Teacher Support Materials 2009

Statistics GCE

Paper Reference SS06

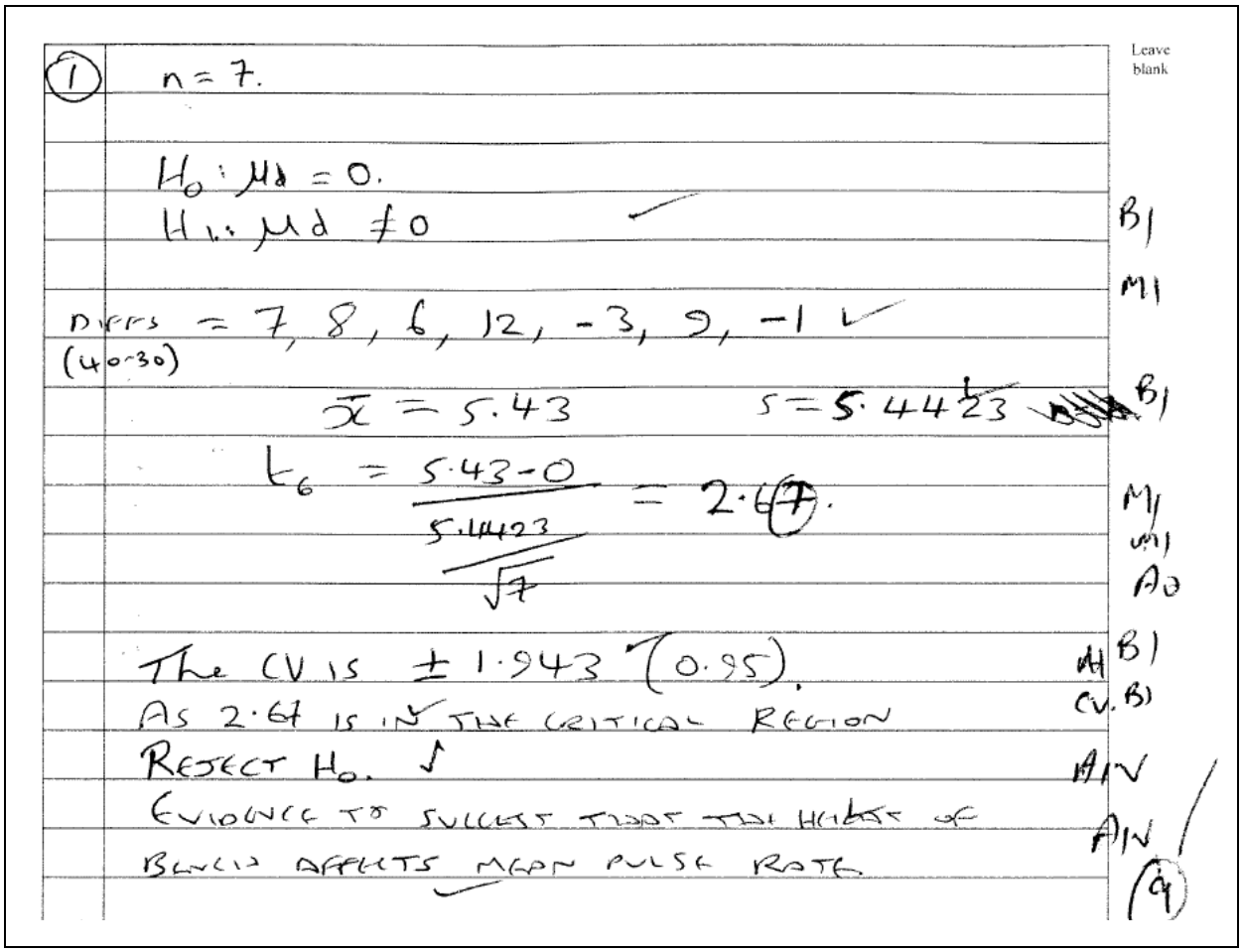
Question 1

An investigation is to be carried out into the effects of exercise on pulse rate. Part of the investigation will involve measuring the pulse rates of volunteers after they have spent three minutes stepping on and off a bench. Before proceeding, the investigator wishes to find out whether the height of the bench is a relevant factor. The pulse rates, in beats per minute, of 7 volunteers after they have stepped on and off benches of heights 30 cm and 40 cm are recorded in the table.

		Height of bench	
		30 cm	**40 cm**
Volunteer	**Waheed**	124	131
	Sonny	118	126
	Debbie	121	127
	Marian	124	136
	Dimitri	137	134
	Sajid	129	138
	Maha	142	141

Carry out a paired *t*-test to examine whether the height of the bench affects the mean pulse rate. Use the 10% significance level. *(10 marks)*

Student Response



Commentary

This candidate has correctly answered the question, but has lost a mark due to a slightly inaccurate test statistic. This might have been due to rounding the mean to three significant figures before using it in a further calculation. However, this is not so in this particular case and it appears to be due to miscopying a figure from the calculator.

Mark scheme

Q	Solution	Marks	Total	Comments
1	W So De Mar Di Sa Mah d 7 8 6 12 −3 9 −1	M1		M1 method for differences
	$H_0: \mu_d = 0$ $H_1: \mu_d \neq 0$	B1		B1 both hypotheses — needs μ or 'population'
	$\bar{x}_d = 5.4286$ $s_d = 5.4423$	B1		B1 5.43 (5.42~5.43) and 5.44 (5.44~5.45)
	$t = \dfrac{5.4286}{5.4423/\sqrt{7}}$	M1		M1 use of $\text{their sd}/\sqrt{7}$
		m1		m1 clearly correct method for t
	$= 2.64$	A1		A1 2.64 (2.63 ~ 2.65) or −2.64
	critical value $t_6 \pm 1.943$	B1 B1		B1 6df B1 1.943 — ignore sign
	Reject $H_0: \mu_d = 0$ — Conclude there is significant evidence of a difference in pulse rates for different bench heights (rate higher for 40cm than for 30cm)	A1√ A1√	10	A1√ correct conclusion their figures — must be compared with correct tail of t. Disallow if contradicted subsequently A1√ correct conclusion in context — allow arithmetic errors or numerically incorrect t value only. Needs previous A1√

Question 2

Each Thursday night a band plays at a jazz club. The bands which have played over the last 19 Thursdays are The Blue River Jazz Band (A), Old Orleans Heat (B), Huddersfield Hot Stompers (C) and The Detroit Teddybears (D).

