST **NOT** BRING THIS DATA SHEET
WITH YOU WHEN YOU SIT THE EXAMINATION.
A CLEAN COPY WILL BE MADE AVAILABLE.

Where does the money go?

In the UK, we have a 'progressive income tax system'. This means that the more you earn, the greater the proportion of your salary that is paid in income tax.

The personal allowance is the amount of salary that you can earn in any financial year that is not taxed. If you earn less than your personal allowance in a financial year, you will not pay any income tax in that particular financial year. In 2011–2012, you paid the basic rate of income tax at 20 % on the first £ 35 000 of salary earned above the personal allowance. If you earned more than £ 35 000 above the personal allowance, you paid income tax on the amount over £ 35 000 at a higher rate of 40 % up to a salary of £ 150 000. If you earned more than £ 150 000, then the salary in excess of that was taxed at an advanced rate of 50 %.

There was an additional complication for very high earners, as for every £ 2 they earned above £ 100 000, they lost £ 1 from their personal allowance. For anyone who earned £ 114 950 or more, the standard personal allowance, of £ 7475, would therefore have been reduced to zero. All of such a person's salary between £ 114 950 and £ 150 000 is taxed at the higher rate and everything above £ 150 000 at the advanced rate.

For a person with the standard personal allowance (£ 7475 in 2011–2012), the ranges in which the different tax rates were applicable are given in **Table 1**.

Table 1

Annual salary and income tax rates for 2011–2012 for someone with the standard personal allowance

Annual salary range for person with standard personal allowance	**Income tax rate**	**Phase of income taxation**
£ 0 – £ 7475	0 %	Tax-free
£ 7475 – £ 42 475	20 %	Basic rate
£ 42 475 – £ 150 000†	40 %	Higher rate
more than £ 150 000	50 %	Advanced rate

†Between a salary of £ 100 000 and £ 114 950, the personal tax allowance for this person is reduced by £ 1 for every £ 2 earned.

Because the different parts of a person's salary, £ S, are taxed at different fixed percentage rates, a graph of their take-home pay, £ I, plotted against £ S gives a number of straight line segments, as shown in **Figure 1**.

The coordinates of the end-points of the line segments can be obtained from the data in **Table 2**, which gives values of I for the key values of S $(0 \leqslant S \leqslant 200\,000)$ for someone with the standard personal allowance of £ 7475 in 2011–2012.

