Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					



Free-Standing Mathematics Qualification Advanced Level June 2010

# **Modelling with Calculus**

6992/2

For Examiner's Use

Examiner's Initials

Mark

Question

1

2

3

4

5

TOTAL

Unit 12

Tuesday 25 May 2010 1.30 pm to 3.00 pm

## For this paper you must have:

- a clean copy of the Data Sheet (enclosed)
- a ruler
- a calculator.

#### Time allowed

• 1 hour 30 minutes

#### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- You may not refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

#### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 60.
- You may use either a scientific calculator or a graphics calculator.



#### Section A

Answer all questions in the spaces provided.

Use Shot put on page 2 of the Data Sheet.

**1** Felipe throws the shot.

The vertical height of the shot, h metres, above O, the point from which it was thrown, can be modelled by the equation

$$h = x - 0.05x^2$$

where x metres is the horizontal distance from point O.

Use this model and calculus to answer the following questions.

- (a) Find the vertical height of the shot above O when x = 8. (1 mark)
- (b) Find  $\frac{dh}{dx}$ . (2 marks)
- (c) Find x when  $\frac{dh}{dx} = 0$ . (2 marks)
- (d) Hence predict the maximum vertical height of the shot above O. (2 marks)
- (e) (i) Find  $\frac{d^2h}{dx^2}$ . (1 mark)
  - (ii) Hence state how this value confirms that the answer to part (d) is the maximum height and not the minimum. (1 mark)
- (f) Felipe lets go of the shot when it is 2 metres above the level of the horizontal ground.

Find the horizontal distance which the shot travels before hitting the ground.

(4 marks)

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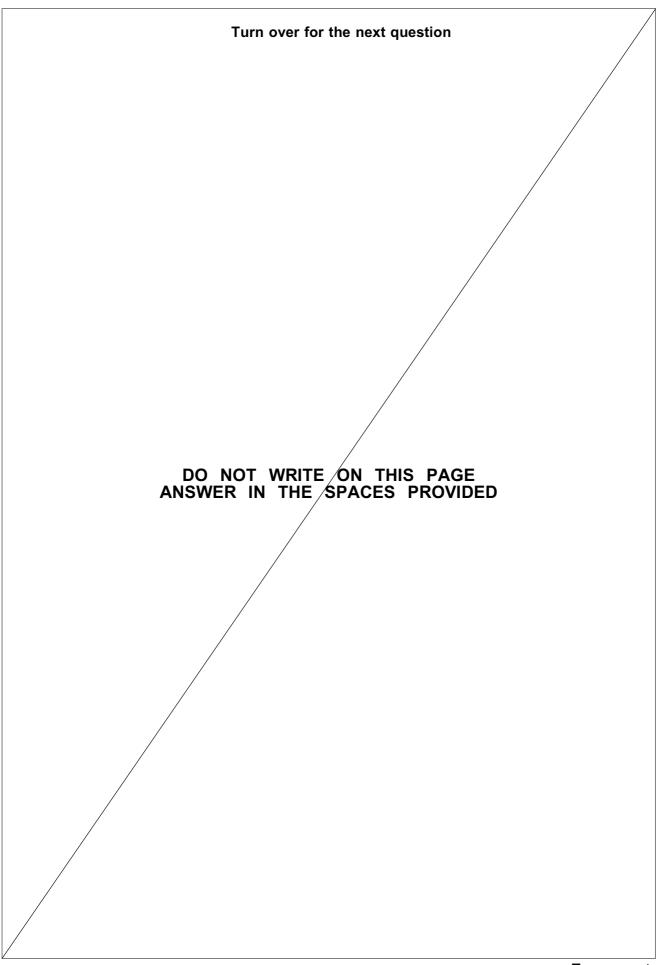


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#### Section B

Answer all questions in the spaces provided.

Use Coffee shop on page 3 of the Data Sheet.

The number of coffees, S, sold per day, 10t days after the shop was opened, may be modelled by the function

$$S = 10t^3 - 60t^2 + 110t + 10$$

for values of t from 0 to 3.3; that is, for the first 33 days that the shop was open.

- (a) Use this model and calculus to answer the following questions.
  - (i) Find t when S has a minimum turning point.

(6 marks)

(ii) Find this minimum value.

(2 marks)

(iii) Find  $\frac{d^2S}{dt^2}$ .