The attendances at the jazz club for the different bands are summarised below.

Band			
A	**B**	**C**	**D**
43	57	32	69
54	46	59	88
38	39	44	96
58		62	72
61		63	84
72			

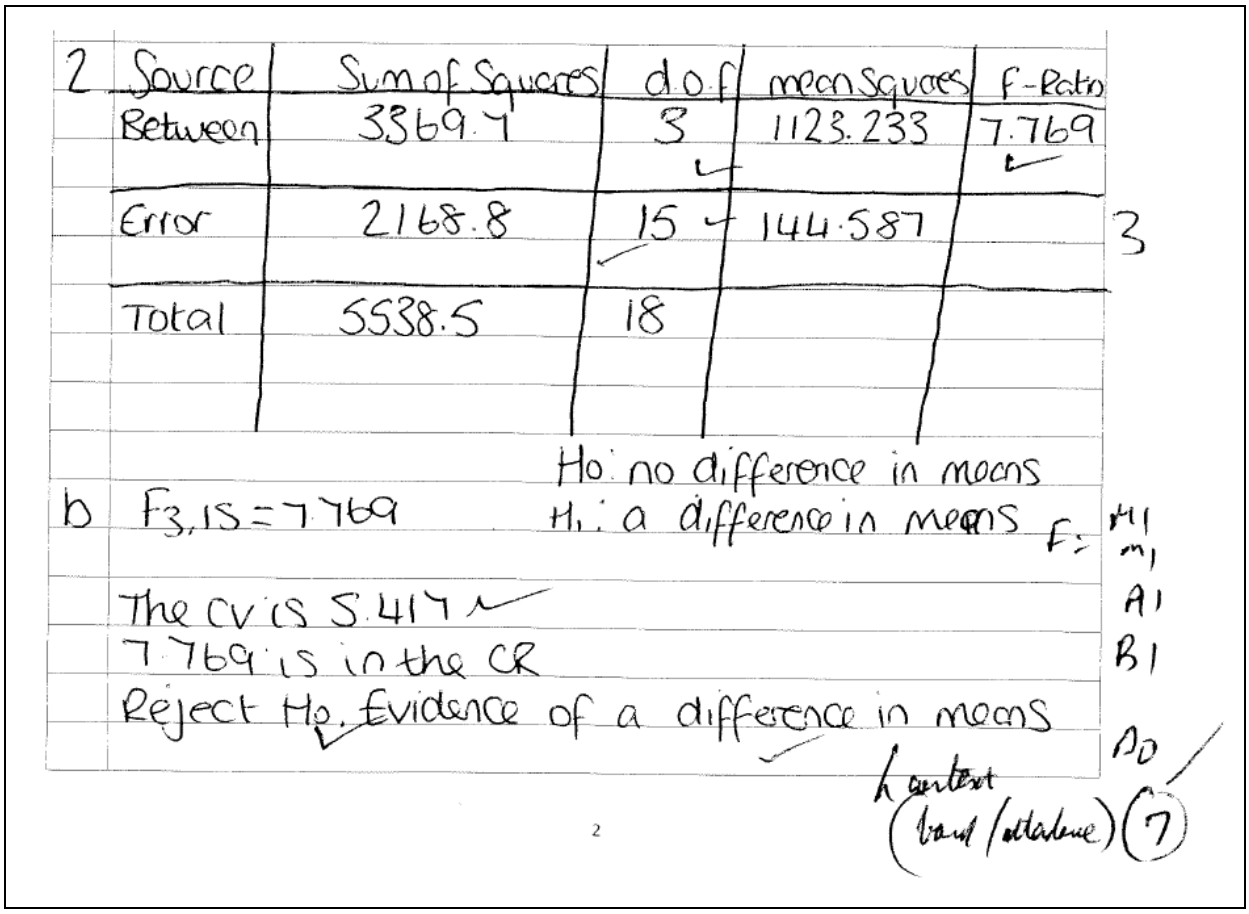
(a) Copy and complete the table below, which follows from an analysis of the data.

Source of variation	Sum of squares	Degrees of freedom
Between bands	3369.7	
Error		
Total	5538.5	

(3 marks)

(b) Hence examine whether the average attendance differs according to which band is playing. Use the 1% significance level. Assume that attendances are normally distributed with constant variance. *(5 marks)*

Student Response



2	Source	Sum of Squares	d.o.f	mean Squares	F-Ratio	
	Between	3369.7	3	1123.233	7.769	
	Error	2168.8	15	144.587		3
	Total	5538.5	18			

Ho: no difference in means

b $F_{3,15} = 7.769$ H1: a difference in means F: H1 m1

The CV is 5.417 A1

7.769 is in the CR B1

Reject Ho, evidence of a difference in means A0

& context

(band / attained) (7)

2

Commentary

The answer is correct, but to gain full marks candidates are required to state their conclusion in context.

Mark Scheme

2(a)	Source SS df MS Bands 3369.7 3 1123.23 Error 2168.8 15 144.59 Total 5538.5 18	B1 B1 M1	3	B1 any correct df B1 all correct df M1 method for error SS
(b)	H_0: No difference between bands H_1: Difference between bands $F = \frac{1123.23}{144.59} = 7.77$ Critical value $F_{3,15}$ is 5.417 Reject H_0 — significant evidence of a difference in average attendance for the different bands.	M1 m1 A1 B1 A1√	5	M1 method for both MS — their df and +ve Error SS m1 method for F, their df — needs both Ms A1 7.77 (7.76~7.78) B1 5.417 or 5.42 A1√ conclusion in context — must be correct df and compared with upper tail of F
	Total		**8**	

Question 3

A road haulage firm frequently undertakes journeys between the firm's depot and a customer's factory. The manager wonders whether it is quicker to use the direct route, D, or a route which is longer but consists mainly of motorway, M.

You are asked to design an experiment to compare these two alternative routes. Six lorries, each with its own driver, are available to you on Wednesday and Thursday, and each will make one journey to the factory each day.

(a) Copy and complete the following table indicating which route, D or M, each driver should use.