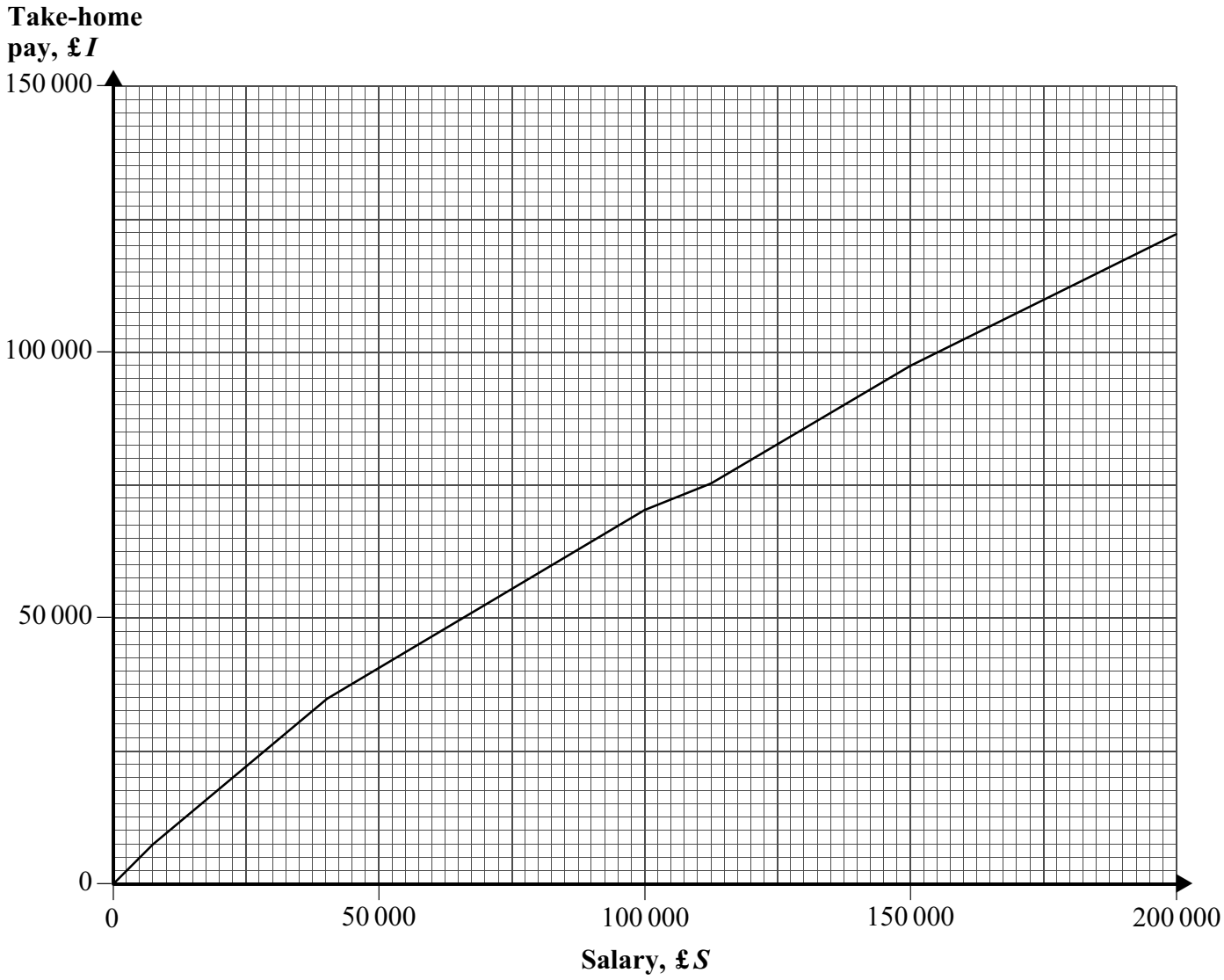
Table 2

Values of I for key values of salary, £S, for someone with the standard personal allowance in 2011–2012

Salary, £S	**Take-home pay, £I**
£0	£0
£7475	£7475
£42 475	£35 475
£100 000	£69 990
£114 950	£75 970
£150 000	£97 000
£200 000	£122 000

Figure 1

Graph showing take-home pay, £I, plotted against salary, £S, for someone with the standard personal allowance of £7475, for 2011–2012 tax rates



Equations for the line segments that represent the different income tax phases in 2011–2012 can be found.

For example:

(i) in the tax-free phase where no income tax is paid $(0 \leqslant S \leqslant 7475)$

$I = S$

(ii) in the basic rate tax phase $(7475 \leqslant S \leqslant 42\,475)$

$$\begin{aligned} I &= 7475 + 0.8(S - 7475) \\ &= 1495 + 0.8S \end{aligned}$$

(iii) in the higher rate tax phase and with a salary below £ 100 000 $(42\,475 \leqslant S \leqslant 100\,000)$

$$\begin{aligned} I &= 7475 + 0.8(42\,475 - 7475) + 0.6(S - 42\,475) \\ &= 7475 + 0.8 \times 35\,000 + 0.6S - 0.6 \times 42\,475 \\ &= 9990 + 0.6S \end{aligned}$$

