General Certificate of Education
June 2005
Advanced Subsidiary Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

# APPLYING MATHEMATICS
# Paper 1

**UOM4/1**

Monday 23 May 2005 Morning Session

**In addition to this paper you will require:**
- a clean copy of the Data Sheet (enclosed);
- an 8-page answer book;
- a ruler;
- a graphics calculator.

Time allowed: 1 hour

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is UOM4/1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of a calculator should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

**Information**
- The maximum mark for this paper is 30.
- Mark allocations are shown in brackets.
- You will be awarded up to 3 marks for your ability to present information accurately using correct notation **and** up to 3 marks for mathematical arguments presented clearly and logically.

Answer **all** questions.

*Use* **Keep taking the tablets** *on the Data Sheet*.

**1** The amount, $C$ milligrams, of paracetamol in a person's body, $t$ hours after taking a single 1000 milligram dose, may be modelled using

$$C = 1000e^{-0.3466t}$$

(a) Find, using this model, the amount of paracetamol in a person's body after 8 hours. *(2 marks)*

(b) How long after taking the paracetamol will it be when the person has 800 milligrams of paracetamol in his body? Show all your working. *(3 marks)*

**2** The model for the decay of paracetamol in a person's body, $C = 1000e^{-0.3466t}$, assumes a half-life of 2 hours.

State **two** factors that will affect this half-life. *(2 marks)*

**3** In deriving $k$ for the model $C = 1000e^{-kt}$, the article substitutes values into the function to give $k = \frac{1}{2}\ln 2$.

Show clearly how this result has been found. *(3 marks)*

**4** You can use the recurrence relation

$$C_{n+1} = 0.25C_n + 1000 \text{ , where } C_0 = 1000$$

to find the peak amount of paracetamol in a person's body at the end of each four-hour period, just after they have taken a 1000 mg dose of the drug.

Explain the significance of the coefficient 0.25 in this expression. *(2 marks)*

**5** Use the recurrence relation $C_{n+1} = 0.25C_n + 1000$, where $C_0 = 1000$, to show that after 12 hours the peak amount of paracetamol in a person's body is 1328 mg, to the nearest milligram. Show clearly your calculations at each stage. *(3 marks)*

**6** A person takes an initial dose of 1000 mg and then a dose equivalent to 1000 mg every 4 hours. The recurrence relation

$$C_{n+1} = (0.5)^{\frac{T}{2}} C_n + 250T, \text{ where } C_0 = 1000$$

can then be used to find the peak amount of drug in the person's body every $T$ hours.

(a) Write down, and simplify, the recurrence relation when $T = 8$. *(2 marks)*

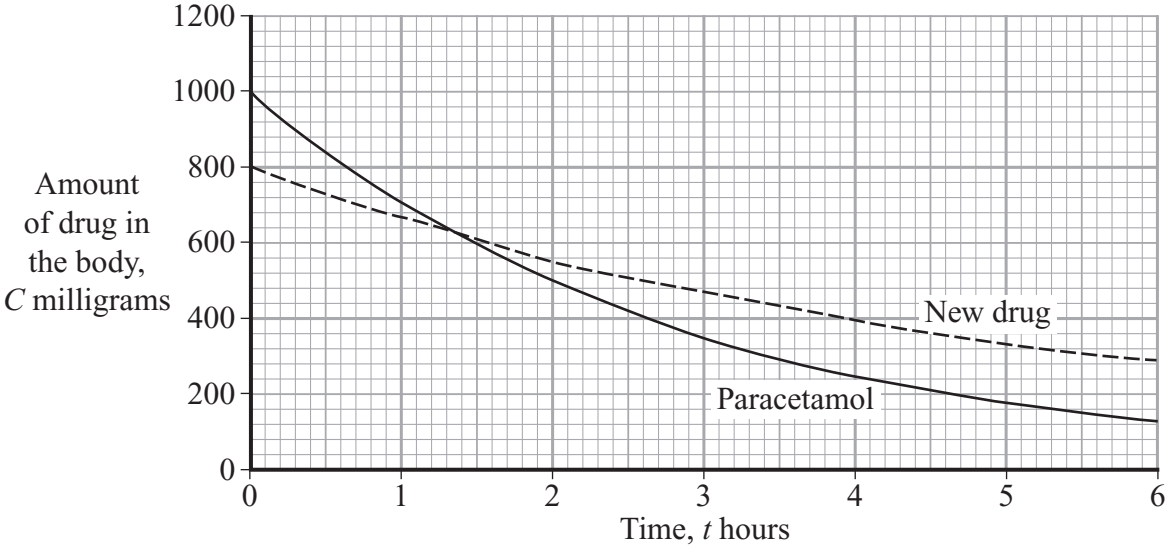
(b) Use your recurrence relation to show that, after 16 hours, this person will have a peak of 2129 mg of paracetamol in their body. *(2 marks)*

**7** The final paragraph of the article, referring to the recurrence relation $C_{n+1} = (0.5)^{\frac{T}{2}} C_n + 250T$, where $C_0 = 1000$, states "You can see from this equation that if you were to leave long intervals between taking doses of the drug, this would mean that the amount of drug left in your body from the previous dose would be small and the dose you would be taking would be large".