(2 marks)

- (iv) Use your answer to part (a)(iii) to confirm that the value found in part (a)(ii) is a minimum value. (2 marks)
- **(b)** The mean number of coffees sold per day is given by

$$\overline{S} = \frac{1}{3} \int_0^3 (10t^3 - 60t^2 + 110t + 10) dt$$

- (i) Use the trapezium rule with three strips to find an estimate for the mean number of coffees sold per day. (5 marks)
- (ii) How would you make the answer obtained from the trapezium rule more accurate?

  (1 mark)
- (iii) Use integration to evaluate

$$\frac{1}{3} \int_{0}^{3} (10t^{3} - 60t^{2} + 110t + 10) dt$$

to find the mean number of coffees sold per day.

(4 marks)

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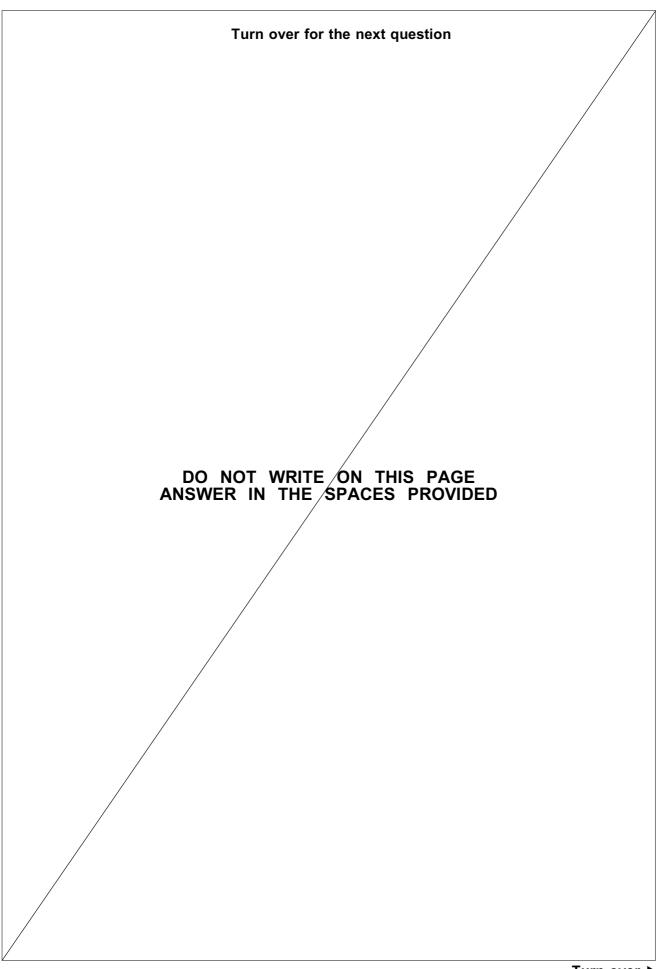


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# Section C

Answer all questions in the spaces provided.

Use Height of a sunflower on page 3 of the Data Sheet.

After t days, the height of the sunflower, h centimetres, satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = k(150 - h)$$

where k is a constant.

(a) (i) Find the general solution for h of this differential equation.

Give your answer in terms of k.

(5 marks)

(ii) Given that, when t = 0, the height of the sunflower was zero, show that  $h = 150(1 - e^{-kt})$ .

(2 marks)

(iii) When t = 30, h = 100. Find k.

(3 marks)

(b) Find the value of t when the height is 120 centimetres.

(3 marks)

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4		The height of another sunflower, $h$ centimetres, after $t$ days of growth is given by
		$h = 150(1 - e^{-0.04t})$
(a	)	Find $h$ , to five decimal places, when:
	(i)	t = 125;
	(ii)	t = 125.1. (2 marks)
(b)		Using your answers to part (a), find an estimate for the rate of increase in height when $t=125$ . (3 marks)
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### **Section D**

Answer all questions in the spaces provided.

Use Experiment with an elastic string on page 4 of the Data Sheet.

The distance, x centimetres, of the weight below O may be modelled by the function

$$x = 15 + 3\cos\frac{\pi}{2}t$$

where t is the number of seconds after the system was set into motion.

- (a) When t = 2, show that x = 12. (1 mark)
- (b) Find an expression for the velocity of the weight,  $\frac{dx}{dt}$ . (2 marks)
- (c) (i) Find the maximum value of  $\frac{dx}{dt}$ .

You may leave your answer as a multiple of  $\pi$  or as a decimal to three significant figures. (2 marks)

(ii) Find two values of t when this occurs. (2 marks)

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