Teacher Support Materials
2009

Statistics GCE

Paper Reference SS03

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Question 1

A clinical nutrition department at a large hospital carried out research into the levels of body fat in females.

The age, x years, and the body fat, y per cent, for each of 10 randomly selected females are given in the table.

Female	x	y
A	23	27.9
B	39	31.4
C	41	25.9
D	49	25.2
E	53	34.7
F	56	32.5
G	57	30.3
H	58	33.0
I	60	41.1
J	61	34.5

(a) Calculate the value of Spearman's rank correlation coefficient between x and y.
(6 marks)

(b) Carry out a hypothesis test, at the 10% level of significance, to determine whether the value you calculated in part (a) indicates an association between x and y.

Interpret your conclusion in context. *(4 marks)*

Student Response

1 (a)

	x	y
A	1	3
B	2	5
C	3	2
D	4	1
E	5	9
F	6	6
G	7	4
H	8	7
I	9	10
J	10	8

$r = 0.6727$ ✓ 6

(b) $H_0: \rho_s = 0$ $H_1: \rho_s \neq 0$ ✓

two tail test

10%

$n = 10$ 3

$0.6727 > 0.5636$ ✓

The value is in the critical region, ∴ Ho is rejected, therefore there is significant evidence to show that there is association between x and y. ✓ 9

E0 context

Commentary

Students who showed the ranks tended to gain full marks in part (a).
In part (b) some students lost marks because their conclusion was not given in context i.e. did not refer to age and body fat.

Mark scheme

Q	Solutions	Marks	Total	Comments
1(a)	<table><tr><th>Rank *x*</th><th>Rank *y*</th></tr><tr><td>10</td><td>8</td></tr><tr><td>9</td><td>6</td></tr><tr><td>8</td><td>9</td></tr><tr><td>7</td><td>10</td></tr><tr><td>6</td><td>2</td></tr><tr><td>5</td><td>5</td></tr><tr><td>4</td><td>7</td></tr><tr><td>3</td><td>4</td></tr><tr><td>2</td><td>1</td></tr><tr><td>1</td><td>3</td></tr></table>	M1 M1 A1		attempt at ranks inconsistent (can be reversed) SC M1M1 B2 for 16 correct
	$r_s =$ 0.673 (3 sf from calc)	B3	6	AWRT B2 0.67 B1 0.7 ft B2 from wrong ranks (small slip) No ranks seen, **SC** 0.67 B4 0.7 B3 **alternative** $d = 2, 3, 1, 3, 4, \ldots, 3, 1, 1, 2$ $\sum d^2 = 54$ B1 $r_s = 1 - \dfrac{6\times 54}{10\times 99} = 0.673$ M1, A1
(b)	H_o Rank orders of age and percentage body fat in females are independent. H_1 Rank orders of age and percentage body fat in females are not independent – there is an association 2 tail 10%	B1		or equivalent
	cv = ±0.5636 $n = 10$	B1		for cv
	test stat $r_s =$ 0.673 $r_s > 0.5636$	M1		for comparison ts/cv **SC** Allow M1 0.593/0.5494 (pmcc)
	Reject H_o . Significant evidence at 10% level to suggest an association between rank orders of age and percentage body fat in females.(or positive association)	E1	4	correct and in context
	Total		**10**	

Question 2

A time of 9 minutes is allocated for the completion of a task on a production line.

The staff working on this production line complained to their line manager that the task took longer than the 9 minutes allocated. The line manager agreed to investigate by taking a sample of 8 measurements of the time taken, in minutes, to complete the task.

The times were as follows:

9.5 8.6 9.2 9.6 8.9 9.7 8.4 9.8

(a) Carry out a Wilcoxon signed-rank test, at the 5% significance level, to investigate whether the average time taken to complete the task is greater than 9 minutes.

Interpret your conclusion in context.

