

A-LEVEL STATISTICS

Statistics 5 – SS05

Mark scheme

6380
June 2014

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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| Q | Solution | Marks | Total | Comments |
|---|--|---|-------|--|
| 1 | $s = 38.2$ or $s^2 = 1460$ $H_0: \sigma = 30$ $H_1: \sigma \neq 30$ $\frac{(n-1)s^2}{\sigma^2} = \frac{11 \times 38.2^2}{30^2}$ $= \frac{16065.66}{30^2}$ $= 17.85$ cv's $\chi^2_{11} = 3.816, 21.92$ $3.816 < 17.8 < 21.92$ accept H_0 <u>Insufficient evidence</u> at the 5% level to doubt that the <u>standard deviation</u> of <u>volumes of milk</u> delivered by the machine <u>is 30ml</u> . | B1 B1 M1 A1 B1 B1,B1 A1 | 8 | awfw 38.2~38.3 or 1450 ~ 1460 accept $\sum(x - \bar{x})^2 = 16100$ (3sf) Both hypotheses o.e.; must use σ M1 “their” $\frac{11s^2}{\sigma^2}$. awfw 17.8 ~17.9 B1 for 11 df; p value 0.170 ~ 0.175 B1 for each χ^2 value; awfw 3.81~3.82 and 21.9~22.0 All correct and statement in context. Comparison with <u>both</u> ends. |

| Q | Solution | Marks | Total | Comments |
|--|---|---|-------|---|
| 2(a) | $H_0: \mu_{females} - \mu_{males} = 1.5cm$ $H_1: \mu_{females} - \mu_{males} > 1.5cm$ | B1 B1 | 2 | B1 : an inequality <u>and</u> 1.5 B1 : both correct |
| (b) | $t.s. = \frac{(8.54 - 6.28 - 1.5)}{\sqrt{\left(\frac{0.6^2}{10} + \frac{0.6^2}{8}\right)}}$ $= 2.67$ $z = 2.3263$ $2.67 > 2.3263 - \text{reject } H_0$ Sufficient evidence at 1% level to suggest that the <u>mean length of female toads</u> is more than 1.5cm greater than the <u>mean length of male toads</u> . | M1 M1 A1 B1 A1 E1dep | 6 | numerator , allow “8.54 – 6.28” seen denominator awfw 2.67~2.675 Ignore sign , p - value : 0.00378 ~ 0.00379 comparison of correct cv with correct t.s; both + ve or both - ve. Statement in context – dependent on previous A1. |
| | | | 8 | |
| Notes: Hypotheses: must use μ or population mean. Allow f and m as suffices but other suffices only if clearly assigned. | | | | |

| Q | Solution | Marks | Total | Comments |
|-----------------|---|-------|----------|---|
| 3(a) (i) | mean = 0 | B1 | 3 | cao |
| | $s.d = \sqrt{\frac{(0.5-(0.5))^2}{12}}$ | M1 | | or $\sqrt{\frac{1}{12}}$; must have square root |
| | $= \frac{1}{\sqrt{12}} = 0.289$ | A1 | | $\frac{1}{\sqrt{12}}$ or awfw 0.288 ~0.289 |
| | | | | s.c. B1 for $\frac{1}{12}$ identified as variance |
| (ii) | 1 – (0.6) or 0.2 + 0.2 | M1 | 2 | B1 for 0.2 or 0.6 seen |
| | = 0.4 | A1 | | |
| (b) | Error more than 0.3 | M1 | 2 | Must be using rectangular distribution. |
| | $P(X > +0.3) = 0.2$ | A1 | | |
| | | | 7 | |

