Teacher Support Materials 2008

Statistics GCE

Paper Reference SS04

Question 1

1 The proportion of blood donors who have blood group AB negative is 1 per cent.

(a) Specify the binomial distribution which describes the number of people who have blood group AB negative in a random sample of 120 blood donors. *(1 mark)*

(b) Use a Poisson approximation to estimate the probability that a random sample of 120 blood donors includes at least 4 people who have blood group AB negative. *(3 marks)*

Student Response

Leave blank

1 a. $X \sim B(120, 0.01)$ ✓ — 1

b. X is r.v n° of people who have blood group AB out of 120 blood donors

$X \sim B(120, 0.01)$

$X \approx Po(np)$

$X \approx Po(1.2)$ ✓ B1 — 1

$P(X \le 4) = 0.99225$

$= 0.992$ ✗

(2)

Commentary

The example illustrates a common error which arose when candidates failed to read the question carefully enough. “At least 4” has been interpreted as “not more than 4”.
The other common error was to state $P(X \le 4) = 1 - P(X \le 4)$.

Mark scheme

1(a)	B(120, 0.01)	B1	**1**	
(b)	X = Number of donors with group AB negative. $X \sim B(120, 0.01) \approx Po(1.2)$ $P(X \ge 4) = 1 - P(X \le 3)$ $=1 - 0.9662$ $= 0.0338$	B1 M1 A1	**3**	B1 for exact binomial (0.0330) awrt
	Total		**4**	

Question 2

2 Katrina is an athlete who competes in the 100 metres hurdles and in the long jump. At the start of one season, she set herself two targets to achieve in competitions during the season.

Target 1: Her mean time for the 100 metres hurdles will be less than 14.5 seconds.

Target 2: The probability that she fails to jump a distance of 5.5 metres or more with her first jump in a long jump competition will be less than 0.25 .

(a) During the season, Katrina's times, in seconds, for the 100 metres hurdles in eight competitions were

14.2	14.6	13.9	14.4	14.6	14.5	14.3	14.1

Assuming that these times may be regarded as a random sample from a normal distribution, carry out a hypothesis test, at the 5% significance level, to investigate whether she achieved Target 1. *(9 marks)*

(b) During the season, Katrina failed to jump a distance of 5.5 metres or more with her first jump in 1 out of 15 long jump competitions.

Carry out a hypothesis test to investigate whether she achieved Target 2. Use an exact distribution and the 5% significance level. *(5 marks)*

Student response

Leave blank

$\bar{x} = 14.325$ ✓

$\sigma = 0.2493$ ✓ B1

$n = 8$

$z =$ ~~4~~

M1 m1

$t_7 = \dfrac{14.5 - 14.325}{0.2493/\sqrt{8}} = 1.99$ ✓ B1

At 5% significance B1 level, the critical value for t_7 is 1.895, ✓ As 1.99 is > 1.895, Ho is rejected. Wrong tail

There is evidence to suggest that the mean time was ~~[illegible]~~ < 14.5 seconds. ✗ 6

b) $H_0: P = 0.25$ ✓

$H_1: P < 0.25$ ✓ B1

$\hat{P} = 1/15 = 0.066$

$z = \dfrac{0.25 - 0.066}{\sqrt{\dfrac{0.25 \times (1-0.25)}{15}}} = 1.65$ ✗

$z = 1.65$ Used normal approximation.

~~[illegible]~~

At 5% Significance level the critical value is 1.6449, As 1.65 is in the critical region, Ho is rejected.

There is evidence to suggest the Probability ✓ is less than 0.25. ✗ 1

(7)

Commentary

Part (a) was generally very well done and it can be seen that this candidate had followed the structure of the hypothesis test correctly for most of the way. The error of losing negative signs for the test statistic and critical value was seen in a large number of scripts. It often led to the candidate considering the wrong tail when evaluating the result. The conclusion stated here matches the candidate's alternative hypothesis ($\mu < 14.5$ which was on the previous page), but does not follow from the preceding work.
Part (b) tested the use of *p*-values in hypothesis testing and specified the use of an exact distribution. Many candidates attempted an inappropriate distributional approximation as illustrated here. The topic seems to be one that many find difficult.