(9 marks)

(b) State **one** assumption that should be made for the test in part (a) to be valid

(1 mark)

Student Response

2. (a) ~~H0~~ H_0 average time taken to complete the task is 9 minutes.

H_1 average time taken to complete the task is greater than 9 minutes ✓ B1

one-tailed test at a significance level of 5%

Differences	Ranks +ve	Ranks −ve	
0.5	4		
−0.4		3	
0.2	2		M1, M1
0.6	5.5		
−0.1		1	
0.7	7		
−0.6		5.5	
0.8 M1	8		
✓	$T^+=26.5$	$T^-=8.5$	M1
	A1	×	

Question number | | Leave blank

B1

critical value is 6 < 8.5

M1 ✓

Therefore we accept H_0 because we have significant evidence that the average time taken to complete the task is 9 minutes. A0 — 8

(b) We have to assume that we eliminated the experimental error by doing the test more than once. ✗ — 0

(8)

Commentary

Candidates who showed ranks and method gained marks if an arithmetic error occurred.
In part (b) the relevant points were that the sample was randomly selected or that the data should be symmetrically distributed.

Mark Scheme

Q	Solutions	Marks	Total	Comments
2.(a)	H_o pop median/mean η , μ = 9 H_1 pop median/mean η , μ > 9 1 tail 5% (d is result – 9)	B1		
	diff: 0.5, -0.4, 0.2, 0.6, -0.1 rank: 4, -3, 2, 5½, -1	M1		For differences (result -9) - ignore signs
	diff: 0.7, -0.6, 0.8 rank: 7, -5½, 8	m1 M1		For ranks For ties
	$T_+ = 4 + 2 + \ldots + 8 = 26\frac{1}{2}$ $T_- = 3 + 1 + 5\frac{1}{2} = 9\frac{1}{2}$	m1 A1		For total attempted from ranks For one correct total
	Test stat $T = 9\frac{1}{2}$ $n = 8$ 1 tail 5% cv = 6 T > 6	B1 M1		For cv Comparison cv/ts (consistent)
	No significant evidence at 5% level to reject H_o . Conclude that there is no significant evidence to suggest that the average time to complete the task is greater than 9 minutes.	E1	9	In context
(b)	Sample was selected at random. or Times to complete the task are symmetrically distributed.	B1	1	Disallow 'normally distributed'
	Total		**10**	

Question 3

A coin expert carries out an analysis to determine the percentage of silver in coins taken from two separate coin mintings during the reign of King Manuel I. The percentages for the coins in a sample from each minting are given in the table.

First Minting	5.8	6.6	6.3	6.9	7.5	7.0	6.7	6.1
Second Minting	6.7	8.8	6.5	8.2	9.4	9.1	8.4	

Carry out a distribution-free test to investigate the claim that coins from the second minting contain a higher percentage of silver than those from the first minting. Use the 5% level of significance and assume each sample to be random. *(10 marks)*

Student Response

Q3 (a) H_0 = no significant difference

~~θ = 52.9~~ – H_1 = second miniting > first minitng

XX B0,0

TF = 52.9

UF = 52.9 – ~~8~~0 (8+1) / 2 = 16.9

no ranks M0,0,0,0

T.S. = 75

75 – ~~8~~7 (7+1) / 2 = ~~3~~ 47

A0

One tailed Test

Critical value = 16.9

Critical region = 13 B1 M0 A0

So we accept H_0 and reject H_1 as our value does not lie on the critical region so there is no significance evidence to reject H_0.

