Version 1.0

Free-Standing Mathematics Qualification
June 2013

Mathematics Advanced Level 6992

(Specification 6992)

Modelling with Calculus

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Question	Solution	Marks	Total	Comments
1(a)	$h = 12 - 5 = 7$ metres	B1	1	
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 12 - 10t$	M1A1	2	
(c)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow$			
	$12 - 10t = 0$	M1		
	$t = \frac{12}{10}$ or 1.2	A1	2	
(d)	when $t = 1.2$			
	$h = 12 \times 1.2 - 5 \times (1.2)^2$	M1		
	$= 7.2$	A1	2	
(e)(i)	$\frac{\mathrm{d}^2h}{\mathrm{d}t^2} = -10$	B1	1	
(ii)	$\frac{\mathrm{d}^2h}{\mathrm{d}t^2}$ is negative; turning point is a maximum	E1	1	
	Total		**9**	

Question	Solution	Marks	Total	Comments
2(a)	$\frac{dn}{dt} = 100t - 4t^3$	M1A1	2	
(b)	$\frac{dn}{dt} = 0 \Rightarrow$	M1		
	$100t - 4t^3 = 0$			
	$t(100 - 4t^2) = 0$			
	$4t(5-t)(5+t) = 0$	M1		$t = 0$ or $t = \frac{0 \pm \sqrt{0+1600}}{-8}$
	= 0 or 5 or –5	A2	4	A1 for two correct lose $t = 0$; $t = 5$, -5 M1A1 M1A1 for any 1 (or 2) correct
(c)	$\frac{d^2n}{dt^2} = 100 - 12t^2$	M1A1	2	ft
(d)	when t =5			or for using $t = -5$. If 0 and 5 above (b) condone only using $t = 5$
	$\frac{d^2n}{dt^2} = -200$	B1		
	this is negative, hence answer is a maximum for t = 5 and -5	E1	2	
(e)	when t = 5,			or for using $t = -5$
	$n = 50 \times (5)^2 - (5)^4 + 4000$	M1		must be using t found in (b)
	= 4625	A1		
	maximum flow is at 6 pm or 8 am	E1	3	do not accept a value of t ; needs both answers if 0 and 5 above in (b), penalise E1 for any 1 value
	Total		**13**	

Question	Solution	Marks	Total	Comments
3(a)	four strips $\Rightarrow$ values of t are –6, –3, 0, 3 and 6 when: $t = -6$, $n = 4504$ $t = -3$, $n = 4369$ $t = 0$, $n = 4000$ $t = 3$, $n = 4369$ $t = 6$, $n = 4504$	B2		B1 for any two correct
	area $\approx$ $\frac{1}{2}\times 3\{4504+4504+2(4369+4000+4369)\}$	M1A1		
	$=\frac{3}{2}(9008+2\times 12\,738)$ = 51 726			
	total number of vehicles is 51 700	A1	5	
(b)(i)	$\int_{-6}^{6}(50t^2-t^4+4000)\mathrm{d}t$			
	$=\left[\frac{50}{3}t^3-\frac{1}{5}t^5+4000t\right]_{-6}^{6}$	B1B1		B1 for two correct
	= (3600 – 1555.2 + 24 000) – (–3600 + 1555.2 – 24000)	M1		
	= 52 089.6			
	= 52 100	A1	4	accept 52 090
(ii)	$\therefore$ average number per hour is $\frac{52090}{12}$			
	or $\frac{52100}{12}$	M1		or $\frac{51726}{12} = 4310$
	= 4340 (or 4341)	A1	2	ft from (a) or (b)
	Total		**11**	

Question	Solution	Marks	Total	Comments
4(a)(i)	$\frac{dc}{dt} = -\frac{1}{25}(20-3) = -\frac{17}{25}$ or -0.68	B1	1	accept $-\frac{17}{25}$; not $-\frac{1}{25}(17)$
(ii)	Cola is cooling at less than 1° per minute	E1	1	oe needs minute
(b)	$\frac{dc}{dt} = -\frac{1}{25}(c-3)$			
	$\int \frac{dc}{c-3} = -\int \frac{1}{25} dt$	M1		
	$\ln(c-3) = -\frac{1}{25}t + d$	A1A1		
	$\frac{1}{25}t = d - \ln(c-3)$			
	when $t = 0, c = 23 \Rightarrow d = \ln 20$			
	$\frac{1}{25}t = \ln\frac{20}{c-3}$	B1	4	
(c)	$\frac{1}{25}t = \ln 10$	M1		
	$t = 57.56$			
	$t = 57.6$	A1	2	
(d)	$e^{\frac{1}{25}t} = \frac{20}{c-3}$	M1		condone sign error
	$c - 3 = 20e^{-\frac{1}{25}t}$	A1		
	$c = 3 + 20e^{-\frac{1}{25}t}$	A1	3	
(e)	when $t = 6, c = 3 + 20e^{-\frac{6}{25}}$	M1		
	$= 3 + 15.73$			
	$= 18.7°$	A1	2	
(f)(i)	as t becomes very large, c approaches 3	B1	1	
(ii)	as t becomes very large, $\frac{dc}{dt} = 0$	B1	1	
	Total		**15**	

Question	Solution	Marks	Total	Comments
5(a)(i)	4.03638	B1		
(a)(ii)	3.49813	B1	2	penalise once if not to 5dp
(b)	$\frac{3.49813 - 4.03638}{0.1}$	M1		condone $\frac{\text{(i)} - \text{(ii)}}{0.1}$ for M1
	$= -5.38$	A1	2	
	Total		**4**	
6(a)	when $t = 3, n = 110 + 50\sin\frac{\pi}{2}$			
	$= 110 + 50 \times 1$	B1		
	number is 160	B1	2	B1 for $\sin\frac{\pi}{2} = 1$
(b)(i)	when $t = 9$,			
	$\sin\frac{3\pi}{2} = -1$ which is a minimum value	E1		needs minimum
	hence number of birds is a minimum	E1	2	
(ii)	the model predicts next minimum when			
	$\sin\frac{\pi}{6}t = -1$	B1		
	the next minimum point is when $t = 21$	B1	2	
(c)	$\frac{dn}{dt} = 50.\frac{\pi}{6}\cos\frac{\pi}{6}t$	B1		$\frac{\pi}{6}$ **or** $\cos\frac{\pi}{6}\ t$
	$= \frac{25\pi}{3}\cos\frac{\pi}{6}t$	B1	2	all correct
	Total		**8**	
	TOTAL		**60**	