		Wednesday	**Thursday**
Driver	**1**		
	2		
	3		
	4		
	5		
	6		

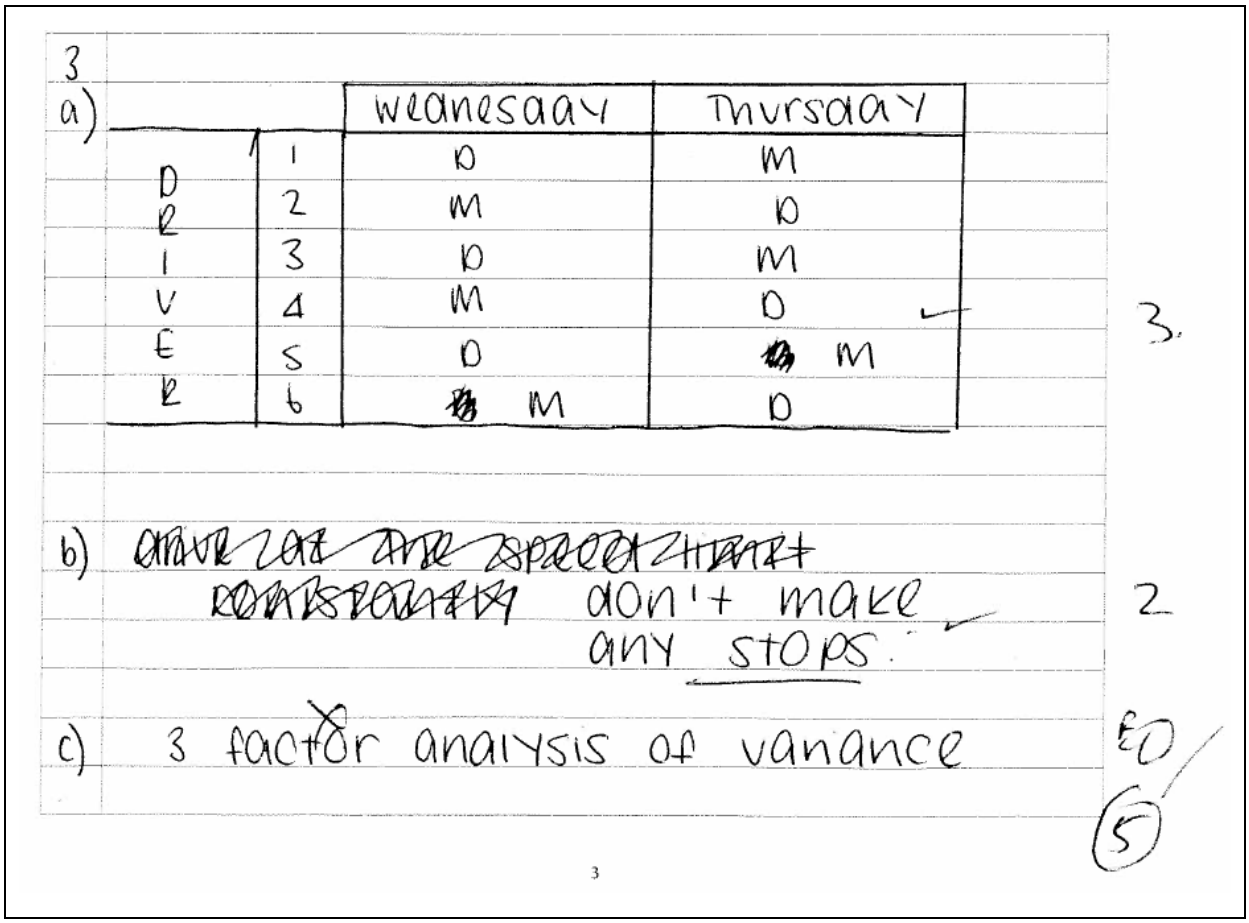
For example, if you wish driver 1 to travel by the direct route on Wednesday, put a D in the top left hand rectangle of your table. Continue until you have filled in all twelve rectangles. *(3 marks)*

(b) Each driver is to be told to start their journey at the same time on Thursday as they did on Wednesday.

Suggest a further instruction that you might give the drivers which would help to ensure a fair comparison between routes. *(2 marks)*

(c) Suggest an appropriate statistical analysis to be carried out after the journey times have been collected. *(2 marks)*

Student Response



3

a)

		Wednesday	Thursday
Driver	1	D	M
	2	M	D
	3	D	M
	4	M	D
	5	D	M
	6	M	D

3.

b) driver and speed don't make any stops.

2

c) 3 factor analysis of vanance

ED

(5)

3

Commentary

The design of experiment is fine and the instruction, although brief, is relevant and straight to the point. There are three factors involved in the experiment - route plus two blocking factors - day and driver. However, a 3-factor analysis of variance as suggested in part (c) would not be possible. A latin square would require the same number of levels of each factor and a completely crossed design (although beyond the specification) would require data for all possible combinations of the three factors.

Mark Scheme

3(a)	Wednesday Thursday 1 D M 2 M D 3 D M 4 M D 5 D M 6 M D	B1 B1 B1	3	B1 6Ms 6Ds B1 paired B1 3Ms 3Ds each day
(b)	Don't take a break/ take same number and length of breaks Drive as quickly as possible consistent with safety and speed limits etc	E2,1	2	E1 any reasonable point E1 clearly explained Disallow drive same speed
(c)	Paired *t*-test	E1 E1	2	E1 paired E1 *t*-test Allow sign test, Wilcoxon signed-rank test Disallow 2-Factor A of V, unless some explanation included
	Total		7	