1

(1)

Leave blank

Commentary

Candidates who showed no ranks and/or method on the answer paper lost most of the marks

Mark Scheme

Q	Solutions	Marks	Total	Comments
3	H_o Samples are taken from identical populations	B1		or equivalent in words implying pop averages same/ 2nd greater
	H_1 Samples are not taken from identical populations – population average percentage silver higher in second minting.	B1		
	5% 1 tail			
	Ranks			(Alternative method acceptable)
	First: 1, 2, 3, 5, 6½, 8, 9, 10 Second: 4, 6½, 11, 12, 13, 14, 15	M1		Attempt at M-Whitney – ranks as one group (can be reversed)
		m1		Ties
	T_{1st} = 44½ T_{2nd} = 75 ½ m = 8 n = 7	m1		For total attempt
	$U_{1st} = 44\frac{1}{2} - \frac{8\times 9}{2} = 8\frac{1}{2}$			
	$U_{2nd} = 75\frac{1}{2} - \frac{7\times 8}{2} = 47\frac{1}{2}$	m1		For U formula correct
	Test stat U = 8½	A1		Either U correct
	cv = 13 $n = 7$ $m = 8$ 1 tail 5%	B1		correct/relevant cv used
	U = 8½ < 13	M1		comparison with U (consistent)
	Significant evidence to reject H_o and conclude that the percentage of silver was higher in the second minting.	A1	10	
	Total		**10**	

Ranks table:

First	Second
1	4
2	6½
3	11
5	12
6½	13
8	14
9	15
10	

Question 4

An eye clinic treats a large number of adult patients who have one normal eye but suffer from glaucoma in the other eye. The thickness, in microns, of the cornea of each eye was measured for each of a random sample of 8 such patients. The results are given in the table.

Patient	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Normal eye	488	478	492	444	436	398	464	476
Eye with glaucoma	484	478	480	426	440	410	458	460

(a) Carry out a sign test, at the 10% level of significance, to investigate whether there is any difference in the average cornea thickness between the normal eye and the eye with glaucoma. *(6 marks)*

(b) Later it was discovered that the measurements from 5 other randomly selected adult patients had been lost. However, it is known that all 5 patients had a lower cornea thickness in the eye with glaucoma than in the normal eye.

Use this additional information, together with the information given in the table, to carry out a sign test, at the 5% level of significance, to investigate whether there is evidence that the average cornea thickness of the normal eye is greater than that of the eye with glaucoma. *(5 marks)*

Student Response

4) a) H_0: population median difference = 0
H_1: population median difference ≠ 0 (Normal - glaucoma) ✓
two tailed test
10% significance level

Patient 1 2 3 4 5 6 7 8
Normal-glaucoma + • + + − − + + ✓ ✓

$x \sim B(7, 0.5)$
$P(x \geq 5) = 1 - P(x \leq 4) = 1 - 0.7734$
$= 0.2266$ ✓ → test statistic

Critical region ≤ Critical value
$0.2266 > 0.05$ ✓

Value not in critical region. So accept H_0. There is no significant evidence at a 10% significance level to indicate any difference in the average cornea thickness between the normal eye and the eye with glaucoma. ✓

6

b) H_0: population median difference = 0
H_1: population median difference (normal > glaucoma) > 0
one tailed test ✓
5% significance level

* Patients 1 2 3 4 5 6 7 8 9 10 11 12 13

+ · + + − − + + + + + + +

$X \sim B(12, 0.5)$ ✓

$P(X \geq 10) = 1 - P(X \leq 9)$ ✓

$= 1 - 0.9807$ ✓

$= 0.0193$ test statistic

critical region ≤ critical value

test statistic ≤ 0.05 ✓

5

Value in critical region. Reject H_0. There is significant evidence at a 5% significance level to indicate that the average cornea thickness at the normal eye is greater than that at the eye with glaucoma. ✓

✓ (11)

Commentary

Few candidates managed the two sign test but several produced excellent solutions identifying the tests stat and comparing it to the correct probability for the Binomial distribution. Good candidates realised that the zero should be excluded so B(7, 0.5) should be used.
In part (b), many candidates correctly added 5 to their test value thus obtaining n = 12 and compared the relevant B(12, 0.5) probability to 0.05.