| Q | Solution | Marks | Total | Comments |
|-----|--|--|-------|--|
| 4 | $\bar{x}=511.5$ | B1 | 7 | cao |
| | $s = 11.7, s^2 = 137$ or $\sum(x - \bar{x})^2 = 1230$ | B1 | | awfw 11.69~11.71, 136~ 137 or 1230~1231 |
| | $v = 9$ | B1 | | B1 for 9 df |
| | $t_9 = 2.262$ | B1 | | |
| | CI: $511.5 \pm 2.262 \times \frac{11.7}{\sqrt{10}}$ | M1 | | M1 : Use of their $\frac{s}{\sqrt{10}}$ |
| | | m1 | | m1 : method for interval ; allow incorrect t ₉ value |
| | [503,520] or 511.5 ± 8.36 | A1 | | awfw (503.0~503.2, 519.8 ~520 or $511.5 \pm 8.362 \sim 8.365$) |
| | (b) $\chi_9^2 = 3.325, 16.919$ | B1 | | either |
| | CI limits for variance: $\frac{9 \times 11.7^2}{16.919} \quad \frac{9 \times 11.7^2}{3.325}$ | M1 m1 | | M1: method for interval – condone one small slip eg $10s^2$ m1: correct expression - condone use of 4.168 and 14.584 for (incorrect) χ_9^2 values |
| | CI for sd: [8.53,19.2] | m1 A1 | | m1: method for sd A1both : awfw 8.52~8.54,19.2~19.4 |
| (c) | There is no reason to doubt that Ahmed is filling his punnets with a mean of 515 grams of strawberries as 515grams is within Ahmed’s confidence interval | B1 ft | 5 | B1ft: comment about “their” CI in (a) |
| | | E1dep | | E1 dep comment in context dep. A1 in (a) accept eg: “Ahmed is filling his punnets with a sufficient <u>mean</u> weight of strawberries” or “Ahmed’s <u>mean</u> packed weight is OK/good/suitable” |
| | E1dep | E1dep : below CI dep A1 in (b) | | |
| | E1dep | E1dep : comment on CI in (b) in context dep A1 in (b) ; accept eg Ahmed’s packing is too <u>variable</u> in weight or “Ahmed’s punnets need to be more <u>consistent</u> in weight ” | | |
| | | | 4 | |
| | | | 16 | |

| Q | Solution | Marks | Total | Comments |
|------|--|--------|---|---|
| 5(a) | $s_k = 0.680$ $s_a = 0.646$ $(s_k^2 = 0.462$ $s_a^2 = 0.418)$ | B1 | 7 | awfw 0.679 ~ 0.680, 0.646 ~ 0.647 (0.462 ~ 0.463, 0.417 ~ 0.418) |
| | $H_0: \sigma_k^2 = \sigma_a^2$ $H_1: \sigma_k^2 \neq \sigma_a^2$ | B1 | | Both hypotheses. Use of other suffices must be clearly defined. |
| | t.s. $F = \frac{0.680^2}{0.646^2} = 1.107$ | M1 | M1 : using their s^2 | |
| | df 6,8 c.v. = 4.652 | A1 | A1 : awfw 1.10 ~ 1.11 | |
| | Accept H_0 , there is no significant evidence that the samples come from populations with different variances. | B1, B1 | p- value 0.868 ~ 0.869 | |
| | | E1dep | E1: conclusion in context, dependent on A1 for t.s. and B1 for cv | |
| (b) | $\bar{x}_k = 9.77$ $\bar{x}_a = 9.14$ | B1 | Both means awfw (9.77 ~ 9.78, 9.14 ~ 9.15) | |
| | $H_0: \mu_k = \mu_a$ $H_1: \mu_k > \mu_a$ | B1 | Both hypotheses. Use of other suffices must be clearly defined | |
| | Pooled variance: $s_p^2 = \frac{6 \times 0.680^2 + 8 \times 0.646^2}{7 + 9 - 2} = 0.437$ | M1A1 | awfw 0.436 ~ 0.437 ($s_p = 0.660 \sim 0.661$) | |
| | $t = \frac{9.77 - 9.14}{\sqrt{0.437(\frac{1}{7} + \frac{1}{9})}} = 1.88$ | M1 | M1 (numerator) (accept 9.14 - 9.77) | |
| | cv $t_{14} = 1.345$ | m1 | m1 (denominator – ft on their S_p^2 but must have 1/7 and 1/9.) | |
| | 1.88 > 1.345 reject H_0 | A1 | A1 : awfw 1.88 ~ 1.89, ignore sign | |
| | | B1 | ignore sign p-value : 0.0403 ~ 0.0404 | |
| | | A1dep | A1dep; comparison and conclusion; dependent on A1 for t.s. and B1 for cv. (signs must be consistent with the alternative hypothesis.) | |
| | evidence at 10% level that Kanwar's car travels, on average , more miles to the litre than Ashok's car | E1dep | E1 conclusion in context; dependent on previous A1 | |
| (c) | Not reasonable: miles travelled per litre of petrol does not depend on distance travelled. | E1 | | |
| | | | 10 | |
| | | | 1 | |
| | | | 18 | |