Question 4

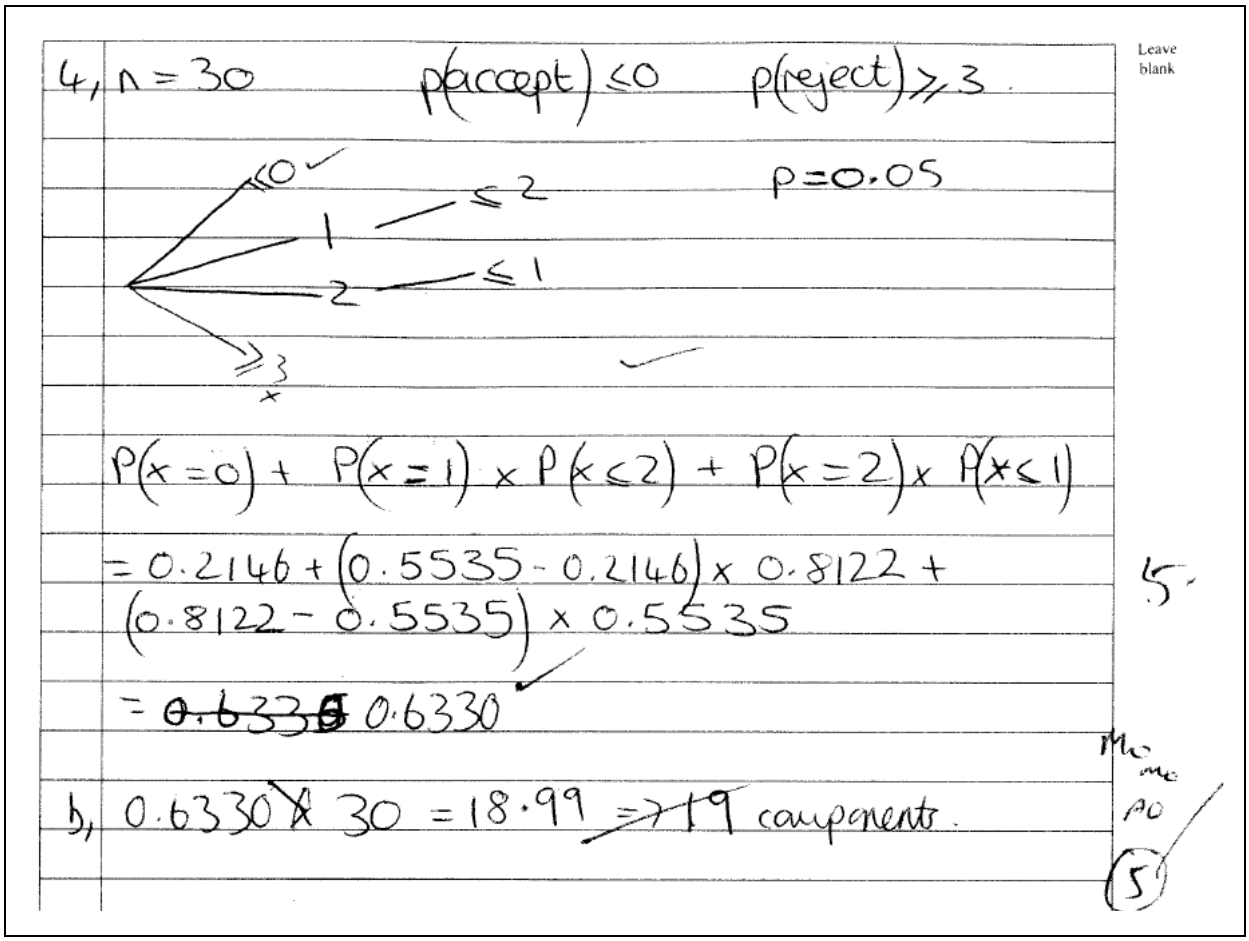
The following double sampling plan is applied to batches of components.

Test a random sample of 30 components.
Accept the batch if no non-conforming components are found.
Reject the batch if 3 or more non-conforming components are found.
Otherwise, test a further random sample of size 30 and accept the batch if a total of 3 or fewer (out of 60) non-conforming components are found.

(a) Calculate the probability of accepting a batch containing 5% non-conforming components. *(5 marks)*

(b) Calculate the expected number of components tested if a batch containing 5% non-conforming components is tested. *(3 marks)*

Student Response



Commentary

Part (a) has been answered correctly.

In part (b) the candidate has realised that the answer will involve a probability multiplied by 30 but this is insufficient to gain any marks. 30x0.6330 + 30 although still incorrect would have been sufficient to earn one mark.

Mark Scheme

Q	Solution	Marks	Total	Comments
4(a)	1st 0 1 1 1 2 2 2nd 0 1 2 0 1	M1		M1 reasonable attempt at double sampling
	P(Accept) = P(0) + P(1) × P(2 or fewer) + P(2) × P(1 or fewer)	m1		m1 method their attempt
	= 0.2146 + 0.3389 × 0.8122 + 0.2587 × 0.5535	B1 M1		B1 use of B(30, 0.05) M1 completely correct method
	= 0.633	A1	5	A1 0.633 (0.632~0.634)
(b)	E (number tested)			
	= 30 + 30 × P(1 or 2)	M1		M1 reasonable attempt at method
	= 30 + 30 × 0.5976	m1		m1 completely correct method
	= 47.9	A1	3	A1 47.9 (47.8~48)
	Total		**8**	

Question 5

[**Figure 1**, printed on the insert, is provided for use in this question.]

An importer uses an automatic machine to pack brown sugar into bags of nominal weight 1000 grams.

A supermarket chain buys large batches of these bags. When a batch is delivered, each of a random sample of 6 bags is weighed and the batch is rejected if the mean weight is less than 1001 grams. The weights may be assumed to be normally distributed with standard deviation 2.9 grams.

(a) Find the probability of accepting a batch containing bags with mean weight:

(i) 998 grams;

(ii) 1004 grams. *(4 marks)*

(b) Using your results from part (a), together with the data in the following table, draw the operating characteristic on **Figure 1**.

Mean weight, grams	999	1000	1001	1002	1003
Probability of acceptance	0.046	0.199	0.500	0.801	0.954

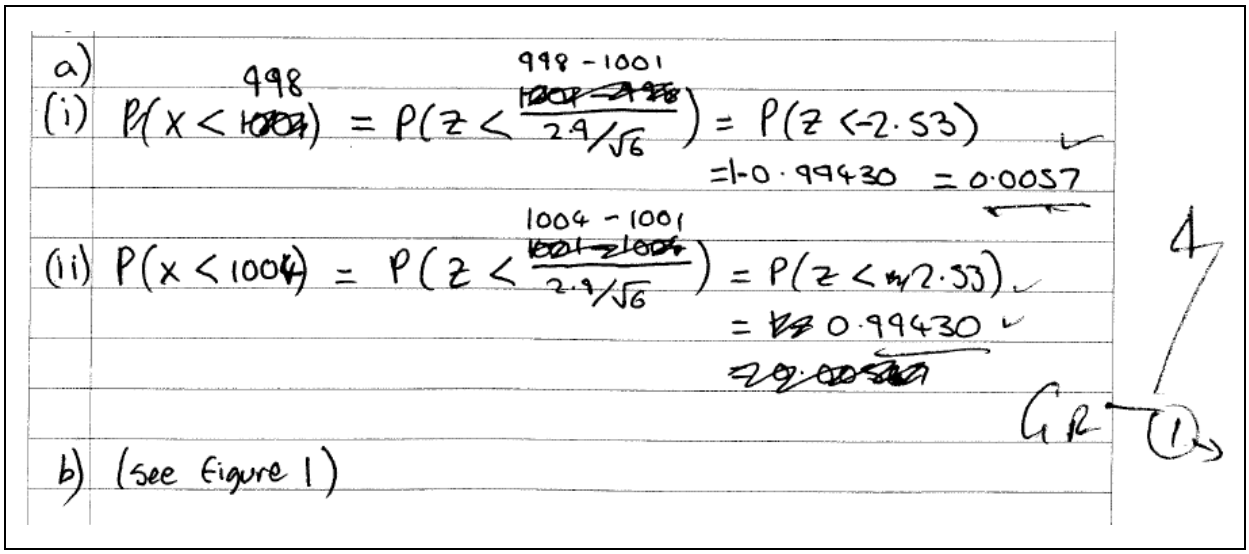
(2 marks)