(iv) for someone with a salary of just over £ 100 000, their tax allowance, £ T, in terms of their salary, £ S, is given by $(100\,000 \leqslant S \leqslant 114\,950)$

$$\begin{aligned} T &= 7475 - \frac{(S - 100\,000)}{2} \\ &= 57\,475 - \frac{S}{2} \end{aligned}$$

their take-home pay, £ I, is related to S and T by the equation

$$\begin{aligned} I &= T + 0.8 \times 35\,000 + 0.6 \times (S - 35\,000 - T) \\ &= 0.4T + 0.2 \times 35\,000 + 0.6S \\ &= 0.4T + 7000 + 0.6S \end{aligned}$$

substituting for T gives

$$\begin{aligned} I &= 0.4\left(57\,475 - \frac{S}{2}\right) + 7000 + 0.6S \\ &= 22\,990 - 0.2S + 7000 + 0.6S \\ &= 29\,990 + 0.4S \end{aligned}$$

In considering the case of tax rates and personal allowance for 2011–2012, you have met the key ideas of our income tax system. However, the government minister in charge of the nation's finances, the Chancellor of the Exchequer, may change the system each financial year while taking account of increased salaries by changing factors such as the personal tax allowance and the tax rates.

Table 3 gives the more general equations for take-home pay, £ I, in terms of salary, £ S, up to £ 100 000. The table applies to someone with a tax-free allowance of £ A where £ B is the income taxed at the basic rate. The final column gives a formula for I in terms of S.

Table 3

General equations giving take-home pay, £I, in terms of salary, £S, for the UK model of taxation

Annual salary for person with standard personal allowance (£A)	**Income tax rate (percentage rate expressed as a decimal)**	**Phase of income tax**	**Formula for take-home pay, £I, in terms of salary, £S**
£0 − £A	0	Tax-free	$I = S$
£A − £$(A + B)$	r	Basic rate	$I = A + (1 - r)(S - A)$
£$(A + B)$ − £100 000	t	Higher rate	$I = A + (1 - r)B + (1 - t)\left[S - (A + B)\right]$

The tax rates for the financial year 2012–2013 are given in **Table 4**. As you can see, compared with the previous year, the personal tax allowance was increased by £630 to £8105 but the amount on which the basic rate of income tax is paid was reduced by the same amount, to £34 370. This change had the effect that in both financial years, the threshold above which the higher rate income tax was payable was the same.

Table 4

Annual salary and income tax rates for 2012–2013 for someone with the standard personal allowance

