# Teacher Support Materials 2008 

## Maths GCE

## Paper Reference SSO3

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Dr Michael Cresswell, Director General.

## Question 1

A manufacturer of an electrical appliance wants to adjust one of the components used in the appliance. The effect that the adjustment would have on the resistance of the component is investigated.

The manufacturer selects, at random, 8 components. Each component has its resistance, in ohms, measured before and after the adjustment.

The results of the investigation are shown in the table.

| Component | A | B | C | D | E | F | G | H |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Before | 38 | 42 | 44 | 35 | 44 | 36 | 44 | 42 |
| After | 41 | 49 | 42 | 40 | 43 | 40 | 46 | 50 |

(a) Carry out a Wilcoxon signed-rank test, at the 5\% significance level, to investigate whether or not the average resistance of the component is changed by the adjustment. Interpret your conclusion in context.
(9 marks)
(b) (i) Give one reason why a Wilcoxon signed-rank test might be preferred to a sign test in carrying out an investigation similar to the one carried out in part (a).
(1 mark)
(ii) Describe one situation in which it would not be possible to carry out a Wilcoxon signed-rank test but it would be possible to carry out a sign test.
la. $H_{0}$-Population Median $=0$
$H_{1}$ - Population median $\neq 0$
2 tailed test at $5 \%$ (before - after)

|  | + | rank |  |
| :---: | :---: | :---: | :---: |
| -3 |  | 4 |  |
| -7 |  | 7 |  |
| 2 | 2.5 |  |  |
| -5 |  | 6 |  |
| 1 | 1 | 5 |  |
| -4 |  | 2.5 |  |
| -2 |  | 8 |  |
| -8 |  | 3.5 | 32.5 |

$$
\text { total }^{-}=32.5
$$

$$
\text { total }{ }^{+}=3.5
$$

lest Statistic $=3.5$.

$$
\text { Critical value }=4 \text {. }
$$

test statistic < 4
Ho is rejected significant evidence that the Population median oles not $=0$ and that there is a difference between before get after resistant.
bi) : I more depth.
ii) If the data was not numerical, it would not be possible ta case a wicoxon signed pork test.

## Commentary

Many candidates made a very good effort at this question and the majority showed the differences between pairs of values and the rank values used. Several incorrectly ranked -8 with rank 1. Candidates should understand the difference with the smallest absolute value is assigned rank 1.
Hypotheses were usually correctly stated and conclusions were generally fairly well done and in context. The solution shown for part (a) gained full marks and is clearly laid out. In part (b) (i) most candidates had a good idea of the required comment but many did not express themselves clearly enough to gain the mark.
In part (b) (ii) there were some excellent solutions with clear examples given but many candidates gained only 1 mark as they were not specific enough in their explanation. This candidate did not clearly express the reason or give an example.

## Mark scheme

1(a)

$\mathrm{T}_{+}=3+7+\ldots+8=321 / 2$
$\mathrm{T}_{\text {- }}=21 / 2+1=31 / 2$

Test stat T $=31 / 2 \quad n=81$ tail $5 \%$
$\mathrm{n}=8 \mathrm{cv}=4$
$\mathrm{T}<4$

Significant evidence at 5\% level to reject $H_{o}$ and conclude that the average resistance differs after the adjustment ( higher)
(b)(i) Wilcoxon signed rank test takes into account the magnitude of the
differences not simply whether they are + or -
(ii) When the data is not symmetrically distributed so Wilcoxon signed-rank cannot be carried out.

Data given only as signs/preferences so only sign test possible - no numerical differences can be evaluated

In context

Correct reasoning and explained well

## Question 2

. A road safety organisation obtained the annual number of road deaths, $x$ per 100000 of the population, and the number of motor vehicles, $y$ per 1000 of the population, for countries in the EU.
The table gives the results for a random sample of 10 countries in the EU.

| Country | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{H}$ | $\mathbf{I}$ | $\mathbf{J}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{x}$ | 5.9 | 6.1 | 6.3 | 8.0 | 8.4 | 10.2 | 10.5 | 12.8 | 14.8 | 19.3 |
| $\boldsymbol{y}$ | 559 | 528 | 518 | 650 | 487 | 607 | 754 | 597 | 496 | 480 |

(a) Calculate the value of Spearman's rank correlation coefficient between $x$ and $y$.

## marks)

(b) Carry out a hypothesis test, at the $10 \%$ level of significance, to determine whether the value you calculated in part (a) indicates an association between the annual number of road deaths per 100,000 of the population and the number of motor vehicles per 1,000 of the population for countries in the EU.


## Commentary

Part (a) was answered correctly by many candidates but a significant number found the product moment correlation coefficient in error. Some candidates successfully obtained the coefficient from a calculator but many detailed the use of the formula. Ranks were generally quoted thus gaining method marks even if the final answer was incorrect. The candidate shown gave all ranks and method and gained full marks.
In part (b), the critical value was usually quoted correctly but often candidates compared a negative correlation coefficient with the positive critical value. This candidate obtained an incorrect critical value although a comparison between a negative ts with a negative cv was made.
Conclusions were often wrong indicating a lack of understanding of the critical region. The conclusion in context stated often did not make sense, for example "road deaths are not associated with cars". This candidate displays excellent wording for the conclusion in context.