Explain how you can deduce this from the recurrence relation. *(3 marks)*

**8** The diagram below shows **Figure 2** with an additional curve. This curve represents the decay of a new drug.

Describe how the initial dose and half-life of the new drug compare to those of paracetamol. *(2 marks)*




**END OF QUESTIONS**

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**APPLYING MATHEMATICS** **UOM4/1PM**
**Paper 1**

**PRELIMINARY MATERIAL**

**DATA SHEET**

**To be issued to candidates between Monday 9 May and Monday 16 May 2005**

**REMINDER TO CANDIDATES**

YOU MUST **NOT** BRING THIS DATA SHEET
WITH YOU WHEN YOU SIT THE EXAMINATION.
A CLEAN COPY WILL BE MADE AVAILABLE.

**Keep taking the tablets**

When you have a headache, you may take tablets containing drugs such as paracetamol, aspirin or ibuprofen to relieve the pain. You are usually advised to take an initial dose of one or two tablets and then repeat this once every four hours. How, then, does the total amount of the drug in your body vary with time?

In the period immediately after you take any dose of tablets, the drug is absorbed into your body, quickly reaching maximum concentration. The amount of the drug then reduces exponentially (see the sketch in **Figure 1**). The rate of decay depends on many factors: for example, your age, sex, size (height and weight), metabolism and so on.

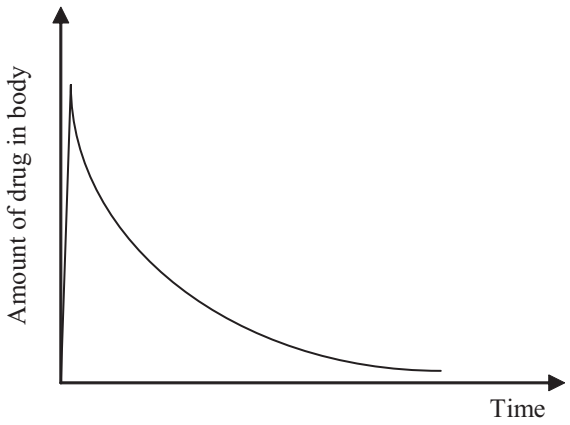


**Figure 1:** Sketch graph showing how the amount of a single dose of a drug in your body varies with time.

For a typical person, the half-life of paracetamol is about two hours. This means that every two hours the amount of paracetamol in their body will be halved. Assume that this typical person takes two tablets of paracetamol, each of which contains 500 milligrams of the drug, and this is immediately absorbed into their body. The amount, $C$ milligrams, of paracetamol in the person's body after $t$ hours is therefore given by a function of the form $C = 1000e^{-kt}$.

You can find the value of $k$ using information about the half-life of paracetamol, giving $k = \frac{1}{2}\ln 2 = 0.3466$

**Figure 2** shows a graph of the function $C = 1000e^{-kt}$ for $0 \leq t \leq 6$, from which you can see that after one half-life (two hours) 500 milligrams of paracetamol remain in this person's body; after two half-lives, this reduces to 250 milligrams; and after a further half-life, this reduces to 125 milligrams.