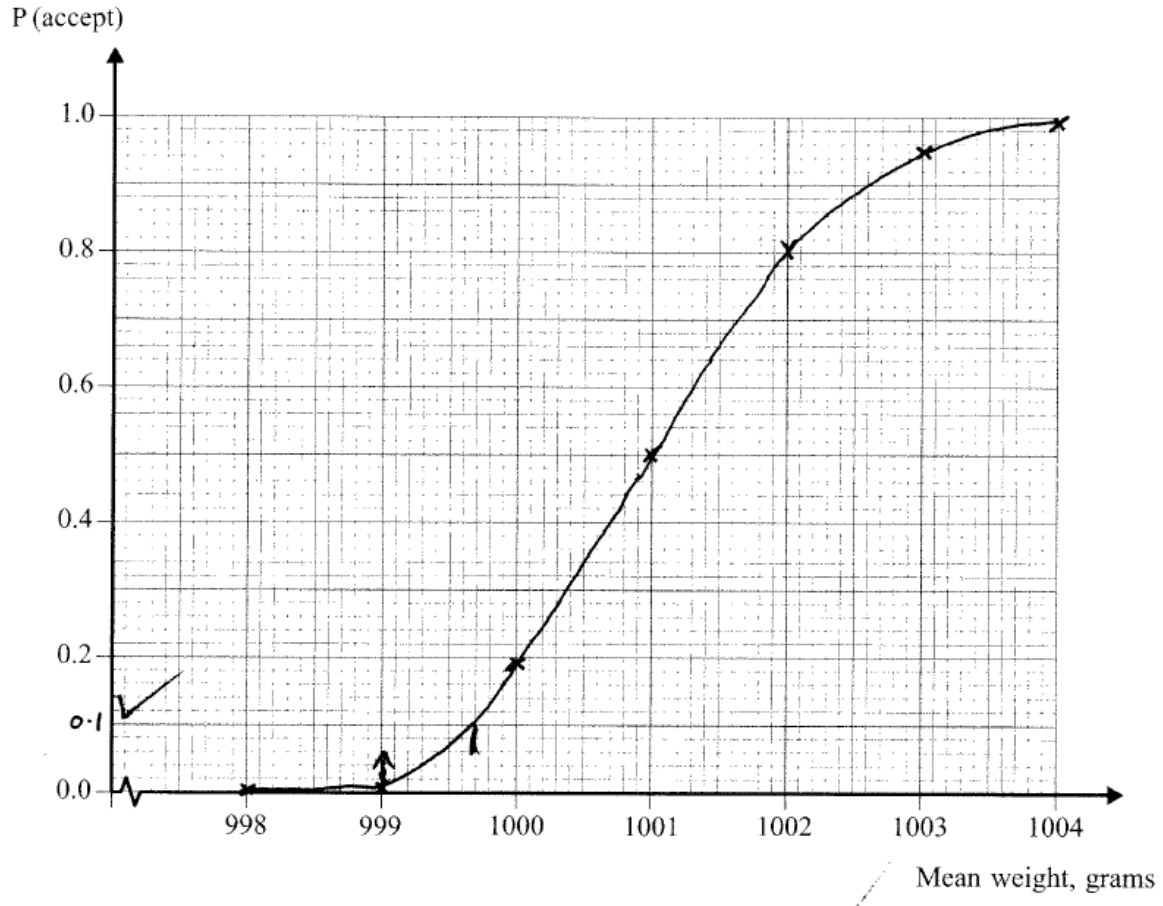
(c) Find from your operating characteristic, or otherwise, the mean weight of a batch which has a probability of 0.9 of being rejected. *(2 marks)*

(d) It is decided to continue to reject batches if the sample mean weight is less than 1001 grams, but to increase the sample size. The probability of rejecting a batch consisting of bags with mean weight 999.5 grams is to be at least 0.95.

How large a sample will be necessary in order to achieve this? *(5 marks)*

Student Response





M1 A0 (1)

c) 999·4 grams (correct point on graph) M1 A0

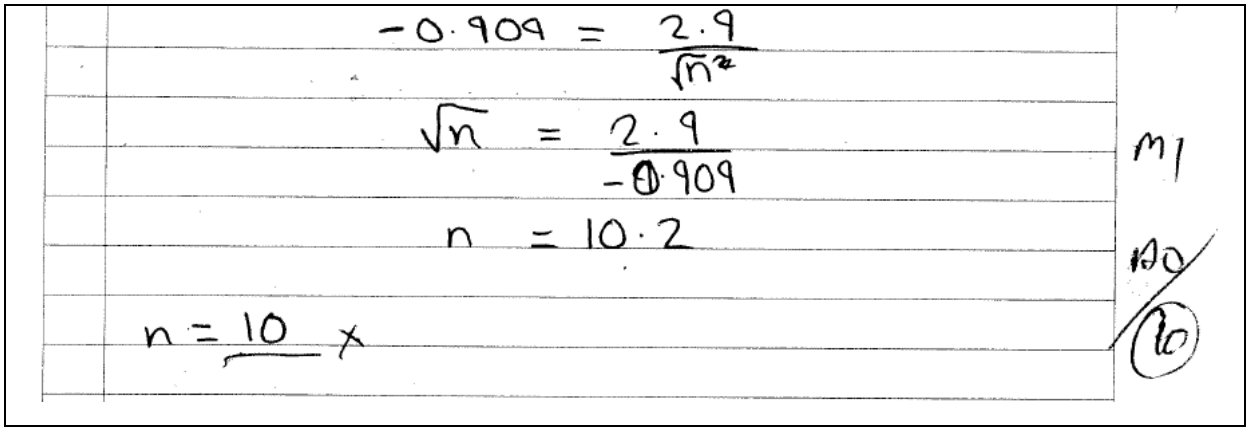
Leave blank

$\bar{x} = 999.5 \quad p = 0.95 \quad s = 2.9 \quad n = ?$

$P(x < 999.5) = P\left(z < \frac{999.5 - 1001}{2.9/\sqrt{n}}\right) = P(z < 1.65) = 0.95$ M1