Mark Scheme

Q	Solutions	Marks	Total	Comments
4(a)	H_0 η_d = 0 H_1 $\eta_d \neq 0$ 2 tail 10%	B1		
	Signs + . + + − − + +	M1		Signs (allow signed differences)
	$5^+/2^-$ signs – test values	A1		test stat correct and identified
	Binomial (7, 0.5) model	M1		Binomial model used and probability attempted
	P(≥ 5 +) = P(≤ 2 -) = 0.227 > 0.05 or P(≥ 5 +) = P(≤ 2 -) = 0.453 > 0.10 one tail test	m1		Comparison of Binomial probability with 0.05 (or 0.1) Identified correct critical region with probability given also M1m1
	Accept H_o There is not sufficient evidence, at the 10% level, to suggest that the average cornea thickness differs between the normal eye and the eye with glaucoma.	E1	6	Interpretation in context
4(b)	H_0 η_d = 0 H_1 η_d > 0 1 tail 5%	B1		One tail – either way if consistent
	$10^+/2^-$ signs – test values Binomial (12, 0.5) model	B1		test stat identified ft incorrect ts from (a)
	P(≥ 10 +) = P(≤ 2 -) = 0.0193 < 0.05	M1		Binomial model used and probability attempted
	one tail test	m1		Comparison of Binomial probability with 0.05 Identified correct critical region with probability given also M1m1
	Reject H_o . There is sufficient evidence, at the5% level, to suggest that the average cornea thickness is greater for the normal eye than for the eye with glaucoma.	A1	5	**SC** n = 8 in part(a) Allow part(b) n =13 M1, M1 for 0.0112 < 0.05
	Total		**11**	

Question 5

A factory has four identical machines, A, B, C and D, that produce bottle caps. The production manager believes that there are some differences between the average daily outputs of the machines. In order to investigate his belief he decides to select one of the machines at random on each of 21 days and record the number of bottle caps it produces during the day.

The **rank values** of the results are given in the table. A rank value of 1 indicates the lowest production.

Machine A	Machine B	Machine C	Machine D
2½	15	1	8
5	16	2½	9
10	18	4	11
14	20	6	12
17	21	7	13
19			

(a) Carry out a Kruskal-Wallis test, using the 1% significance level, to investigate whether there is any difference between the average daily number of bottle caps produced by the four machines. *(12 marks)*

(b) The maintenance engineer at the factory has money available to replace one of the four machines.

Identify, with a reason, which machine you would advise him to replace. *(2 marks)*

Student Response

Leave blank

5 a) Ho: No difference between the average daily (ranks of) numbers of bottle caps produced by the four machines

H₁: There is difference between at least two of the average daily (ranks of) numbers of bottle caps produced by the four machines.

$T^A = 67.5 \quad T^B = 90 \quad T^C = 20.5 \quad T^D = 53$

$$H = \frac{12}{21(22)} \times \left(\frac{(67.5)^2}{6} + \frac{(90)^2}{5} + \frac{(20.5)^2}{5} + \frac{(53)^2}{5} \right) - 66$$

$= 12.577$

1% sig. level, two tailed

~~CV = 11.345~~

degrees of freedom = 3.

$11.345 < 12.577$

Ho rejected.

∴ we have sig. evidence that there is difference of at least two of the average daily (ranks of) numbers of bottle caps produced by the four machines. 12

b) Machine C since it has the lowest ranks of the four, therefore we see that it doesn't produce the amount ~~needed~~ and expected. It produces a lot less than the other 3. 2

14

Commentary

Candidates who referred to population averages or average daily numbers of bottle caps needed to mention a difference of 'at least 2' in their H_1.
Many correct attempts were seen this session on the Kruskal-Wallis test .
Most candidates identified machine C as having the lowest production.

