Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
TOTAL	

Free-Standing Mathematics Qualification
Advanced Level
June 2010

Modelling with Calculus 6992/2

Unit 12

Tuesday 25 May 2010 1.30 pm to 3.00 pm

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a ruler
- a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.
- You may use either a scientific calculator or a graphics calculator.

Section A

Answer **all** questions in the spaces provided.

Use **Shot put** *on page 2 of the Data Sheet.*

1 Felipe throws the shot.

The vertical height of the shot, h metres, above O, the point from which it was thrown, can be modelled by the equation

$$h = x - 0.05x^2$$

where x metres is the horizontal distance from point O.

Use this model and calculus to answer the following questions.

(a) Find the vertical height of the shot above O when $x = 8$. *(1 mark)*

(b) Find $\dfrac{\mathrm{d}h}{\mathrm{d}x}$. *(2 marks)*

(c) Find x when $\dfrac{\mathrm{d}h}{\mathrm{d}x} = 0$. *(2 marks)*

(d) Hence predict the maximum vertical height of the shot above O. *(2 marks)*

(e) (i) Find $\dfrac{\mathrm{d}^2h}{\mathrm{d}x^2}$. *(1 mark)*

(ii) Hence state how this value confirms that the answer to part **(d)** is the maximum height and not the minimum. *(1 mark)*

(f) Felipe lets go of the shot when it is 2 metres above the level of the horizontal ground.

Find the horizontal distance which the shot travels before hitting the ground. *(4 marks)*

QUESTION PART REFERENCE	
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Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

Section B

Answer **all** questions in the spaces provided.

Use **Coffee shop** *on page 3 of the Data Sheet.*

2 The number of coffees, S, sold per day, $10t$ days after the shop was opened, may be modelled by the function

$$S = 10t^3 - 60t^2 + 110t + 10$$

for values of t from 0 to 3.3; that is, for the first 33 days that the shop was open.

(a) **Use this model and calculus** to answer the following questions.

(i) Find t when S has a minimum turning point. *(6 marks)*

(ii) Find this minimum value. *(2 marks)*

(iii) Find $\dfrac{\mathrm{d}^2S}{\mathrm{d}t^2}$. *(2 marks)*

(iv) Use your answer to part **(a)(iii)** to confirm that the value found in part **(a)(ii)** is a minimum value. *(2 marks)*

(b) The mean number of coffees sold per day is given by

$$\overline{S} = \frac{1}{3}\int_0^3 (10t^3 - 60t^2 + 110t + 10)\mathrm{d}t$$

(i) Use the trapezium rule with three strips to find an estimate for the mean number of coffees sold per day. *(5 marks)*

(ii) How would you make the answer obtained from the trapezium rule more accurate? *(1 mark)*

(iii) Use integration to evaluate

$$\frac{1}{3}\int_0^3 (10t^3 - 60t^2 + 110t + 10)\mathrm{d}t$$

to find the mean number of coffees sold per day. *(4 marks)*

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Section C

Answer **all** questions in the spaces provided.

Use **Height of a sunflower** *on page 3 of the Data Sheet.*

3 After t days, the height of the sunflower, h centimetres, satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = k(150 - h)$$

where k is a constant.

(a) (i) Find the general solution for h of this differential equation.

Give your answer in terms of k. *(5 marks)*

(ii) Given that, when $t = 0$, the height of the sunflower was zero, show that $h = 150(1 - \mathrm{e}^{-kt})$. *(2 marks)*

(iii) When $t = 30$, $h = 100$. Find k. *(3 marks)*

(b) Find the value of t when the height is 120 centimetres. *(3 marks)*

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4 The height of another sunflower, h centimetres, after t days of growth is given by

$$h = 150(1 - e^{-0.04t})$$

(a) Find h, to five decimal places, when:

(i) $t = 125$;

(ii) $t = 125.1$. *(2 marks)*

(b) Using your answers to part **(a)**, find an estimate for the rate of increase in height when $t = 125$. *(3 marks)*

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Section D

Answer **all** questions in the spaces provided.

Use **Experiment with an elastic string** *on page 4 of the Data Sheet.*

5 The distance, x centimetres, of the weight below O may be modelled by the function

$$x = 15 + 3\cos\frac{\pi}{2}t$$

where t is the number of seconds after the system was set into motion.

(a) When $t = 2$, show that $x = 12$. *(1 mark)*

(b) Find an expression for the velocity of the weight, $\frac{\mathrm{d}x}{\mathrm{d}t}$. *(2 marks)*

(c) (i) Find the maximum value of $\frac{\mathrm{d}x}{\mathrm{d}t}$.

You may leave your answer as a multiple of π or as a decimal to three significant figures. *(2 marks)*

(ii) Find two values of t when this occurs. *(2 marks)*

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END OF QUESTIONS

There are no questions printed on this page

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