Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
TOTAL	

AQA

Free-Standing Mathematics Qualification
Advanced Level
June 2012

Working with Algebraic and Graphical Techniques 6991/2

Unit 11

Friday 18 May 2012 1.30 pm to 3.00 pm

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.
- You may use either a scientific calculator or a graphics calculator.

Advice
- You do not necessarily need to use all the space provided.

Section A

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Longbows** *on page 2 of the Data Sheet.*

1 The curve of the wood on a longbow can be modelled by the equation

$$y = \frac{x(180 - x)}{270}$$

where y cm is the distance of the wood from the string, and x cm is the distance from one end of the string.

(a) Using this model, complete the table of values opposite. *(2 marks)*

(b) On the grid opposite, draw the graph of

$$y = \frac{x(180 - x)}{270} \text{ for } 0 \leqslant x \leqslant 180$$ *(2 marks)*

(c) Use your graph to find the values of x when $y = 10$. *(2 marks)*

(d) (i) Complete the square to express $x(180 - x)$ in the form $a - (b - x)^2$, giving the values of a and b. *(3 marks)*

(ii) Explain how the values of a and b are connected to the dimensions of the longbow. *(2 marks)*

(e) Another longbow has a string that is 110 cm long, and the maximum distance from the string to the wood is 20 cm.

Write down an equation that models the curve of the wood on this longbow. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

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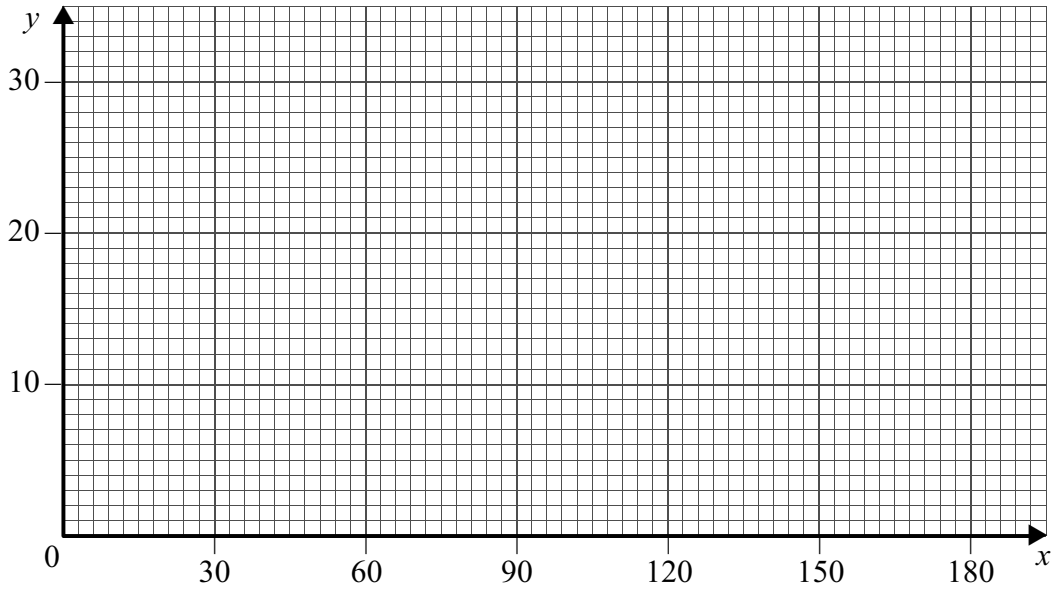
QUESTION PART REFERENCE

Answer space for question 1

x	0	30	60	90	120	150	180
y	0	16.7					



QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

Section B

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Radioactive iodine** *on page 2 of the Data Sheet.*

2 (a) The mass of radioactive iodine, m grams, after time t days can be modelled by the equation

$$m = Ce^{-kt}$$

where C and k are constants.

For this model, $\ln m = \ln C - kt$.