$\frac{999.5 - 1001}{2.9/\sqrt{n}} = 1.65$ B1

$\frac{999.5 - 1001}{1.65} = 2.9 \div \sqrt{n}$ m1



Commentary

The method for parts (a), (b) and (c) is correct but marks have been lost due to inaccurate plotting of the points. A direct method has been almost correctly undertaken in part (d) [trial and improvement was another possible method] but the candidate has not realised that in order to achieve the objective 10.2 should be rounded up to 11.

Mark Scheme

5(a)(i)	$z = \frac{1001-998}{2.9/\sqrt{6}} = 2.534$	M1 m1		M1 use of $2.9/\sqrt{6}$
(ii)	P(accept) = 1 – 0.994 = 0.006 $z = \frac{1001-1004}{2.9/\sqrt{6}} = -2.534$ P(accept) = 0.994	 m1 A1	 4	m1 method for either z — ignore sign m1 completely correct method both probabilities — allow interchanged A1 0.006 (0.005 ~ 0.006) and 0.994 (0.994 ~ 0.995)
(b)	on insert	M1 A1	 2	M1 method for graph A1 reasonably accurate plot — by eye
(c)	999.5	M1 A1	 2	M1 method — needs M1 in (b) A1 999.5 (999.3~999.6)
(d)	$\frac{999.5-1001}{2.9/\sqrt{n}} < -1.6449$ $\sqrt{n} > 1.6449 \times \frac{2.9}{1.5}$ $n > 3.180^2$ $n = 11$	M1 B1 m1 m1 A1	 5	M1 reasonable attempt at expression (generous) B1 1.6449 (1.64 ~ 1.65) m1 correct expression — allow <,>,= m1 method for manipulation of expression A1 11 or at least 11
	Total		**13**	

Question 6

[**Figures 2 and 3**, printed on the insert, are provided for use in this question.]

A food factory produces bottles of salad cream. Samples of size 4 are taken at hourly intervals and their contents are checked. The target volume for the contents is 400 ml and a standard deviation of 2.3 ml is considered satisfactory. The volumes may be assumed to be normally distributed.

Figure 2 shows upper and lower warning (95%) and action (99.8%) limits on a chart for means.

(a) Add to **Figure 3** upper and lower warning and action limits for standard deviations. *(3 marks)*

(b) The volumes, in ml, of contents in the last seven samples are shown below.

	Sample						
	1	**2**	**3**	**4**	**5**	**6**	**7**
	399 401 401 397	393 395 396 397	398 397 400 399	401 402 398 397	400 394 398 400	402 395 399 396	395 397 400 404
Mean, $\bar{x}$	399.5	395.25	398.5	399.5	398.0	398.0	
Standard deviation, s	1.91	1.71	1.29	2.38	2.83	3.16	

(i) Calculate the values of $\bar{x}$ and s for sample 7. *(1 mark)*

(ii) Plot the seven values of $\bar{x}$ and s on your charts. *(2 marks)*

(iii) Comment on the performance of the process over the last seven hours. *(3 marks)*

(c) Sample 8 had mean $\bar{x} = 398.0$ and standard deviation $s = 4.55$. State, with a reason, the action, if any, you would advise as a result of this sample. *(2 marks)*

(d) Currently, the mean volume of contents is 396 ml with a standard deviation of 2.3 ml.

(i) Find the proportion of bottles with contents outside tolerances of 392 ml to 408 ml. *(2 marks)*

(ii) State whether or not it would be possible to consistently meet these tolerances. Explain your answer. *(2 marks)*

Student Response

6

Leave blank

a) A - 2.3 × 0.09 = 0.207
W - 2.3 × 0.27 = 0.621
W - 2.3 × 1.76 = 4.048
A - 2.3 × 2.33 = 5.359

(4 groups) 3

b)

i- $\bar{x}$ = 399
s = 3.916

1

GR (1)

ii- all ~~processes~~ samples are within the warning limits for both the mean and the standard deviation* (except sample 2.) However the mean is always below the target and the standard deviation seems to be rising therefore they should consider taking another sample to investigate it futher. £1 £1 £1

* so the performance is adequate

c) mean is within warning limits however standard deviation is between upper warning and action limit, take another sample to investigate futher. 2.