Mark Scheme

Q	Solutions	Marks	Total	Comments
5(a)	Ho Samples are taken from identical populations	B1		or $H_0 \quad \eta_A = \eta_B = \eta_C = \eta_D$
	H_1 Samples are not taken from identical populations – population average bottle cap productions differ for the 3 machines. 1 tail 1%	B1		H_1 at least two of $\eta_A, \eta_B, \eta_C, \eta_D$ do differ
	Ranks Machine A: 2½, 5, 10, 14, 17, 19 Machine B: 15, 16, 18, 20, 21 Machine C: 1, 2½, 4, 6, 7 Machine D: 8, 9, 11, 12, 13			
	$T_A = 67½ \quad T_B = 90 \quad T_C = 20½ \quad T_D = 53$ $n_A = 6 \quad n_B = 5 \quad n_C = 5 \quad n_D = 5$	M1		Totals
	$\sum_{i=1}^{m} \frac{T_i^2}{n_i} = \frac{67.5^2}{6} + \frac{90^2}{5} + \frac{20.5^2}{5} + \frac{53^2}{5}$	m1		Method for $\sum_{i=1}^{m} \frac{T_i^2}{n_i}$
	$= 3025.225$	m1		n_i correct
	$H = \frac{12}{21 \times 22} \times 3025.225 - (3 \times 22)$	m1		test stat H
	$= 12.58$	A1		12.3 – 12.9
	Critical value from $\chi_3^2 = 11.345$	B1		3 df
	$H > 11.345$	B1		cv
		M1		comparison cv/ts
	Sig evidence to reject H_0 and conclude that samples are not from identical populations	A1		
	Significant evidence at the 1% level to suggest that the population average bottle cap productions differ for the 4 machines. At least two machines have different averages	E1	12	Difference in context Mention of 'at least two'
(b)	Machine C had the lowest average rank score so it would seem likely that machine C is the lowest producer of bottle caps and therefore this machine is the most obvious one to suggest for replacement	B1		Machine C
		E1	2	Reason – must refer to the lowest average score or lowest production
	Total		**14**	

Question 6(a)

An Institute for Health and Welfare carried out an investigation into ladder-related falls during 2004/05.

(a) The place of occurrence of the fall and the sex of the person who fell were recorded and the results are summarized in **Table 1**.

Table 1

Place of occurrence of fall \ Sex	Male	Female	Total
Home	1269	393	1662
School, other institution or sport area	35	5	40
Trade or service area	76	16	92
Industrial or construction area	156	6	162
Farm	31	6	37
Total	1567	426	1993

(i) Test, at the 1% level of significance, whether the place of occurrence of a fall is independent of the sex of the person who falls. *(10 marks)*

(ii) By comparing observed and expected frequencies, identify, in context, **two** important facts. *(2 marks)*

(iii) Make one further general statement regarding the observed frequencies in **Table 1**. *(1 mark)*

Student Response

Leave blank

(6) Ho : place of occurence of a fall is independend of sex

H, : the place of occurence of a fall is not independend of sex.

1% sign. level.

~~Expected values:~~

Expected values:

	Male	Female
Home	1307	355
School/sportarea	31	9
Trade/service area	72	20
Industrial/cons.	127	35
farm	29	8

✓✓ A0 ✗

$ts = \sum\left(\frac{(O-E)^2}{E}\right) = 39.8$ ✓ M1 A1 implied

$CV = 13.277$ ✓✓ $ts > CV$ ✓

Reject Ho, there is significant evidence to assume that the place of the fall is not independent of the sex of the person who fell. ✓

9

(ii) The expected values for female falls ~~[illegible]~~ were greater than the observed. Also there were more male falls in industrial and construction area than female falls observed. x 0

(iii) In 2004/05 there where more males falling from ladders in any area than of women. 1

Commentary

Expected values should be given to at least 1 dp or marks will be lost.
In part (a) (ii) reference to the comparison of expected and observed frequencies is essential and marks will not be gained otherwise.
In part (a) (iii) any comment on the observed frequencies is acceptable.