(i) Complete the table of values opposite. *(2 marks)*

(ii) On the grid opposite, plot $\ln m$ against t.

Draw a line of best fit on your graph. *(3 marks)*

(iii) Write down the value of C and use your graph to find the value of k. *(3 marks)*

(b) Another model for the mass of radioactive iodine is given by the equation

$$m = 12 \times 2^{-\frac{t}{8}}$$

(i) Find the mass of radioactive iodine predicted by this model after 3 days. *(2 marks)*

(ii) Find the time predicted by this model for the mass of radioactive iodine to decay to 5 grams. *(3 marks)*

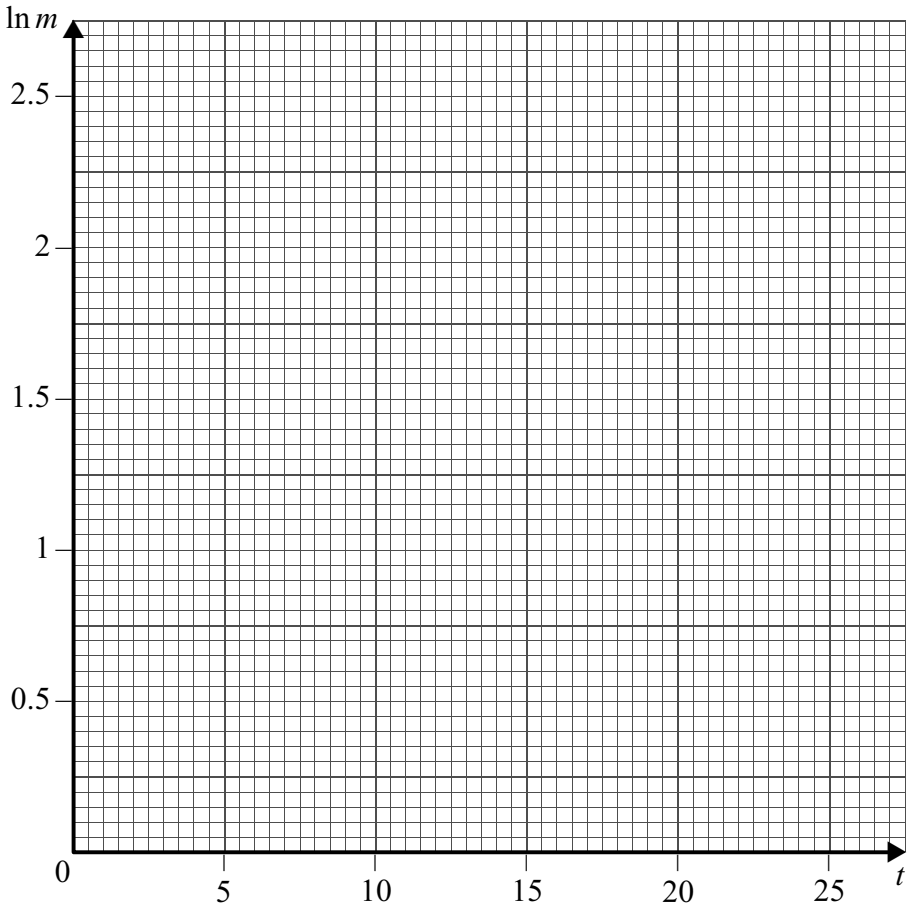
QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

t	0	5	10	15	20	25
m	12	7.79	5.06	3.28	2.13	1.38
$\ln m$	2.48					



QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

Section C

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Garden waste** *on page 3 of the Data Sheet.*

3 In one year, the amount of garden waste, A tonnes, for each month, t, is modelled by the local council using the equation

$$A = 245 + 165\cos(30t + 150)^\circ$$

Values of t correspond to the month of the year.
For example, January is the first month of the year and so $t = 1$.
For February, $t = 2$.