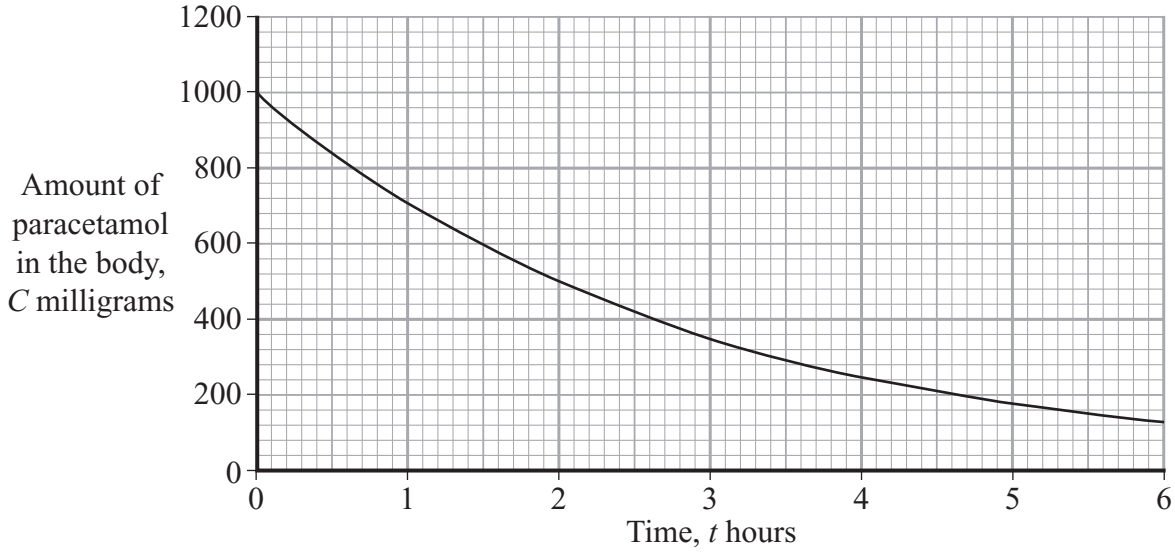


**Figure 2:** Exponential decay of a dose of paracetamol in a typical person's body.

This does not, however, take account of a person's taking further doses of the drug. For painkillers such as paracetamol, it is likely that you will be advised to take the same dose every four hours. Of course, when you take your second dose, there still remains some of the original dose in your body; this might suggest that, over time, the amount will continue to grow. The graph in **Figure 3** shows a simplified model of how the amount of paracetamol in a typical person's body will vary with time over the first 12 hours after they have taken a first dose of 1000 mg followed by further 1000 mg doses every four hours.

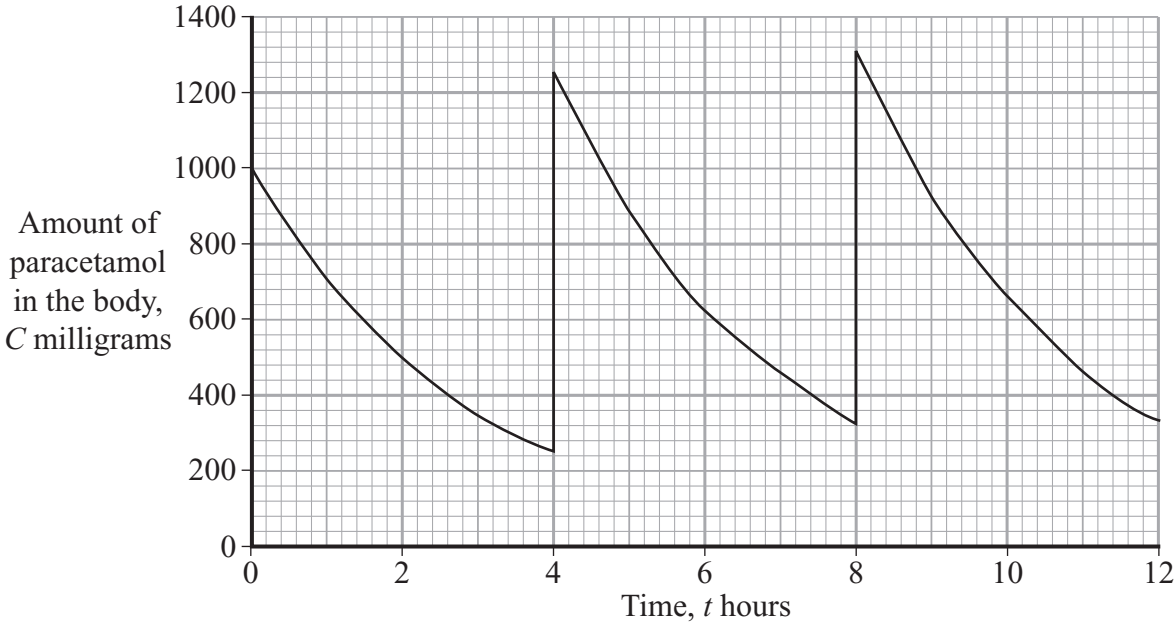


**Figure 3:** Amount of paracetamol in a typical person's body when taking successive doses of 1000 mg every four hours.

As you can see, according to this model, the amount of drug in this person's body reaches a local peak every four hours. A recurrence relation will allow you to investigate how these peaks vary with time. The relation

$$C_{n+1} = 0.25C_n + 1000 \text{ , where } C_0 = 1000$$

gives the peak amount of paracetamol in a person's body at the end of each four-hour period, just after they have taken a 1000 mg dose. The table and graph in **Figure 4** show these peak amounts in someone's body over a period of 24 hours.