Leave blank

d

i- mean = 396

sd = 2.3

$P(X < 392) = P\left(Z < \frac{392 - 396}{\frac{2.3}{\sqrt{4}}}\right)$ X

$= P(Z < -3.48)$

$= 1 - 0.99975$

$= \underline{0.00025}$ X

$P(X > 408) = P\left(Z > \frac{408 - 396}{\frac{2.3}{\sqrt{4}}}\right)$ X

$= P(Z > +10.43)$

$= 1 - 1$

$= \underline{0}$

M0

A0

$0.00025 + 0 = \underline{\underline{0.00025}}$ X

ii- No

$408 - 392 = 16$ ✓

$6\sigma = 6 \times 2.3 = 13.8 < 16$ ✓

Yes because 13.8 is within the tolerance limits. ✓

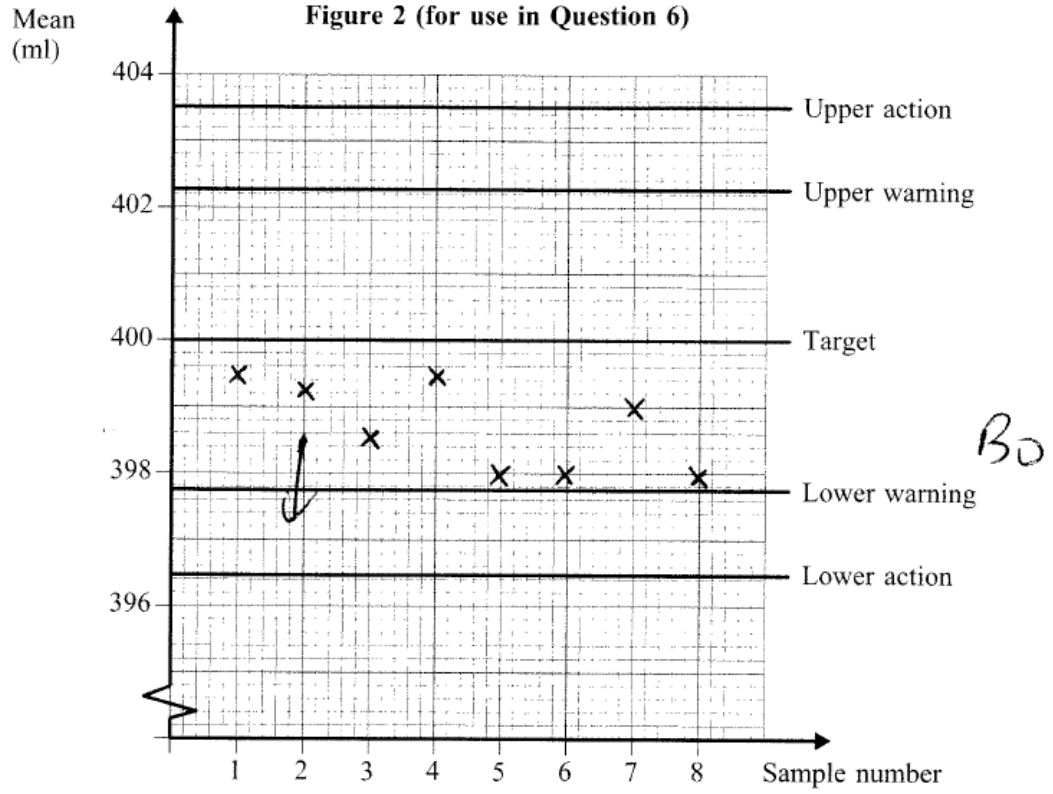
1 point for mean or target

E1

E0

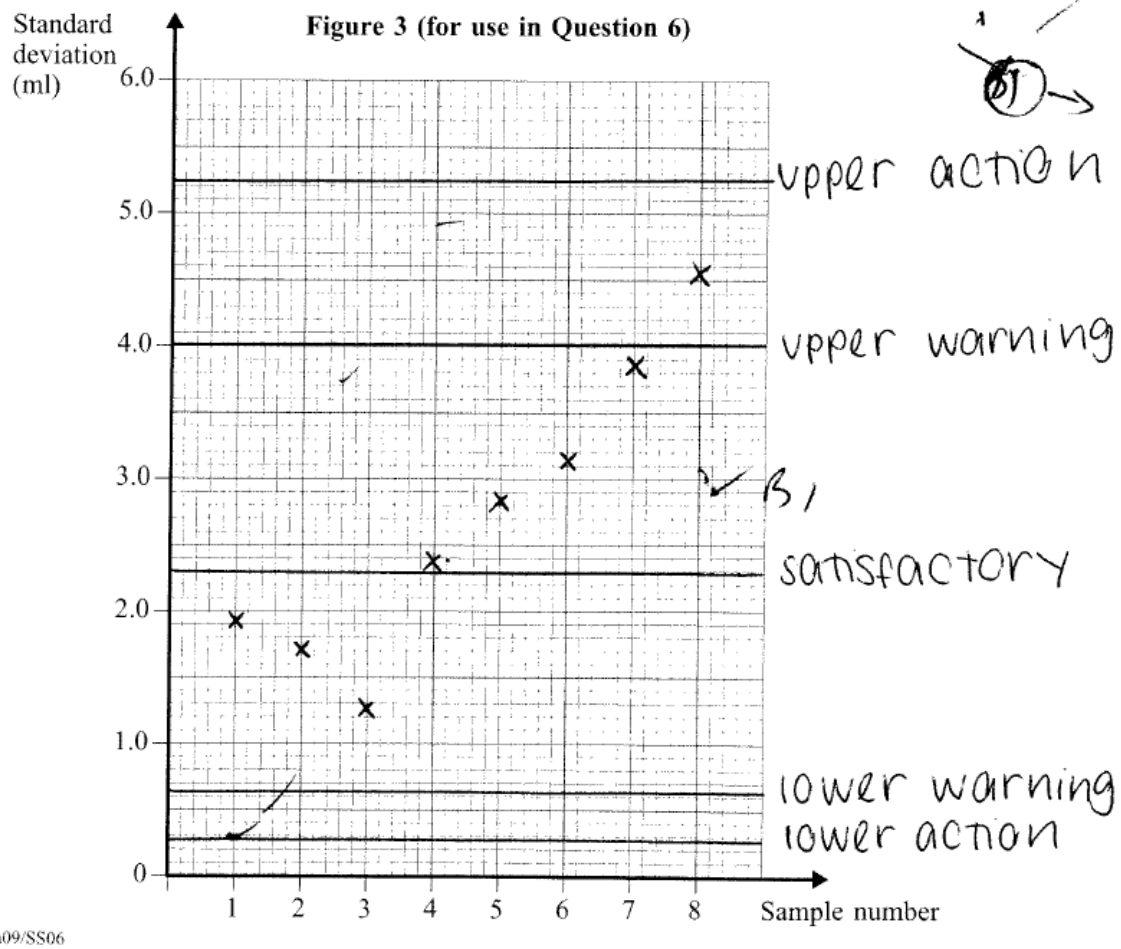
(11)

Figure 2 (for use in Question 6)



B0

Figure 3 (for use in Question 6)



B1

Commentary

The limits for the standard deviation have been correctly calculated and plotted.. Despite an error in plotting the second point on the means chart (which should have been below the lower action limit) the candidate has made sufficient relevant points in part (b)(iii) to gain all the available marks.

Part (c) is also answered correctly.

Part (d)(i) refers to individual bottles and not the means. Hence the standard deviation used should be 2.3. The square-root of 4 is not required.

In part (d)(ii) the candidate has carried out a relevant calculation, but not explained it well or commented that it is only possible to consistently meet the tolerances provided the mean is kept on target.