Mark Scheme

2(a)	$H_0 : \mu = 14.5$ $H_1 : \mu < 14.5$	B1		Both.
	$\bar{x} = 14.325, \quad s = 0.2493$	B1		14.3 to 14.33; accept 0.249
	$\nu = 8 - 1 = 7$	B1		
	$t_{crit} = -1.895$	B1		Ignore sign.
	Test statistic = $\dfrac{14.325 - 14.5}{\frac{0.2493}{\sqrt{8}}}$	M1 m1		Use of formula for ts. Their sd divided by $\sqrt{8}$.
	= -1.985(6)	A1		-2 to -1.9
	-1.985(6) < -1.895	E1		
	There is evidence at the 5% significance level to claim that Katrina has achieved Target 1.	A1√		ft on ts and cv. Depends on M1 and m1.
			9	
(b)	$H_0 : p = 0.25$ $H_1 : p < 0.25$	B1		Both
	X = Number of failed attempts. Under H_0, $X \sim B(15, 0.25)$	B1		May be implied.
	$P(X \leq 1) = 0.0802$	B1		
	0.0802 > 5% so H_0 cannot be rejected.	M1		Attempt to reach conclusion by comparing probability with 5%.
	There is not enough evidence at the 5% level to claim that Katrina has achieved Target 2.	A1	**5**	
	Total		**14**	

Question 3

3 Every spring, bluebell plants flower in a field. It has been found that the mean number of bluebell plants in one square metre of the field is 23.

(a) (i) State **two** necessary conditions for the number of bluebell plants in one square metre of the field to be modelled by a Poisson distribution. *(2 marks)*

(ii) Assuming that the necessary conditions are satisfied, use a distributional approximation to find the probability that a square metre of the field contains fewer than 30 bluebell plants. *(5 marks)*

(b) The local council constructs a footpath next to the field. Roy, a local resident, is concerned about the possible effects of trampling, picking of bluebell flowers and digging up of bluebell plants. The footpath is opened at the beginning of a year. In the spring of the following year, Roy selects an area of 4 square metres in the field and finds a total of 79 bluebell plants inside this area. He claims that there has been a reduction in the number of bluebell plants per square metre of the field.

(i) Assuming that a Poisson model is still appropriate, construct an approximate 95% confidence interval for the mean number of bluebell plants per 4 square metres of the field. *(4 marks)*

(ii) Use your confidence interval to assess whether Roy's claim is justified. Assume that the area Roy examined was randomly selected from the field. *(3 marks)*

(iii) Give **two** reasons why the confidence interval that you constructed in part (b)(i) is approximate rather than exact. *(2 marks)*

Student Response

3b. i) $23 \times 4 = 92$

$\lambda = 92$

✓B1 ✓B1

$92 \pm 1.96\sqrt{92}$ M1

$73.2 \rightarrow 110.8$ // ✗

✓B1

3

ii) 79 is within the confidence interval, so there is no strong evidence to suggest ✓ that the number of bluebells has reduced. But, 79 is towards the lower end of the confidence interval, so it would no be unreasonable to take another sample. ✓ E1

2

Commentary

Many made a good attempt at the probability in part (a) of the question, with errors mainly confined to missing or incorrect continuity corrections.
This example of part (b) illustrates the fact that most candidates knew how to construct a confidence interval for the (large) mean of a Poisson distribution based on a single value.
It also demonstrates the confusion that was seen in a lot of scripts. The candidate has essentially set up critical values for a hypothesis test to investigate whether the mean number of bluebells has changed. This was not what the question asked for, but demonstrated enough technique and understanding to gain most of the available marks.
Most scored at least one mark in part (b) (iii), usually by pointing out that the standard deviation was estimated from the sample value.