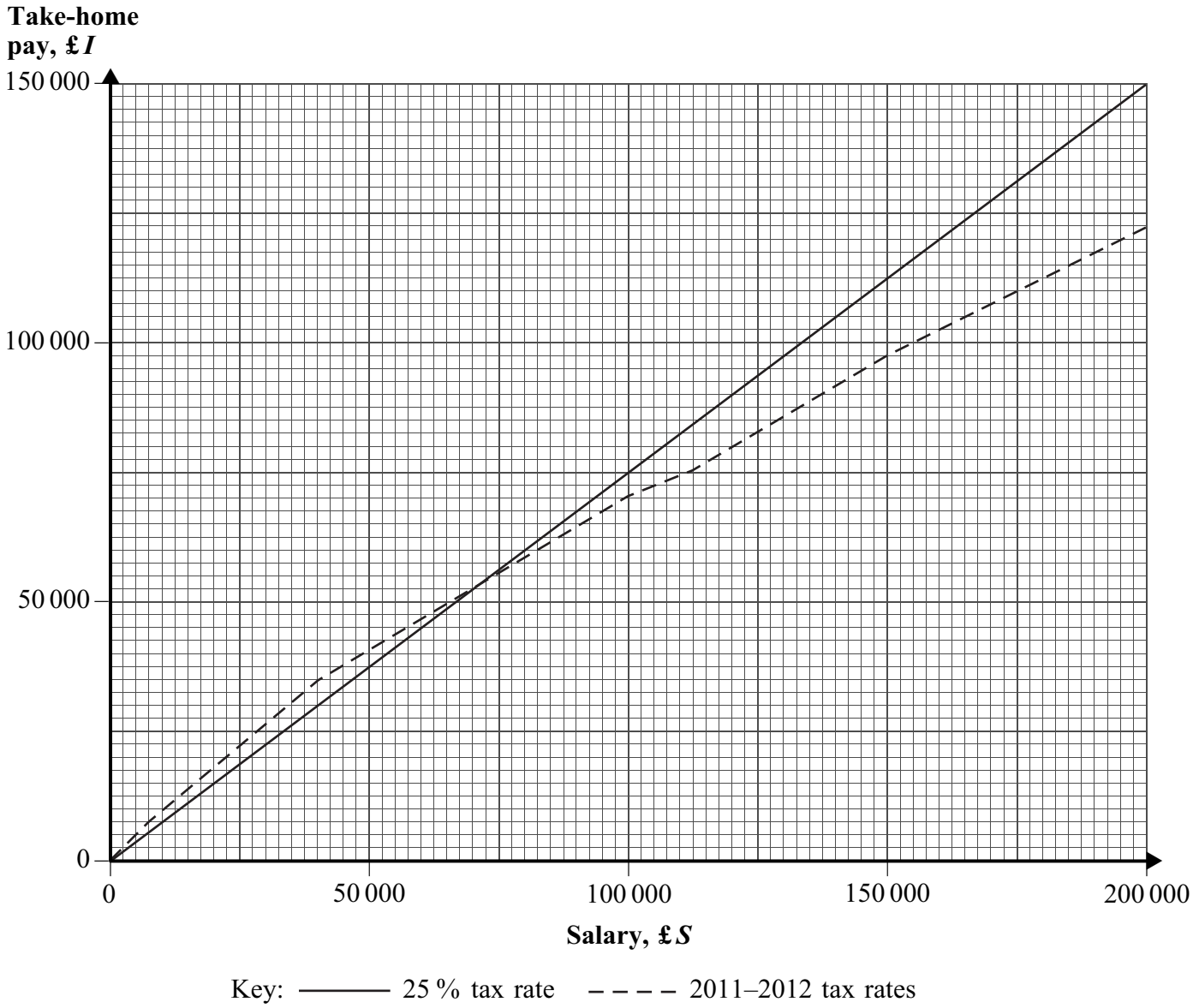
Annual salary range for person with standard personal allowance	**Income tax rate**	**Phase of income taxation**
£0 − £8105	0 %	Tax-free
£8105 − £42 475	20 %	Basic rate
£42 475 − £150 000†	40 %	Higher rate
more than £150 000	50 %	Advanced rate

†The personal allowance is reduced by £1 for every £2 earned when income is more than £100 000.

What would be the effect of having an income tax system that was non-progressive? For example, what would be the effect of having a constant rate of income tax for all salaries? The graph in **Figure 2** shows the situation for an income tax rate of 25 % applied to all salaries, together with a graph of the actual situation in 2011–2012.

Figure 2

Graph showing take-home pay, £I, plotted against salary, £S, for 2011–2012 tax rates and for a single constant income tax rate of 25 %