(a) What is the maximum amount of garden waste predicted by this model? *(1 mark)*

(b) In which month does this maximum occur? *(2 marks)*

(c) What is the minimum amount of garden waste predicted by this model? *(1 mark)*

(d) In which month does this minimum occur? *(2 marks)*

(e) In which months does this model predict the value of A will be 300 tonnes? *(4 marks)*

(f) For the function $245 + 165\cos(30t + 150)^\circ$, state:

(i) the amplitude; *(1 mark)*

(ii) the period. *(1 mark)*

(g) The actual amount of garden waste recycled in June is 400 tonnes.

Calculate the percentage error in using the model to predict the value of A in June. *(3 marks)*

(h) Describe fully the transformations that map the graph of $A = \cos t^\circ$ onto the graph of $A = 245 + 165\cos t^\circ$. *(2 marks)*

(i) Describe fully the transformations that map the graph of $A = \cos t^\circ$ onto the graph of $A = \cos(30t + 150)^\circ$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 3

Section D

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Aircraft drag** *on page 3 of the Data Sheet.*

4 (a) On an aircraft, the induced drag, D thousand newtons, is given by the equation

$$D = \frac{k}{v^2}$$

where v m/s is the speed of the plane, and k is a constant.

Find k if $D = 20$ when $v = 220$. *(2 marks)*

(b) The graph of D against v is shown opposite.

Find the gradient of the curve when $v = 100$. *(2 marks)*

(c) What are the units of the gradient? *(1 mark)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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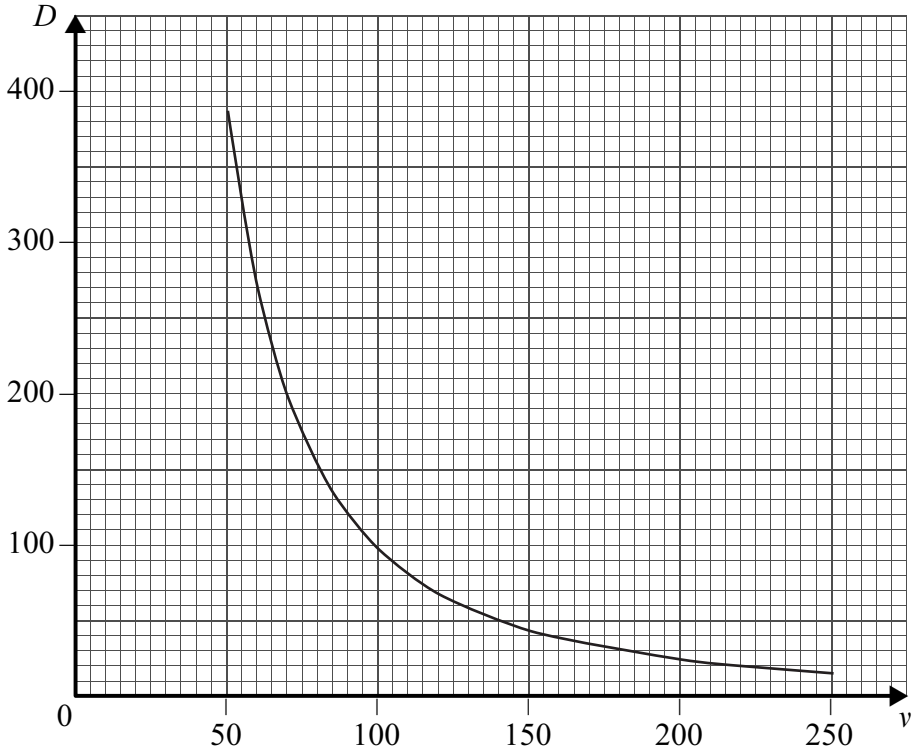

5 (a) For another aircraft, the parasitic drag, P thousand newtons, is given by the equation

$$P = \frac{v^2}{4000}$$

On the set of axes below, draw the graph of $P = \frac{v^2}{4000}$ for $0 \leqslant v \leqslant 160$. *(3 marks)*

(b) On the same set of axes, draw the graph of the induced drag given by the equation

$$D = \frac{10\,000}{v^2} \text{ for } 20 \leqslant v \leqslant 160$$ *(3 marks)*

(c) (i) The fuel consumption of an aircraft is least when the induced drag is equal to the parasitic drag.