Mark Scheme

Q	Solutions	Marks	Total	Comments
6(a)(i)	H_o No association between place of occurrence of fall and sex of person who falls. H_1 Association exists between place of occurrence of fall and sex of person who falls.	B1		
	1 tail 1%			
	Expected freqs: Male / Female **Home**: 1306.75 / 355.25 **School, other**: 31.45 / 8.55 **Trade and serv**: 72.34 / 19.66 **Industrial etc**: 127.37 / 34.63 **Farm**: 29.09 / 7.91	M1 m1 A1		For method for E for 3 correct All correct (1dp required minimum except 1307/355)
	ts = $\sum \frac{(O-E)^2}{E}$ $= \frac{(1269-1306.75)^2}{1306.75} + \frac{(393-355.25)^2}{355.25}$ ……….$+ \frac{(6-7.91)^2}{7.91}$	m1		ts sum with correct denominators
	= 38.5	A1		for ts in range 34 - 43
	df = 4 1% cv = 13.277	B1 B1		for df = 4 for cv
	ts > 13.277	m1		for comparison ts/cv
	Significant evidence to reject H_o and conclude that there is an association between place of occurrence of fall and sex of person who falls.	A1	10	
(ii)	Far fewer than expected females have ladder-related falls in the Industrial or Construction area and far more males than expected do have falls in this area. More females were observed to have falls at home than would be expected.	E1 E1	2	Any two points made
6(a)(iii)	Many more males involved in ladder-related falls than females. Most falls occurred at home.	E1	1	Any one point made

Question 6(b)

The number of patients admitted to hospital during 2004/05 for the three admission categories involving ladder-related falls for males and for females was also recorded. The results are summarized in **Table 2**.

Table 2

Admission category \ Sex	Male	Female
Direct ladder-related fall	62.1%	50.9%
Transfer following ladder-related fall	12.3%	10.5%
Other incident also involving fall from ladder	25.6%	38.6%
Total admissions	227	57

(i) Use the information in **Table 2** to construct a contingency table with frequencies that could be analysed to investigate whether there is an association between admission category and the sex of the person who falls. *(3 marks)*

(ii) For the contingency table in part (b)(i), the value of $\sum \frac{(O-E)^2}{E}$ is 3.83, correct to 3 significant figures.

Test, at the 5% level of significance, whether admission category is independent of the sex of the person who falls. *(4 marks)*

Student Response

6 b) i)

Admission category \ Sex	Male	Female	Total
Direct ladder-related fall	140.967	29.013	169.98
Transfer following ladder-related fall	27.921	5.985	33.906
Other incident also involving ladder	58.112	22.002	80.114
Total admissions.	227	57	284

A0 2

ii) H_0 independent of sex   1% sig level.

H_1 not independent of sex   B1

degree of freedom $= (3-1)(2-1) = 2$.

$\sum \frac{(O-E)^2}{E} = 3.84$ 2

Critical Value $= 9.210$ B0 M1

A0

Test statistic does not exceed critical value so there is no evidence to doubt H_0 and we conclude that the admission ~~category~~ category is independent of the sex of the person who falls

14

Commentary

In part (i) frequencies that are observed should be given as integers or marks will be lost.

(b)(i)	**Direct**: Male 141, Female 29; **Transfer**: Male 28, Female 6; **Other**: Male 58, Female 22	M1		% of 227 or % of 57
		m1		4 correct (not necessarily integers)
		A1	3	all correct and integers
(ii)	H_o No association between admission category and sex of person who falls. H_1 Association exists between admission category and sex of person who falls.	B1		
	ts = 3.84 df = 2 5% cv = 5.991	B1		
	3.84 < 5.991 No significant evidence to reject H_o. Conclude that there is no evidence of association between admission category and sex of person who falls.	M1, A1	4	

	Male	**Female**
Direct	141	29
Transfer	28	6
Other	58	22