Mark Scheme

3(a)(i)	Plants randomly distributed. Constant average density over the field. Independent occurrence of plants. P(two plants in same position) = 0. Size of plant can be treated as negligible.	B2,1,0	**2**	B1 each for any two distinct conditions. Must be in context for full marks.
(ii)	$X \sim \text{Po}(23) \approx \text{N}(23, 23)$ $\text{P}(X < 30) = \Phi\left(\frac{29.5 - 23}{\sqrt{23}}\right)$ $= \Phi(1.355) = 0.912$	B1 M1 m1 A1 A1√	**5**	m1 attempted cc. correct cc. 0.911 to 0.914;√ on no cc. (0.928) or 30.5 (0.941)
(b)(i)	Normal approximation with sd = $\sqrt{79}$ z = 1.96 95% confidence limits are $79 \pm 1.96\sqrt{79}$ giving (61.6, 96.4)	B1 B1 M1 A1	**4**	May be implied. Up to 3 if 92 used instead of 79. awrt.
(ii)	Original mean per 4m^2 was 92 OR CI for mean per m^2 is (15.4, 24.1) Original mean lies within 95% CI for new mean. Not enough evidence to support Roy's claim.	B1 E1 B1	**3**	Maximum of 2 if roles of 92, 79 reversed.
(iii)	Normal approximation used. SD estimated from count of plants in sample area. Assumption that area was randomly selected may not be justified.	E2,1,0	**2**	Any two.
	Total		**16**	

Question 4

4 Patients who attend a particular clinic are prescribed drug A, which they must take over a long period of time. It is found that 25 per cent of them suffer stomach pains as a side effect of the drug. The consultant in charge of the clinic wants to investigate whether this side effect is equally likely to occur when an alternative drug, B, is taken.

The consultant prescribes drug B for 50 existing patients and 19 of them suffer stomach pains as a side effect.

(a) Construct an approximate 99% confidence interval for the proportion of patients taking drug B who suffer stomach pains as a side effect. Assume that the 50 patients given drug B are a random sample of existing patients. *(6 marks)*

(b) Use your confidence interval to assess whether patients are equally likely to suffer stomach pains as a side effect when taking drug B as they are when taking drug A. *(2 marks)*

(c) It is later found that the consultant had explained the purpose of the investigation to all existing patients, and had asked for volunteers to try drug B. Explain how, if at all, this information might affect the assessment that you made in part (b). *(2 marks)*

Student Response

b) Patients are equally as likely to suffer stomach pains as a side effect This is because 0·25 is in the 99% confidence interval ✓ ~~Also the upper bound is~~
Too definite E1 — 1

c) It may affect it because it is not random. The person may now think that they are going to suffer stomach pains. ✓ This means that it is not independent any more. This will certainly have an affect on the outcome. The patient may psychologically now think ✓ that they are suffering pains when there actually not. ✓ E1 — 1

Commentary

There were few difficulties with finding the confidence interval in part (a).
In part (b), candidates were asked for an assessment of whether patients were equally likely to suffer from stomach pains as a side effect when taking drug B as when taking drug A. Many, like this one, made a too definite statement without considering the range of values in the confidence interval. While it cannot be ruled out that the two proportions are equal, it is not certain.
Most were able to make a sensible comment about a flaw in the method used by the consultant, but few explained how this might affect their assessment. A good many thought that a new experiment was being set up rather than a revised analysis of the results already found.

4(a)	$\hat{p} = \frac{19}{50} = 0.38$	B1		
	Normal approximation with sd $\sqrt{\frac{0.38 \times (1 - 0.38)}{50}}$	M1 A1		Correct values substituted.
	$z = 2.5758$	B1		Accept 2.58
	99% confidence limits for p are $0.38 \pm 2.5758 \times \sqrt{\frac{0.38 \times 0.62}{50}}$	M1		
	giving (0.203, 0.557)	A1	**6**	awrt
(b)	There is not enough evidence to say there is a difference in the proportion of patients suffering the side effect as	B1		
	25% = 0.25 lies within the confidence interval.	E1	**2**	
(c)	Patients who suffer stomach pains more likely to volunteer than those who do not. Pre-knowledge could cause imaginary pains.	E1		Likely effect on sample or effect of knowing purpose of trial.
	The confidence interval could indicate a reduction in proportion if sample drawn mainly from population of those suffering the side effect. Pre-knowledge may have exaggerated sample proportion.	E1	**2**	Possible effect on assessment.
	Total		**10**	

Question 5

5 Many of the passengers who use a town bus route have travel passes which they must show to the driver, who then issues them with a ticket. Those who do not have travel passes must buy a ticket from the driver.

The time, X seconds, taken by the driver to serve a passenger who has a travel pass is normally distributed with mean 5.8 and standard deviation 1.4.

The time, Y seconds, taken by the driver to serve a passenger who does not have a travel pass is normally distributed with mean 18.5 and standard deviation 3.6.

At a particular bus stop, two passengers get on the bus. One of them has a travel pass and the other does not.

(a) Find the probability that the total time taken to serve the two passengers is less than 30 seconds. *(4 marks)*

(b) By considering the variable $Y - 3X$, find the probability that it takes more than three times as long to serve the passenger without a travel pass as it does to serve the passenger with a travel pass. *(7 marks)*

Student Response

b) $Y - 3X$ $\quad X\ (5.8, 1.4^2)$

$3X$ mean $= 5.8 \times 3$ ✓ B1

$= 17.4$

$3X\ (17.4, (\sqrt{5.88})^2)$

$Y\ (18.5, 3.6^2)$

s.d $= \sqrt{1.4^2 + 1.4^2 + 1.4^2}$

$= \sqrt{5.88}$ ✗

$Y - 3X$ mean $= 18.5 - 17.4$ ✓

$= 1.1$ ✓ M1

sd $= \sqrt{3.6^2 + (\sqrt{5.88})^2}$ ✓ B1

$= \sqrt{18.84}$

$= \dfrac{1.1}{\sqrt{18.84}}$ $\quad \lambda = 0.25$ $\quad \Rightarrow 0.59871$

M0

Commentary

This seems to be a love it or hate it topic. Many candidates sailed through it with no problems and most made a good attempt at part (a). A lot of problems arose in dealing with the variance of $Y - 3X$ in part (b). The example shows the very common error of evaluating $\text{Var}(3X) = 3 \times \text{Var}(X)$. In this case the resulting variance was added to that of Y so another method mark was earned. Many, however, went on to subtract the variances. This candidate could have picked up another two method marks for using the distribution found, but evidence of an intention to find $P(Y - 3X > 0)$ was required.

Mark Scheme

5	$X \sim N(5.8, 1.4^2)$ $Y \sim N(18.5, 3.6^2)$			
(a)	$X + Y \sim N(24.3, 14.92)$	B2		B1 mean; B1 sd.
	$P(X + Y < 30) = \Phi\left(\dfrac{30 - 24.3}{\sqrt{14.92}}\right)$	M1		
	$= \Phi(1.476) = 0.930$	A1		0.929 to 0.931
			4	
(b)	$3X \sim N(3 \times 5.8, 3^2 \times 1.4^2)$			
	$= N(17.4, 17.64)$	B2		B1 mean; B1 sd.
	$Y - 3X \sim N(18.5 - 17.4, 3.6^2 + 17.64)$	M1		Means subtracted; sds added.
	$= N(1.1, 30.6)$	A1		cao
	$P(Y > 3X) = P(Y - 3X > 0)$	M1		
	$= 1 - \Phi\left(\dfrac{0 - 1.1}{\sqrt{30.6}}\right)$	m1		
	$= 1 - \Phi(-0.199) = 0.579$	A1		
			7	
	Total		**11**	

Question 6

6 The Woodways Trust maintains a nature trail which is open to the public. There are two car parks next to the nature trail, and motorists using a car park are asked to place a donation in an 'honesty box'.

The Trust puts up a notice in each car park in an attempt to increase the total amount that it receives in donations from motorists.

The notice in car park A displays a photograph of a popular celebrity and says: '*Please give generously to support the Woodways Trust*'.

The notice in car park B says: '*Please support the Woodways Trust. Suggested donation: £1*'.

(a) Before the notices were put up, the proportion of motorists who made a donation was 40 per cent in both car parks.

(i) After the notice was put up in car park A, 33 out of a random sample of 60 motorists made a donation.

Carry out a hypothesis test, at the 5% significance level, to investigate whether the proportion of motorists using car park A who made a donation was more than 40 per cent. *(8 marks)*

(ii) After the notice was put up in car park B, 18 out of a random sample of 45 motorists made a donation. Explain why no hypothesis test is necessary to investigate whether the proportion of motorists using car park B who made a donation was more than 40 per cent. *(2 marks)*

(b) Before the notices were put up, the mean amount given by motorists who made a donation was 42 pence in each car park.

(i) After the notice was put up in car park A, the amounts, x pence, given by a random sample of 10 motorists who made a donation may be summarised as follows:

$$\bar{x} = 59.5 \qquad s = 19.21$$

Assuming that the amounts given by motorists using car park A who made a donation may be modelled by a normal distribution with mean μ, construct a 95% confidence interval for μ. *(4 marks)*

(ii) A 95% confidence interval for the mean amount, in pence, given by motorists who made a donation using car park B after the notice was put up was found to be (49.5, 76.3).

Use this information and your results from parts (a) and (b)(i) to compare the effectiveness of the two notices in increasing the **total** amount received by the Trust from car park donations. *(4 marks)*

(iii) Explain why a normal distribution is unlikely to be a suitable model for the amounts given by motorists using car park B who made a donation after the notice was put up. *(2 marks)*

Student Response

Question number

Leave blank

b)

$\bar{x} \pm z \times \frac{\sigma}{\sqrt{n}}$.

$\bar{x} = 5.95$

$z = 95\% = 97.5\% = 1.96$ ✓ ✗

$\sigma = 19.21$

$n = 10$

$59.5 \pm 1.96 \times \frac{19.21}{\sqrt{10}}$. ✓ M1 m1

2

47.5935, 71.4065. ✗

ii A = 47.5935, 71.4065
B = 49.5 , 76.3

Both signs ~~are~~ ~~[illegible]~~ have confidence intervals carpark A has a slighty lower confidence intervals I am 95% confident that the mean lies between 47.6 ~~and~~ and 71.4. Car Park b has a higher confidence interval of 49.5 ; 79.3 Therefore this shows that car park B's sign is more efficient as the lowest mean is higher than A's lowest mean. Therefore they will be more money donation in car park B

Not significantly different.

0

Commentary

Part (a) was well done with many completely correct hypothesis tests. In part (ii), most stated that the sample proportion was exactly 40% but thought this meant that the population could not be more than 40%. Clearly it could, but no hypothesis test would reveal this. Most acceptable explanations stated that the test statistic would be 0 leading to acceptance of H_0.
In part (b), a lot of candidates treated the value of *s* given for the sample as a known population standard deviation and so used a *z*-value as the critical value, as shown in the example. The technique for finding the confidence interval was well known.
There were few really good attempts at evaluating the effectiveness of the two notices. Many commented that the proportion of people giving a donation had increased in A but not in B, and that the mean donation had increased in both car parks. There was a common belief that amounts donated had risen more in B than in A, in spite of the large overlap of the intervals. Some interpreted the confience intervals as showing the range of individual donations. The use of bold type for the word "total" was intended to encourage candidates to link the proportion donating and the mean amount donated, but few did this.

6(a)(i)	H_0: p = 0.4 H_1: p > 0.4 Under H_0, $X \sim B(60, 0.40)$ $\approx N(24, 14.4)$ z = 1.6449 Test statistic = $\dfrac{32.5 - 24}{\sqrt{14.4}}$ = 2.24 OR = $\dfrac{33 - 24}{\sqrt{14.4}}$ = 2.37 2.24 (2.37) > 1.6449 so there is enough evidence at the 5% significance level to claim that the proportion who make a donation is greater than 40%.	B1 B2 B1 M1 A1 E1 A1√	8	Both. B1 normal; B1 parameters. Accept 1.64; 1.645. OR s.e.= $\sqrt{\dfrac{0.4 \times 0.6}{60}}$ (M1A1) $ts = \dfrac{0.55 - 0.40}{\sqrt{\dfrac{0.4 \times 0.6}{60}}} = 2.37$ (M1A1) Exact binomial: 0.0133 < 5% gets full marks. ft on ts and cv.
(ii)	$\dfrac{18}{45} = 0.4$ so sample proportion is exactly 40%. Hypothesis test would lead to accepting H_0 at any sensible significance level.	B1 E1	2	
(b)(i)	$\nu = 9$; $t_{crit} = 2.262$ 95% confidence limits for μ are $59.5 \pm 2.262 \times \dfrac{19.21}{\sqrt{10}}$ giving (45.8, 73.2)	B1 M1 m1 A1	4	sd divided by $\sqrt{10}$ awrt
(ii)	Both notices seem to have increased the mean donation to about the same level. Evidence of increased proportion of motorists donating in A but not in B. Total likely to have increased more in A than in B.	B1 B1 B1 B1	4	
(iii)	£1 recommendation could lead to truncation. Could be high frequency for £1 but mean ≠ £1 so distribution not symmetrical. Recommended donation means amount given not random/independent variable.	E2,1,0	2	
	Total		**20**	