| ***n*** | **No. of hours, *t*** | $C_n$ |
|---|---|---|
| 0 | 0 | 1000 |
| 1 | 4 | 1250 |
| 2 | 8 | 1313 |
| 3 | 12 | 1328 |
| 4 | 16 | 1332 |
| 5 | 20 | 1333 |
| 6 | 24 | 1333 |

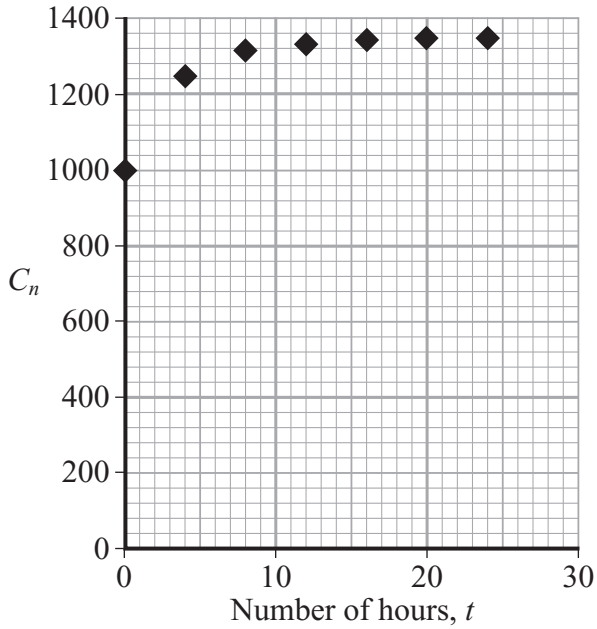


**Figure 4:** Table and graph showing the peak amounts of paracetamol in a person's body when a 1000 mg dose is taken initially, followed by further 1000 mg doses every four hours.

A question that you might ask is, "What would happen if, after taking a first dose of 1000 mg, the person were to take twice the dose but only half as frequently, that is 2000 mg every eight hours?"

| ***n*** | **No. of hours, *t*** | $C_n$ |
|---|---|---|
| 0 | 0 | 1000 |
| 1 | 8 | 2063 |
| 2 | 16 | 2129 |
| 3 | 24 | 2133 |
| 4 | 32 | 2133 |
| 5 | 40 | 2133 |
| 6 | 48 | 2133 |

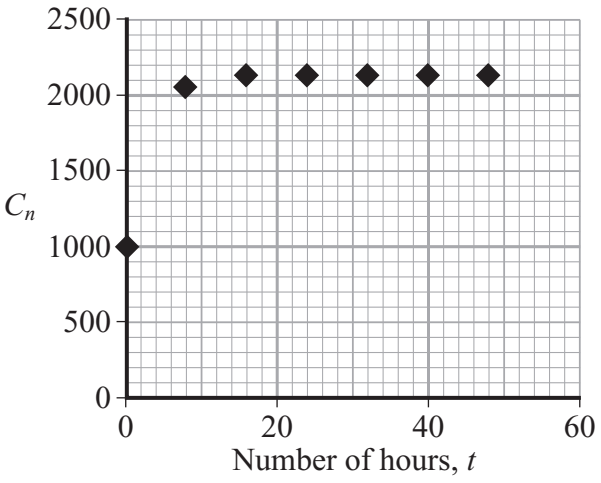


**Figure 5:** Table and graph showing the peak amounts of paracetamol in a person's body when an initial 1000 mg dose is taken, followed by 2000 mg doses every eight hours.

As you can see from the table and graph in **Figure 5**, the peak amount of the drug in this person's body becomes steady at a higher value of 2133 mg compared with the steady value of 1333 mg reached by a person taking 1000 mg every four hours.

For the general case of a person who takes

- an initial dose of 1000 mg and
- subsequent doses every $T$ hours equivalent to 1000 mg every 4 hours,

the peak amount of drug, $C_n$ mg, in his or her body every $T$ hours can be found using the recurrence relation

$$C_{n+1} = (0.5)^{\frac{T}{2}} C_n + 250T, \text{ where } C_0 = 1000$$

You can see from this equation that if you were to leave long intervals between taking doses of the drug, this would mean that the amount of drug left in your body from the previous dose would be small and the dose you would be taking would be large. Of course, taking a large dose of any drug could be very dangerous. The implication of this is clear: you should follow the instructions given to you by your doctor or enclosed with any medication.

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