Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
TOTAL	

AQA

Free-Standing Mathematics Qualification
Advanced Level
June 2011

Modelling with Calculus 6992/2

Unit 12

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.
- You may use either a scientific calculator or a graphics calculator.

Do not write outside the box

Section A

Answer **all** questions in the spaces provided.

Use **Tides at Ilfracombe** *on page 2 of the Data Sheet.*

1 The height of the water, h cm, can be modelled by the equation

$$h = 16t^2 - 193t + 780$$

for values of t from 0 to 12, where t is the time in hours after 6 am.

(a) **Use this model and calculus** to predict the minimum height of the water which occurred that day. *(6 marks)*

(b) (i) Sketch a graph of the equation

$$h = 16t^2 - 193t + 780 \quad \text{for} \quad 0 \leqslant t \leqslant 18$$ *(2 marks)*

(ii) State one problem with using this model. *(1 mark)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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Section B

Answer **all** questions in the spaces provided.

Use **Shares** *on page 3 of the Data Sheet.*

2 The value of the shares, £ v, may be modelled by the function

$$v = 16 - 2t + 5t^2 - 2t^3$$

for values of t from 0 to 3.

Use this model and calculus to answer the following questions.

(a) Find the values of t at the turning points of this function. *(5 marks)*

(b) Find $\frac{d^2v}{dt^2}$. *(2 marks)*

(c) Find the maximum value of the shares, confirming that this value is a maximum. *(4 marks)*

(d) Find the value of t when $\frac{d^2v}{dt^2} = 0$ and state the relevance of this to an investor. *(3 marks)*

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3 (a) (i) Use the trapezium rule with three strips to find an estimate for

$$\int_0^3 (16 - 2t + 5t^2 - 2t^3)\,\mathrm{d}t \qquad \textit{(5 marks)}$$

(ii) Explain how a more accurate value could be found. *(1 mark)*

(b) Use integration to find the value of

$$\int_0^3 (16 - 2t + 5t^2 - 2t^3)\,\mathrm{d}t \qquad \textit{(5 marks)}$$

(c) Hence, using the model $v = 16 - 2t + 5t^2 - 2t^3$, find the mean value of the shares during the first three months of 2010. *(2 marks)*

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Section C

Answer **all** questions in the spaces provided.

Use **Rollercoaster** *on page 4 of the Data Sheet.*

4 The height, h centimetres, of the rollercoaster above the ground may be modelled by the function

$$h = 70 + 40\cos\left(\frac{\pi}{3}x\right)$$

where x metres is the horizontal distance from the starting point O.

(a) Find the height of the rollercoaster predicted by the model when Daniel is 6 metres horizontally from O. *(2 marks)*

(b) Find an expression for $\frac{\mathrm{d}h}{\mathrm{d}x}$. *(3 marks)*

(c) Find the maximum value of $\frac{\mathrm{d}h}{\mathrm{d}x}$.

Give your answer to three significant figures. *(3 marks)*

QUESTION PART REFERENCE

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Section D

Answer **all** questions in the spaces provided.

Use **Radioactive decay** *on page 4 of the Data Sheet.*

5 The mass, m grams, of a radioactive substance decreases with time, t hours.

The rate of change of the mass is directly proportional to the mass at that time.

This can be expressed by the differential equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -km$$

where k is a positive constant.

(a) Find the general solution for m of this differential equation. *(5 marks)*

(b) When $t = 0$, the mass of the radioactive substance was 40 grams.

Show that $m = 40\,\mathrm{e}^{-kt}$. *(2 marks)*

(c) The half-life of the radioactive substance is 6 hours; ie when $t = 6$, the mass of the radioactive substance is 20 grams.

Find k. *(3 marks)*

(d) Find the mass of the radioactive substance when $t = 18$. *(3 marks)*

(e) Find the value of t when the mass is 2 grams. *(3 marks)*

QUESTION PART REFERENCE

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END OF QUESTIONS

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