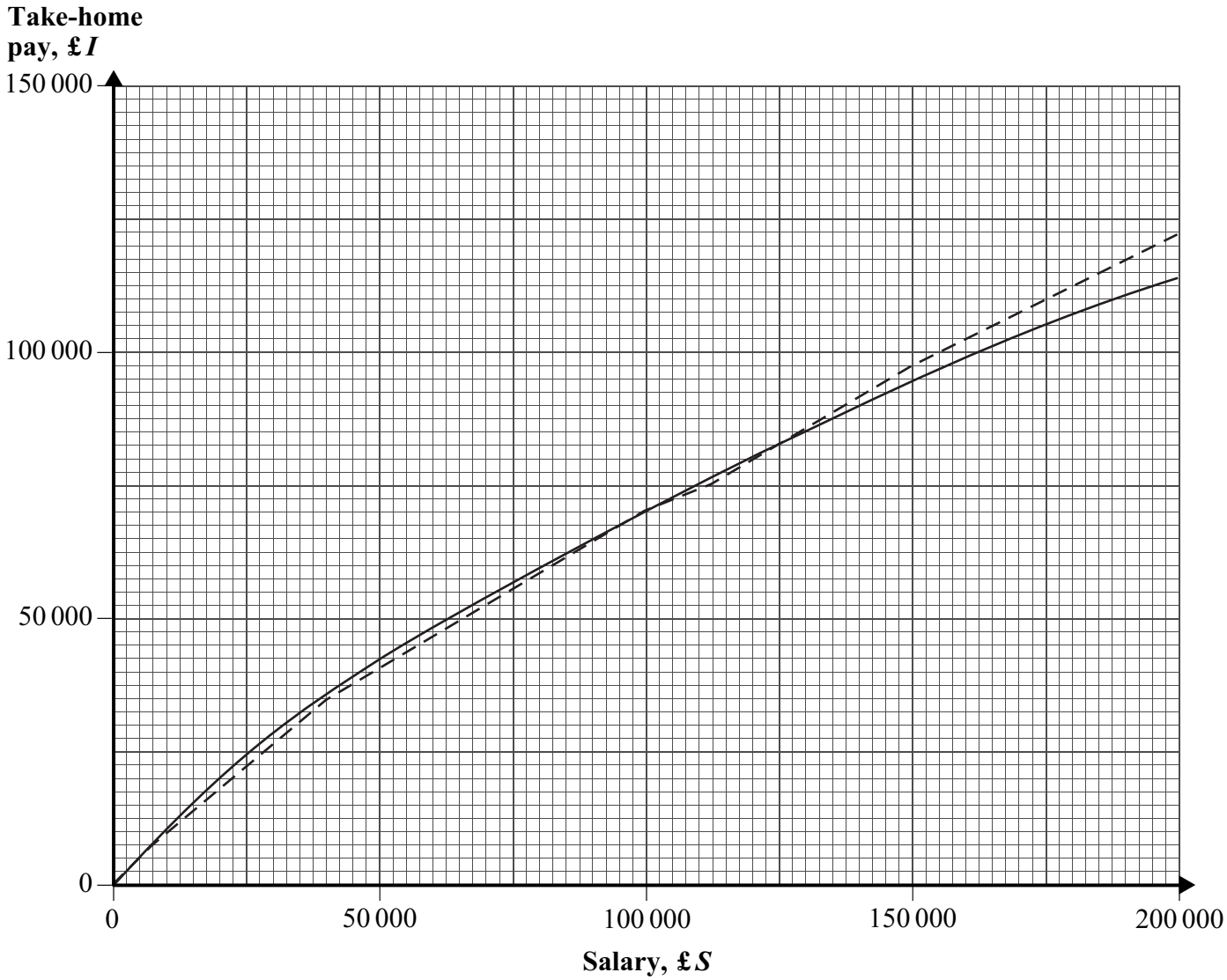


As you can see, if this were to be the system, those on low incomes would lose out, as their income after paying tax would be decreased, whereas those on higher incomes would gain.

The graph of **Figure 3** illustrates that it appears that take-home pay, £I, might perhaps be modelled using a single function of salary, £S. For both 2011–2012 and 2012–2013, the function $I = 350\sqrt{(S + 20\,000)} - 50\,000$ appears to give a good approximation to the linear functions used. Overall, this would have the effect of being fairer in that the proportion of your salary paid in income tax would increase continuously with salary.

Figure 3

Graph showing take-home pay, £I, plotted against salary, £S, for 2011–2012 tax rates and modelled by the function $I = 350\sqrt{(S + 20\,000)} - 50\,000$



Key: ——— Model tax rate – – – – 2011–2012 tax rates

As you can see, although using only relatively straightforward mathematical ideas, taxation is a complex business. It is important that you understand these ideas if you want to know why your actual income is less than your salary. However, this is just part of the story as you will also pay another proportion of your salary in an additional tax known as National Insurance. You may also contribute to a pension scheme and repay a student loan.

END OF DATA SHEET

There are no data printed on this page