General Certificate of Education
June 2008
Advanced Subsidiary Examination

APPLYING MATHEMATICS
Paper 1

UOM4/1

Friday 16 May 2008 9.00 am to 10.00 am

For this paper you must have:
- an 8-page answer book
- a clean copy of the Data Sheet (enclosed)
- a graphics calculator
- a ruler.

Time allowed: 1 hour

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is UOM4/1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

Information
- The maximum mark for this paper is 30.
- The marks for questions are shown in brackets.

Answer **all** questions.

Use **Population growth** *on the Data Sheet.*

1 A sum of money is invested and receives compound interest. The growth can be expressed by the recurrence relation $P_{n+1} = 1.025P_n$, where P_n is the amount of money at the end of the n^{th} year, and P_{n+1} is the amount of money one year later.

(a) (i) State the interest rate as a percentage in this case. *(1 mark)*

(ii) Make a copy of the table below.

n, year number	P_n, amount at end of n^{th} year
0	£200.00
1	£205.00
2	
3	
4	
5	

Complete your table using the recurrence relation $P_{n+1} = 1.025P_n$ to show how much an initial investment of £200 is worth at the end of each successive year for the first 5 years.

You should give values to the nearest penny, requiring you to work with values correct to at least three decimal places. *(2 marks)*

(b) This growth can also be modelled by the exponential function $P = P_0 e^{kt}$, where P is the value of the investment after t years, P_0 is the initial investment and k is a constant.

(i) Sketch a graph of P plotted against t that shows clearly all significant features. *(2 marks)*

(ii) Find k, using the method of considering growth in the first year, as suggested in the article. *(3 marks)*

2 The article explains that for a population that doubles every 25 years, '$2P_0 = P_0 \times e^{25k}$ leading to $k = 0.0277$'.

Show clearly how this value of k can be obtained. *(3 marks)*

3 Scotland currently has a population growth rate of 0.4%.

(a) Use the 'rule of 70' to find how long it will take the population to double in size. *(2 marks)*

(b) Suggest a reason why, in reality, this is unlikely to be the case. *(1 mark)*

4 The data in **Figure 2** show that the population of the world approximately doubled between 1960 and 2000. The 'rule of 70' gives the annual percentage growth rate over this period as approximately 1.75%.

(a) Use this growth rate together with $(1 + r) = e^k$ to confirm that the value of k in the exponential model is given by $k = 0.0173$. *(3 marks)*

(b) (i) The exponential model

$$P = 3e^{0.0173(t-1960)}$$

predicts the population in the calendar year t.

Find the population this predicts for the year 2050. *(2 marks)*

(ii) Explain why this value is not the same as the value suggested in the table of **Figure 2** in the article. *(2 marks)*

5 Find a linear model for the population, P billions of people, of the world in year t, assuming that the population of the world was 3 billion in 1960 and 6 billion in 2000. *(4 marks)*

6 Use the recurrence relation

$$P_{n+1} = P_n + 0.035P_n\left(1 - \frac{P_n}{9.8}\right)$$

and a value of the world's population $P_0 = 3$ (billion) in 1960 to find the population of the world in 1961. *(3 marks)*

7 (a) State the limiting value of the world's population according to the logistic model

$$P_{n+1} = P_n + 0.035P_n\left(1 - \frac{P_n}{9.8}\right)$$ *(1 mark)*

(b) Explain, making reference to the logistic model, why you chose this value. *(1 mark)*

END OF QUESTIONS

There are no questions printed on this page