Mark Scheme

6(a)	Upper action $2.33 \times 2.3 = 5.359$ Upper warning $1.76 \times 2.3 = 4.048$ Lower warning $0.27 \times 2.3 = 0.621$ Lower action $0.09 \times 2.3 = 0.207$ + graph	M1 m1 A1	 3	M1 method for upper limits m1 method for all limits A1 accurate plot by eye Allow B1 if values for range charts used or if incorrect sample size (eg 7) used — but not both
(b)(i)	$\bar{x} = 399.0 \quad s = 3.92$	B1	1	399 CAO and 3.92 (3.91~3.92)
(ii)	on graph	B1 B1	 2	B1 accurate plot of means — by eye B1 accurate plot of sd — by eye
(iii)	Means — all within warning limits except sample 2 which is below lower action limit. Action appears to have been taken successfully. all 7 below target sd — all between warning limits but variability appears to be increasing over last 5 samples.	E1 E1 E1	 3	an E mark for any sensible point — maximum 2 for each chart. Maximum 3 in total.
6(c)	Sd between warning and action limits. Take another sample immediately if still above warning limit take action.	E1√ E1	 2	E1√ sd **between** warning and action E1 take another sample immediately
(d)(i)	$z_1 = \frac{392-396}{2.3} = -1.739$ $z_2 = \frac{408-396}{2.3} = 5.217$ proportion outside tolerance $1 - 0.959 = 0.041$	 M1 A1	 2	 M1 method — allow upper limit not considered A1 0.041 (0.04~0.042)
(ii)	Tolerance width $16 = \frac{16}{2.3} \approx 7$sd Possible to meet tolerances consistently provided mean on target.	E1 E1	 2	E1 possible to meet tolerances as width $> 6\sigma$; needs some calculation E1 provided mean is on target
	Total		**15**	

Question 7

Examiners return scripts to an awarding body using prepaid envelopes. The awarding body is considering new designs for the envelopes and asks four experienced examiners to rate each of three designs by giving them a mark out of 100. A perfect design would score 100.

The results are as follows.

		Design		
		P	**Q**	**R**
Examiner	**John**	23	33	42
	Gill	46	37	79
	Gwen	56	44	80
	Neil	54	60	75

You may assume that $\sum_i \sum_j x_{ij}^2 = 36\,721$.

(a) Carry out a 2-factor analysis of variance and test for a difference between designs of envelope. Use the 5% significance level and proceed on the basis that any necessary assumptions are satisfied. *(11 marks)*

(b) Compare the three designs of envelope, including a recommendation as to which one the awarding body should use. *(3 marks)*

Student Response

$T_J = 98$ $T_G = 162$ $T_{GW} = 180$ $T_N = 189$

7 $n_J = 3$ $n_G = 3$ $n_{GW} = 3$ $n_n = 3$

a) ~~$SS_T = 36721 -$~~

$T_P = 179$ $T_Q = 174$ $T_R = 276$ $T = 629$

$n_P = 4$ $n_Q = 4$ $n_R = 4$ $n = 12$

$$SS_T = 36721 - \frac{629^2}{12} = 3,750.92$$ ✓ M1

$$SS_R = \left(\frac{98^2}{3} + \frac{162^2}{3} + \frac{180^2}{3} + \frac{189^2}{3}\right) - \frac{629^2}{12}$$

$= 1,686.25$ ✓ M1

Leave blank

$$SSc = \left(\frac{179^2}{4} + \frac{174^2}{4} + \frac{276^2}{4}\right) - \frac{629^2}{12}$$

$$= 1,653.17$$ ✓ M1

H0: difference between designs of envelope. B0
H1: no difference between designs of envelope

Source of variation	Sum of Squares	degrees of freedom	Mean Square	F-ratio
SSc - Designs	1653.17	4-1=3	1653.17/3 = 551.06	551.06/58.79 = 9.373
SSR - Examiner	1,686.25	3-1=2	1686.25/2 = 843.125	843.125/58.79 = 14.34
Errors	4,11.5	~~12~~-5=7 ✗	411.5/7 = 58.79	
Totals	3,750.92	12		

M1 M1 A0.

$F_{3,7} = 4.347$ ✗ at 5% sig level. B0

9.373 is in crit region. ✓ A1✓
Reject H0. ^
Evidence there is a difference between designs of envelope. A1✓ 4/4

b) I recommend they use design R ~~because~~ it got the most points. E1

1 E0 E0

9

Commentary

The candidate has correctly calculated the sums of squares but has become confused over degrees of freedom - both interchanging designs and examiners and using one too many for errors and totals. Apart from this, the analysis is correct.

In part (b) design R is correctly identified as the best but this is not linked to the conclusion in part (a) that there is a significant difference between designs.

Mark Scheme

7(a)	P Q R Total J 23 33 42 98 Gi 46 37 79 162 Gw 56 44 80 180 N 54 60 75 189 Total 179 174 276 629			
	Total SS = $36721-\frac{629^2}{12} = 3750.92$	M1		M1 method for total SS
	Between designs SS $= \frac{179^2}{4}+\frac{174^2}{4}+\frac{276^2}{4}-\frac{629^2}{12} = 1653.17$	M1		M1 method for between designs or examiners SS (generous)
	Between examiners SS $= \frac{98^2}{3}+\frac{162^2}{3}+\frac{180^2}{3}+\frac{189^2}{3}-\frac{629^2}{12}$ $= 1686.25$	M1		M1 method for between designs and examiners SS
	Source SS df MS Designs 1653.17 2 826.58 Examiners 1686.25 3 Error 411.5 6 68.58 Total 3750.92 11	M1 B1 M1		M1 method for error SS — their figures B1 df error M1 method for MS — designs and error — their SS and df
	H_0: No difference between designs H_1: Difference between designs $F = \frac{826.58}{68.58} = 12.1$ Critical value $F_{2,6}$ is 5.143 Reject H_0 — significant evidence of difference between designs	m1 A1 B1 A1√ A1√	11	m1 method for F — requires all previous Ms A1 12.1 (12 ~ 12.2) B1 5.143 A1√ conclusion — must be compared with upper tail of F A1√ in context — previous A mark required **Special case** If designs and error SS interchanged, allow M and B but not A marks
7(b)	Results show significant evidence that not all means equal. Hence Q (lowest mean/total) differs from R(highest mean/total).	E1		E1 significant evidence of difference or Q different from R
	However means/totals suggest P and Q similar.	E1		E1 P and Q similar
	Recommend choose design R.	E1	3	E1 Choose R
	Total		**14**	