Find the speed that gives the least fuel consumption for the aircraft in part **(b)**. *(2 marks)*

(ii) Another aircraft has its least fuel consumption at a speed of 200 m/s. Its parasitic drag is given by the equation $P = 0.0004v^2$.

Find the equation for its induced drag. *(2 marks)*

QUESTION PART REFERENCE

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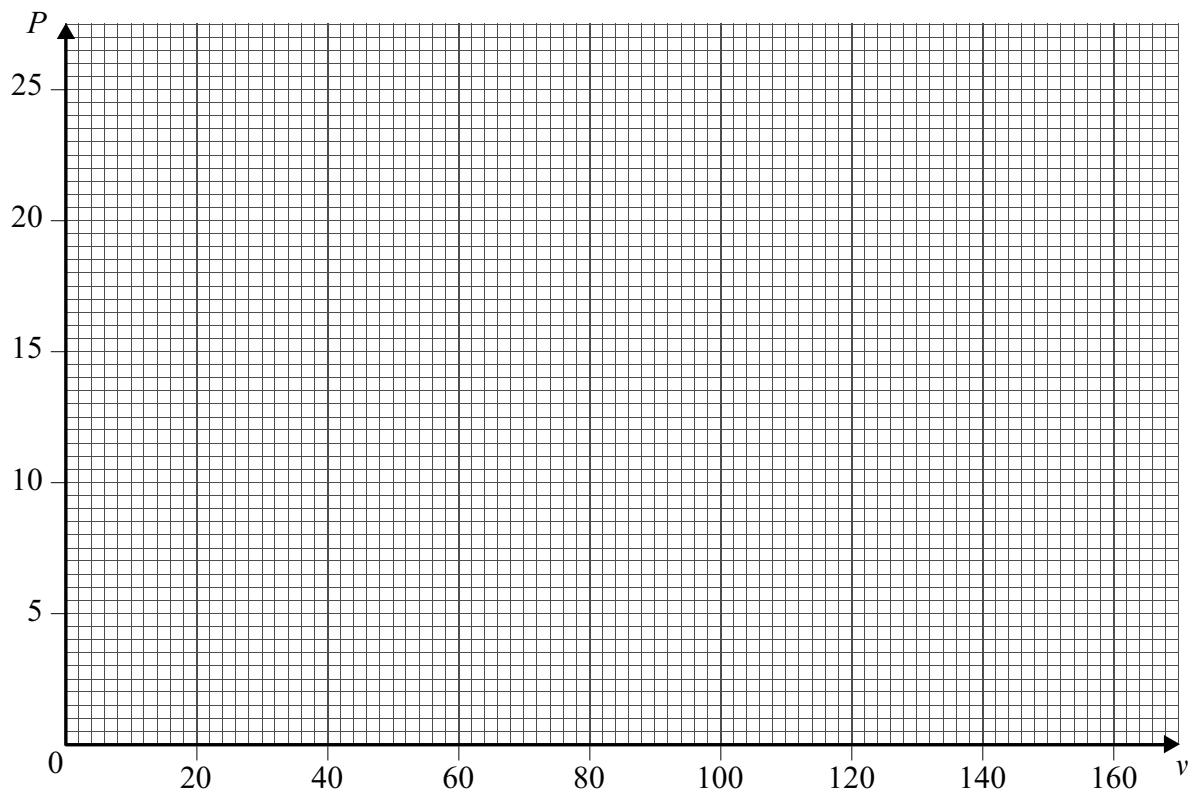

QUESTION PART REFERENCE

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