| Q | Solution | Marks | Total | Comments | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---------|---|--------------|-----------|--|--------|----------------|---------------|-----|-----|--------|--------|----------|--------|-----|----|--------|-------|----------|--------|-----|----|--------|-------|---------|--------|------|----|--------|-------|----------|--------|------|----|--------|-------|---------|--------|------|----|--------|-------|---------|--------|-----|---|--------|------|--------|--------|--------------|--|--|--|----------------|--|----|----|--|
| 6(a)(i) | $P(X > 2) = 1 - 0.6767$ | B1 | 3 | B1: $\lambda = 2$ used | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $= 0.323$ | M1 | | M1: $1 - 0.6767$ (allow $1 - 0.4060$) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (ii) | Emails must be : independent occur at a constant rate occur at random | A1 | 2 | awfw (0.323~0.324) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | B1 | | B1: Any 2 correct assumptions | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (b) | H_0 : data can be modelled by an exponential distribution with parameter $\lambda = \frac{2}{15}$ H_1 : data cannot be modelled by an exponential distribution with parameter $\lambda = \frac{2}{15}$ | E1 | | E1: assumptions in context | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | B1 | | B1 both hypotheses (accept mean = 7.5) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1"> <thead> <tr> <th>x</th> <th>O</th> <th>P(x)</th> <th>E</th> <th>$(O - E)^2$</th> <th>$(O - E)^2/E$</th> </tr> </thead> <tbody> <tr> <td>0 -</td> <td>124</td> <td>0.3297</td> <td>105.50</td> <td>342.3394</td> <td>3.2450</td> </tr> <tr> <td>3 -</td> <td>82</td> <td>0.2210</td> <td>70.72</td> <td>127.3028</td> <td>1.8002</td> </tr> <tr> <td>6 -</td> <td>51</td> <td>0.1857</td> <td>59.43</td> <td>71.1355</td> <td>1.1969</td> </tr> <tr> <td>10 -</td> <td>31</td> <td>0.1283</td> <td>41.04</td> <td>100.8778</td> <td>2.4578</td> </tr> <tr> <td>15 -</td> <td>16</td> <td>0.0659</td> <td>21.07</td> <td>25.7311</td> <td>1.2211</td> </tr> <tr> <td>20 -</td> <td>11</td> <td>0.0512</td> <td>16.37</td> <td>28.8767</td> <td>1.7636</td> </tr> <tr> <td>>30</td> <td>5</td> <td>0.0183</td> <td>5.86</td> <td>0.7413</td> <td>0.1265</td> </tr> <tr> <td colspan="3" style="text-align: right;">Total</td> <td></td> <td>11.8110</td> <td></td> </tr> </tbody> </table> | x | O | P(x) | E | $(O - E)^2$ | $(O - E)^2/E$ | 0 - | 124 | 0.3297 | 105.50 | 342.3394 | 3.2450 | 3 - | 82 | 0.2210 | 70.72 | 127.3028 | 1.8002 | 6 - | 51 | 0.1857 | 59.43 | 71.1355 | 1.1969 | 10 - | 31 | 0.1283 | 41.04 | 100.8778 | 2.4578 | 15 - | 16 | 0.0659 | 21.07 | 25.7311 | 1.2211 | 20 - | 11 | 0.0512 | 16.37 | 28.8767 | 1.7636 | >30 | 5 | 0.0183 | 5.86 | 0.7413 | 0.1265 | Total | | | | 11.8110 | | M1 | 11 | M1: method for probabilities ; at least one value correct. |
| | | x | O | P(x) | E | $(O - E)^2$ | $(O - E)^2/E$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 0 - | 124 | 0.3297 | 105.50 | 342.3394 | 3.2450 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| | | 15 - | 16 | 0.0659 | 21.07 | 25.7311 | 1.2211 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 20 - | 11 | 0.0512 | 16.37 | 28.8767 | 1.7636 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | >30 | 5 | 0.0183 | 5.86 | 0.7413 | 0.1265 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | Total | | | | 11.8110 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| m1 | m1 expected values ft “p” × 320 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| m1 | m1 combining classes “30 and over” | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| m1 | m1 attempt at $(O - E)^2$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| m1 | m1 dividing $(O - E)^2$ by E and summing – at least 2 values seen. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A1 | All correct and awfw 11.8 ~12.0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B1ft | df – ft their number of classes - 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B1 | awfw 10.6 ~ 10.7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | p-value 0.0663~ 0.0666 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | E1 conclusion in context dependent on A1 for ts and B1 for cv. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| c) | Data cannot be modelled adequately by an exponential distribution and hence the Poisson model is unlikely | E1dep | | E1 : accept either Poisson model is unlikely or data lacks fit to Exp(2/15) dependent on a χ^2 test in (b). | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | E1dep | | E1 dep on first E1 - invalid | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | The answer in a(i) is not likely to be valid. | E1dep | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 18 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |