

Teacher Support Materials 2008

Maths GCE

Paper Reference SSO3

Copyright © 2008 AQA and its licensors. All rights reserved. Permission to reproduce all copyrighted material has been applied for. In some cases, efforts to contact copyright holders have been unsuccessful and AQA will be happy to rectify any omissions if notified.

SS03

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334). Registered address: AQA, Devas Street, Manchester M15 6EX. *Dr Michael Cresswell*, Director General.

Question 1

A manufacturer of an electrical appliance wants to adjust one of the components used in the appliance. The effect that the adjustment would have on the resistance of the component is investigated.

The manufacturer selects, at random, 8 components. Each component has its resistance, in ohms, measured before and after the adjustment.

The results of the investigation are shown in the table.

Component	Α	В	С	D	Ε	F	G	Н
Before	38	42	44	35	44	36	44	42
After	41	49	42	40	43	40	46	50

(a) Carry out a Wilcoxon signed-rank test, at the 5% significance level, to investigate whether or not the average resistance of the component is changed by the adjustment. Interpret your conclusion in context.

(9 marks)

(b) (i) Give **one** reason why a Wilcoxon signed-rank test might be preferred to a sign test in carrying out an investigation similar to the one carried out in part (a).

(1 mark)

(ii) Describe **one** situation in which it would **not** be possible to carry out a Wilcoxon signed-rank test but it would be possible to carry out a sign test.

(2 marks)

SS03

lQ.	Ho-Population Median = 0
	H Population redian 70 (before - after) 2 tailed test at 5% (before - after)
	ronk + / =
	-3 4
	2 2.5
	-4 5
	-8 8
	total = 32.5
	total + = 3.5
	test statistic = 3.S.
1	Critical value = 4.
	Lest statistic < 4
	Applation Median obes not = 0 and that there
	is a difference between the two before and affer
-	Testoane.
b	1) & More depth.
) If the data was not numerical, it would
Ĺ	i la marila la rise a wilcoror signed

SS03

Many candidates made a very good effort at this question and the majority showed the differences between pairs of values and the rank values used. Several incorrectly ranked -8 with rank 1. Candidates should understand the difference with the smallest absolute value is assigned rank 1.

Hypotheses were usually correctly stated and conclusions were generally fairly well done and in context. The solution shown for part (a) gained full marks and is clearly laid out. In part (b) (i) most candidates had a good idea of the required comment but many did not express themselves clearly enough to gain the mark.

In part (b) (ii) there were some excellent solutions with clear examples given but many candidates gained only 1 mark as they were not specific enough in their explanation. This candidate did not clearly express the reason or give an example.

1(a)	H_o pop median/mean diff $\eta_d = 0$	B1		
	H ₁ pop median/mean diff $\eta_d \neq 0$			
	1 tail 5% (d is after – before)			For differences (before–after)
	diff 3 7 -2 5 -1	M 1		or (after - before) - ignore
	rank 4 7 -2 ¹ / ₂ 6 -1			signs
		M 1		For ranks
		m1		For ties
	rank 5 2½ 8			
	$T_{+} = 3 + 7 + \dots + 8 = 32\frac{1}{2}$	m1		For total attempted
	$1_{-}=2\frac{1}{2}+1=3\frac{1}{2}$	A1		For one correct total
	Test stat $T = 3\frac{1}{2}$ $n = 8$ 1 tail 5%	111		
	n = 8 cv = 4	B1		For cv
	1 < 4	MI		Comparison cv/ts
	Significant evidence at 5% level to reject H and conclude that the average	E 1		In contact
	resistance differs after the adjustment	EI		III context
	(higher)		_	
(b)(i)	Wilcoxon signed rank test takes into		9	
(0)(1)	account the magnitude of the	E1		
	differences not simply whether they are		1	
	+ OF –			
(ii)	When the data is not symmetrically			
	distributed so Wilcoxon signed-rank	54		~
	cannot de carried out.	BI E1		Correct reasoning and explained well
	Data given only as signs/preferences so		2	enplumed wen
	only sign test possible – no numerical			
	differences can be evaluated			

Question 2

. A road safety organisation obtained the annual number of road deaths, x per 100 000 of the population, and the number of motor vehicles, y per 1000 of the population, for countries in the EU.

The table gives the results for a random sample of 10 countries in the EU.

Country	Α	В	С	D	Ε	F	G	Η	Ι	J
x	5.9	6.1	6.3	8.0	8.4	10.2	10.5	12.8	14.8	19.3
у	559	528	518	650	487	607	754	597	496	480

(a) Calculate the value of Spearman's rank correlation coefficient between *x* and *y*.

marks)

(b) Carry out a hypothesis test, at the 10% level of significance, to determine whether the value you calculated in part (a) indicates an association between the annual number of road deaths per 100,000 of the population and the number of motor vehicles per 1,000 of the population for countries in the EU.

(5 marks)

(6

	van	c	un, up,				
	(Concession)	r,	C2	d	d²	31462	and a state
Za	A	1	6	-5	25	acert .	1
	B	Z	5	-3	9	(152. X 18)	1 - 194
	C	3	4	-1	5 31 - 5	5 1 au ^{2*}	а
	b	4	9	-5	25		5-6-2
	E	5	2	3	9	Topola	2010
	F	6	8	-2	4	1 1 1 6 5 5 1	100
	05	7	. 10	-3	9		< 5
	H	8	7	s - 3 1	l	345.52	
	I	ч	3	6	36	and water to	Sec. 4
	J	10	1	9	81	anthe TVT T	in the
		mini	5 10/07	= 0	200	ALL LOS YEAR	60 m
	Hory	720	\$ r.=	1- 6Ed2	1 = 1.	64200	= 1 - 1.21
	An	2 :	- 0.21	2/2/		10 5 (10 3-1)	
				1		4-2	
5	Ho:	0=0	/ _				1
	H.:	Pto 6	two tail	ed tast	00=0	2.10	
	C. U :	:0.60	IST BO	1-	xt t	+	+
	-0	.21217	-2.8485 N	NO Dan	: + :	-	
	acce	ot Ha	reject 6	I, K	2	a human to all	10 10 1
	no si	ani fic	arterido	nce at u	in the	T DED	a hant
	there	is not	encrucho	midence	ta sa	that the	10 1. 0
	ossosi	ation	between	num ber	ch coo	deaths	curd -
	ct w	oter.	repicter.		1 100	El	A
	BLATT			Ren Bar	Leuters	EL	N
	1.10						

Part (a) was answered correctly by many candidates but a significant number found the product moment correlation coefficient in error. Some candidates successfully obtained the coefficient from a calculator but many detailed the use of the formula. Ranks were generally quoted thus gaining method marks even if the final answer was incorrect. The candidate shown gave all ranks and method and gained full marks.

In part (b), the critical value was usually quoted correctly but often candidates compared a negative correlation coefficient with the positive critical value. This candidate obtained an incorrect critical value although a comparison between a negative ts with a negative cv was made.

Conclusions were often wrong indicating a lack of understanding of the critical region. The conclusion in context stated often did not make sense, for example "road deaths are not associated with cars". This candidate displays excellent wording for the conclusion in context.

2. (a)				
	Country A B C D E x ronk 1 2 2 4 5			attempt at ranks
	x rank = 1 2 3 4 5	M1		(can be reversed)
	Y Tank 0 5 4 9 2 Country F G H I J	M1		for 16 correct
	$x \operatorname{rank} 6 7 8 9 10$	1011		for to contect
	yrank 8 10 7 3 1	A1		
				alternative
				<i>d</i> = 5, 3, 1, 5, 3, 2, 3, 1, 6, 9
				$\sum d^2 = 200 \qquad \qquad B1$
	r = -0.212(3 sf from calc)			6×200
	$r_{s}^{2} = 0.212(3.51 \text{ Hom cale})$	B3		$r_s = 1 - \frac{10 \times 99}{10 \times 99} = 1 - 1.212$
			6	= - 0.212 M1, A1
(b)				
(U)	H_0 Rank orders of annual road deaths			
	and number of motor vehicles are	B1		or alternatives
	independent.			H _o No association
	U. Donk orders of annual road deaths			H ₁ Association
	H_1 Rank orders of annual road deams and number of motor vehicles are not			
	independent – there is an association			
	•			
	2 tail 10%			
	$cy = \pm 0.5636$ $n = 10.2$ tail 10%	D1		
	$ev = \pm 0.5650$ $n = 10.2$ tail 10/0	DI		
	test stat $r_s = -0.212$			
	$r_{s} > -0.5636$	M1		for cv
	Accept H No significant evidence at			
	10% level to suggest an association	A1		for comparison ts/cv
	between rank orders of annual road			$r_s = 0.212 / cv = 0.5636$
	deaths and number of motor vehicles	E1	5	$r_s = -0.212 / cv = -0.5636$
	for countries in the EU			

Question 3

. (a) A long term trial was carried out into the effectiveness of giving accident victims with serious head traumas a steroid drug in addition to other treatments. In the trial, 1061 victims were randomly assigned to be given the steroid drug and the remainder were given a drug with no active ingredient (a placebo).

The victims either died as a result of their injuries or survived.

The results of the trial are summarised in Table 1.

Table 1	Additional tr giver		
Outcome	Steroid Drug	Placebo	Total
Died	396	422	818
Survived	665	665	1330

Carry out a test, using the 5% level of significance, to investigate whether the survival of accident victims with serious head traumas is independent of the additional treatment given.

(10 marks)
 (b) A trial was carried out into the effectiveness of a new anaesthetic drug. A sample of 500 patients undergoing a minor operation volunteered for the trial. Of these patients, 250 were randomly assigned to be given the standard anaesthetic drug and the remaining 250 were given the new anaesthetic drug.

The level of consciousness of each patient, 30 minutes after the operation was completed, was recorded as unconscious, semi-conscious or fully conscious. The **percentages** of patients in these levels of consciousness, for those given the standard anaesthetic drug and for those given the new anaesthetic drug are shown in Table 2.

Table 2	Anaesthetic drug used						
Level of Consciousness	Standard (percentage)	New (percentage)					
Unconscious	52	36					
Semi-conscious	36	46					
Fully conscious	12	18					

(i) Using the 1% level of significance, carry out a χ^2 test for association between the drug used and the level of consciousness 30 minutes after the operation was completed.

(10 marks)

(ii) Interpret your conclusion in part (a)(i) in the context of the question.

(2 marks)

300	Ho = Sur	vival of vi	ctims with 1	need transm	a is independent	ct additional	1 treatmen
	HI, = SUIUI	uch of victin	ns with necci	traumas is	, not independent "	of additional	trechnent
	Two faile	d test at .	sulo signific	ance.	\checkmark		
							je.
		Steriod	Placebo	Total			
	Died	396	422	818			
	Survived	665	665	1330			
	Total	1061	F801	2148			
	observed	Expected	(10-	EI-0.5)2			
	396	404.05	; ;	E 0.14	11		
	665	656.95		U.087 L	/		
	422	413,95	1.	0.137			
	665	637.0	s	1.183			
				1.548. f	10		
	Test stat	istic = 1.50	48 1	Degrees of 1	iveedom = 1		
ć	Criticay	value = 3.8	41 BIM	1 20			
	1.548 = 3	3.841 . 4	ne accept	to as the	re is significant	evidence ho	
	suggest -	mat the si	ruivell of u	ictims wit	n head how ma	is independ	lent
	of the	additional	treatment.		EI M		25.5

Two failed	test at 1º10	significance le	vel		24	Х
Observed	Expected	5 (0-E)2				
52	44	1.4545				
36	41	0.698	SC			
12	15	0.6	ml			
36	44	1.4545	m			
46	<u>ک</u> 4۱	0.6098				
81	15	0.6.				
		5= 5,3286		1	1	
Test stuti	stic = 5.3286	Degrees	of freedom	= 2	(
Critical 1	value = 9.21	v (21)	
S. 3286 <	9.210 50 0	ve Accept the	· m1		(
3						
lore is a	a ani lancer	m de let di		10 × + 0 + 0		

Many candidates stated the hypotheses correctly but often the null and alternative hypotheses were reversed in either part (a) or part (b) or both. Some nonsense statements were common, for example "Head trauma independent of treatment"

"Survival independent of death"

The candidate shown has incorrect, reversed hypotheses in part (b)

Expected frequencies in part (a) were usually correctly evaluated and a sensible attempt at a test statistic with use of Yates' correction was generally seen. Very few applied Yates' correction correctly with the majority finding (O - E - 0.5)². The solution shown has, in error, used (O - E - 0.5)² for some elements of the test statistic and has obtained an incorrect answer. The conclusion shown for part (a) is clear, correct and in context.

In part (b) (i) there were a few excellent solutions but many candidates simply carried out a

 χ^2 test for association using the percentages given and made no effort to evaluate the actual frequencies. This is shown in the solution given where expected values of 44, 41 and 15 are seen.

In part (b) (ii) few candidates referred to observed and expected frequencies to identify a source of association. The conclusion seen in the example solution has incorrectly identified acceptance of the null hypothesis as meaning that there is an association but has not made any attempt to identify any source of that association.

3(a)	H_o No associated drug treatment H_1 Association and drug treatment drug tre	ion betwee used. exists betw nent used.	n survival veen surv	and ival	B1		
	1 tail 5%						
	Died Survived	Steroid 404.05 656.95	Placebo 413.95 673.05		M1 m1		E method All correct
	$ts = \sum \frac{(O - E)}{404.05} + \frac{7.55^2}{413}$	$\frac{\left -0.5\right ^{2}}{E} = \frac{5^{2}}{.95} + \frac{7.55}{.656}$	$\frac{5^2}{95} + \frac{7.5}{673}$	$\frac{55^2}{3.05}$	M1 m1		ts correct denominators Yates' correction
	= 0.456				A1		Range 0.4 – 0.5
	cv df = 1 5% ts < 3.841	41		B1 M1			
	Accept H _o No sig evidenc association bet whether or nor treatment is use	t an val and drug		A1 E1	10		
(b)(i)	 b)(i) H_o No association between the drug used and the level of consciousness H₁ An association exists between the drug used and the level of consciousness 1 tail 1% 						
					B1		
	Drug	Standard	New				
	Unconscious	130	90				
	Semi- conscious	90	115		M1		For attempt to find raw frequencies
	Fully conscious	30	45		A1		4 or more correct

	Expected frequ Drug Level Unconscious	New 110	M1 m1		For one E correct For all E correct ft if original % used	
	conscious Fully conscious	102.5 37.5	102.5 37.5			
	$ts = \sum \frac{(O - E)^2}{E}$ = $\frac{(130 - 110)}{110}$ = 13.3 df = 2 1% ts > 9.21	$\frac{110)^2}{0} +$	M1ft A1 B1 M1 A1	10	ts sum with correct denominators For ts in range 13.0 – 13.6 For cv For comparison ts/cv	
(ii)	Reject H _o Sig evidence to exists between consciousness drug are far les unconscious 30 operation was o versa).	o suggest an drug used a – patients g ss likely to) minutes af completed (a association and level of iven the new be fter their (and vice	E1 E1	2	Sensible correct interpretation in context. Sources of association identified correctly

Question 4

The nicotine content, in milligrams, is measured for a random sample of 16 king-size cigarettes each from a different brand. The brands are either categorised as 'Very Low Tar', 'Low Tar' or no claim is made about tar content.

Very Low Tar	Low Tar	No Claim Made
0.40	0.69	0.86
0.67	0.96	1.06
0.76	1.03	1.12
0.82	1.04	1.26
1.01	1.08	2.03
1.02		

The results are given in the table.

Carry out a distribution- free test, using the 5% significance level, to investigate whether there is any difference in the average nicotine content for cigarette brands categorized as 'Very Low Tar', 'Low Tar' or those for which no claim is made about tar content.

Interpret your conclusion in context.

(13 marks)

	B) Ó
4	Mo no difference in average nicotine content 5705.L.
	Hy is in in in in hootail
	Very Low Tar n=6 Low Tar n=5 No Claim Made a=5
	1 3 6
	2 7 12
	4 10 14
	S 11 15
_	8 13 16
	9
	T' = 29 $T' = 44$ $T'' = 63$
	$H = 12 (7^2 - 3(N+1))$
	$N(N+1) \leq n$
	$= 12 + 29^{2} + 44^{2} + 63^{2} - 3 \times 17$
	16×17 6 65 5
	= 7.287(3sf)
	Vegrees of freedom = 3-1 CV= \$m7.378 BO
	= 2
L	
	accept lieged Mg
1	
	1 2 18 /1 [11]
	Loncuision AD
	1. Los C 1. 5 18 . Mois accepted
	widence to suggest there is no difference in average nicotine
	content. El B
	N'EF

Candidates frequently incorrectly stated the hypotheses and, if referring to population medians, failed to mention that the alternative hypothesis should be that **at least two** of the average nicotine levels from the three cigarette brands differ. The solution shown illustrates this.

The Kruskal Wallis test was carried out successfully by many candidates as seen in the solution considered here but some candidates did not seem to have the confidence to start the test. Most candidates showed their rank values but many made errors in ranking. Critical values were frequently obtained from n = 16 rather than n = 3.

The solution shown has an incorrect cv but one from the correct tables with the correct degrees of freedom so gains a method mark for comparison with the test statistic. The conclusion was explained well in context and most candidates gained one mark. In this case the candidate has followed through an incorrect conclusion with a correct interpretation.

4	H_0 Samples an	e taken from i	identical			or			
	populations			B1					
	H_1 Samples at	e not taken fro	om identical						
	populations –	population av	erage nicotine			H ₀ $\eta_{VLow} = \eta_{Low} = \eta_{Noclaim}$			
	levels differ	1 1	U	B1		H_1 at least two of			
	5% 1 tail					$n_{1} = n_{1} = n_{1}$			
						do differ			
	Ranks					do unici			
	Very Low	Low Tar	No Claim						
	Tar		Made						
	1	3	6			Ranks			
	2	7	12	M1		At least 10 correct			
	4	10	14	m1					
	5	11	15						
	8	13	16						
	9								
				m1		Totals -any one correct			
	$T_{VLow} = 29$	$T_{Low} = 44$	$T_{No\ claim} = 63$	A1		ÿ			
	$n_{VLow} = 6$	$n_{Low} = 5$	$n_{No\ claim}=5$			 			
				m1		test stat $H =$			
	$m_{i}T_{i}^{2} 29^{2}$	2 44 ² 63	2						
	$\sum \frac{i}{i} = \frac{2i}{c}$	$-+\frac{11}{5}+\frac{05}{5}$	- = 1321.17			12 $m T^{2}$			
	$\overline{i=1}$ n_i 0	5 5		A1		$\frac{12}{N(N-1)}\sum_{i=1}^{n-1} - 3(N+1)$			
						$N(N+1) \stackrel{\sim}{\underset{i=1}{\leftarrow}} n_i$			
	н – <u>12</u>	(1321 17 (3	$\times 17) - 7.20$			7.0 -7.5			
	$11 - \frac{1}{16 \times 17}$	(1521.17-(5	×17) = 7.29						
	Critical value	from $\gamma_2^2 = 5$.991 5%	D1					
	H > 5.001	$\chi_2 = c$		BI M1					
	11 / J.991			IVI I					
	Sig evidence t	o reject H. an	d conclude	A 1		Difference in context			
	that samples a	re not from ide	entical	AI					
	nonulations		cintical						
	populations.								
	Significant ex	vidence at the	5% level to			Mention of 'at least two' or a			
	suggest that the	ne population	average	F1		sig difference between			
	nicotine level	differs for the	three			nicotine levels of king-			
	categories of	king-size ciga	rettes.			sizecigarettes for which no			
				1					

It appears that those king-size cigarettes			claim made and those claimed
that have no claim made about tar levels	E1		to have 'Very Low Tar'
have a significantly higher average			to have very how rul .
nicoting level then these claimed to have			
We way Lease Ter?			
very Low Tar .			
		13	

Question 5a

The LDL cholesterol level was measured for each of 16 males living in the USA in 2006. Of these, 8 had been randomly selected from males aged under 30 years and 8 had been randomly selected from males aged over 50 years.

The age and the LDL cholesterol level, in mg/dl, for each male are given in the table.

Male	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Age	29	18	29	28	23	19	21	27	56	54	51	52	71	65	54	76
LDL	121	137	140	159	177	189	191	201	181	196	225	228	234	249	259	339

(a) Carry out a Mann-Whitney *U* test, at the 5% level of significance, to investigate whether, in the USA, males aged under 30 years have, on average, a lower LDL cholesterol level than those aged over 50 years.

(10 marks)

(5a) Ho!	Samples of Identical	populato	from r.s.
	Samplos o identical Under 30 LDL Cr	ve not to populat o Liebro L notestoron L	iken from chs - males nave a lower touel.
nan Mal	d=0.05 k order. e age	one-tai LDL	led.
1 2 3 4	7.5 1. 7.5 6		
5 6 7 8 9	<u>4</u> 2 3 5 r	M1 /	M1 groups
10 10 11 12		raules 9 11 12 13	
		14 15 16	
Toro	$\frac{1}{N^{\mu}} = \frac{1}{N^{\mu}}$	\$ 36 TLON	= 8 MI

SS03

Test Star = ()-5 n 5 -MI 8 60 --2 1250 AD -BO K MO 10 Stat CC.V est 21 D RN 0 Co do VaCIO 0 0 0 01 020 tin d over 12cr OV 5

Commentary

In part (a), few students had the confidence to separate the two age groups and carry out the required The Mann-Whitney test on the LDL levels.

Some candidates made an attempt to sort the data into two groups and some made an effort to rank the data as one group but frequently the ages were ranked as well or were ranked as one group with the LDL levels. This is seen in the solution here.

Hypotheses were well worded in most cases as in the given solution but very few totally correct answers were seen.

5(a)	H _o Samples are taker populations H ₁ Samples are not ta populations (males ag have lower average L 1 tail 5%	n from identical ken from identical ged under 30 years .DL)	B1		Hypotheses referrring to population averages also acceptable
	Under 30 ranks 1 2 3 4 5 7 8 10	Over 50 ranks 6 9 11 12 13 14 15 16	M1 M1		Successful separation of age groups Attempt at M-Whitney - ranks as one group
	$T_G = 1 + 2 + \dots + 10$	= 40	M1		for total attempt
	$T_{R} = 6 + 9 + \dots + 16$ $U_{G} = 40 - \frac{8 \times 9}{2} = 40$ $U_{R} = 96 - \frac{8 \times 9}{2} = 60$	= 96 4 50	M1		for U formula correct
	Test stat U = 4		A1		
	cv = 16 $n = 8$ $m = 8$	3 1 tail 5%	B1		correct/relevant cv used
	U = 4 < 16		M1	10	
	Reject H _o Significant evidence suggest that the avera lower for males aged	at the 5% level to age LDL level is under 30 years.	A1 E1		In context

Question 5b

The median LDL cholesterol level, for males aged between 35 years and 64 years living in the USA, is known to be 223 mg/dl.

A random sample of 9 males, aged between 35 years and 64 years, living in China, each had their LDL level, in mg/dl, measured with the following results:

158 225 164 178 182 184 191 195 231

Carry out a sign test, at the 10% level of significance, to investigate the claim that the median LDL cholesterol level for males aged between 35 years and 64 years is greater for those living in the USA than for those living in China. Interpret your conclusion in context.

(7 marks)



Some excellent solutions were seen in part (b) and the majority of candidates quoted the binomial probability of 0.0898 and showed a comparison with 0.10. The solution shown gives the correct value of 0.898 from the binomial tables but compares to a 2 tail significance level of 5%.

Candidates lost marks if probabilities from the binomial tables were not stated or a critical region was identified without the relevant probability being quoted.

The hypotheses were frequently stated incorrectly with H₁ η > 223 commonly seen as in the solution given here.

Conclusions were often incorrectly stated or poorly worded

5(b)	$H_0 \eta = 223$ $H_1 \eta < 223$ 1 tail 10%	B1		
	Signs - + + $2^+ / 7^-$ signs – test values Binomial (9, 0.5) model $P (\ge 7 -) = P(\le 2 +) = 0.0898 < 0.10$ for one tail test	M1 A1 M1 M1		signs test stat correct and identified Binomial model used and probability attempted Comparison of Binomial probability with 0.10
	Reject H_o . There is sufficient evidence, at the 10% level, to suggest that the median LDL level is greater for males aged 35 to 64 years living in the USA than that for those living in China.	A1 E1	7	Interpretation in context