## Mark scheme

| 2.(a) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Country | A | B | C | D | E | M1 |  | attempt at ranks (can be reversed) |
|  | $x$ rank | 1 | 2 | 3 | 4 | 5 |  |  |  |
|  | $y$ rank | 6 | 5 | 4 | 9 | 2 |  |  |  |
|  | Country | F | G | H | I | J | M1 |  | for 16 correct |
|  | $x$ rank | 6 | 7 | 8 | 9 | 10 |  |  |  |
|  | $y$ rank | 8 | 10 | 7 | 3 | 1 | A1 |  |  |
|  |  |  |  |  |  |  |  |  | alternative $\begin{array}{cc} d=5,3,1,5,3,2,3,1,6,9 \\ \sum d^{2}=200 & \text { B1 } \end{array}$ |
|  | $\mathrm{r}_{\mathrm{s}}=-0.212(3 \mathrm{sf}$ from calc $)$ |  |  |  |  |  | B3 |  | $r_{s}=1-\frac{6 \times 200}{10 \times 99}=1-1.212$ |
|  |  |  |  |  |  |  |  | 6 | $=-0.212 \mathrm{M} 1, \mathrm{~A} 1$ |
| (b) | $\mathrm{H}_{\mathrm{o}}$ Rank orders of annual road deaths and number of motor vehicles are independent. |  |  |  |  |  | B1 |  | or alternatives $\mathrm{H}_{\mathrm{o}}$ No association $\mathrm{H}_{1}$ Association |
|  | $\mathrm{H}_{1}$ Rank orders of annual road deaths and number of motor vehicles are not independent - there is an association |  |  |  |  |  |  |  |  |
|  | 2 tail 10\% |  |  |  |  |  |  |  |  |
|  | $\mathrm{cv}= \pm 0.5636 \quad n=102$ tail $10 \%$ |  |  |  |  |  | B1 |  |  |
|  | $\begin{aligned} \text { test stat } \mathrm{r}_{\mathrm{s}} & =-0.212 \\ \mathrm{r}_{\mathrm{s}} & >-0.5636\end{aligned}$ |  |  |  |  |  | M1 |  | for cv |
|  | Accept $\mathrm{H}_{\mathrm{o}}$ No significant evidence at |  |  |  |  |  |  |  |  |
|  | $10 \%$ level to suggest an association between rank orders of annual road |  |  |  |  |  | A1 |  | for comparison ts/cv $\mathrm{r}_{\mathrm{s}}=0.212 / \mathrm{cv}=0.5636$ |
|  | deaths and number of motor vehicles for countries in the EU |  |  |  |  |  | E1 | 5 | $\mathrm{r}_{\mathrm{s}}=-0.212 / \mathrm{cv}=-0.5636$ |

## Question 3

(a) A long term trial was carried out into the effectiveness of giving accident victims with serious head traumas a steroid drug in addition to other treatments.
In the trial, 1061 victims were randomly assigned to be given the steroid drug and the remainder were given a drug with no active ingredient (a placebo).
The victims either died as a result of their injuries or survived.
The results of the trial are summarised in Table 1.

| Table 1 | Additional treatment <br> given |  |  |
| :--- | :--- | :--- | :--- |
| Outcome | Steroid <br> Drug | Placebo | Total |
| Died | 396 | 422 | $\mathbf{8 1 8}$ |
| Survived | 665 | 665 | $\mathbf{1 3 3 0}$ |

Carry out a test, using the $5 \%$ level of significance, to investigate whether the survival of accident victims with serious head traumas is independent of the additional treatment given.

## (10 marks)

(b) A trial was carried out into the effectiveness of a new anaesthetic drug. A sample of 500 patients undergoing a minor operation volunteered for the trial. Of these patients, 250 were randomly assigned to be given the standard anaesthetic drug and the remaining 250 were given the new anaesthetic drug.

The level of consciousness of each patient, 30 minutes after the operation was completed, was recorded as unconscious, semi-conscious or fully conscious. The percentages of patients in these levels of consciousness, for those given the standard anaesthetic drug and for those given the new anaesthetic drug are shown in Table 2.

| Table 2 | Anaesthetic drug used |  |
| :--- | :---: | :---: |
| Level of <br> Consciousness | Standard <br> (percentage) | New <br> (percentage) |
| Unconscious | 52 | 36 |
| Semi-conscious | 36 | 46 |
| Fully conscious | 12 | 18 |

(i) Using the $1 \%$ level of significance, carry out a $\chi^{2}$ test for association between the drug used and the level of consciousness 30 minutes after the operation was completed.
(ii) Interpret your conclusion in part (a)(i) in the context of the question.

## Student Response

| 300) | $H_{0}$ = Survival of victims with hedd trawma is independent a additional treatment |
| :---: | :---: |
|  | $H_{1}$ = Suiviocal of victims with neacitrawmas is not inclependent of additional treatment |
|  | Two failed test at soio significance. |
|  |  |
|  | Steriod Placebo Total |
|  | Died $396 \quad 422 \quad 818$ |
|  | Survived 6656651330 |
|  | $\begin{array}{llll}\text { Total } & 1001 & 1087 & 2148\end{array}$ |
|  |  |
|  | Obscived Expucted ( $10-E 1-0.5)^{2}$ |
|  | $396 \quad 404.05 \quad E \quad 0.141$ |
|  | $665 \quad 656.95 \quad 0.087$ |
|  | 422 413.95, 0.137 |
|  | $665 \quad 637.05 \quad 1.183$ |
|  | 1.548. AO |
|  | Test statistic $=1.548 \quad$ Degrees of preedom $=1$ |
|  | Critical value $=3.841 \mathrm{Bl}$ MI ®o |
|  | $1.548<3.841 \therefore$ we accept tio as tere is significant evidence to |
|  | suggest that the sivrvival of victioms with head trowma is indey endent |
|  | of the additional treatment. EIM1 |



## Commentary

Many candidates stated the hypotheses correctly but often the null and alternative hypotheses were reversed in either part (a) or part (b) or both. Some nonsense statements were common, for example "Head trauma independent of treatment"
"Survival independent of death"
The candidate shown has incorrect, reversed hypotheses in part (b)
Expected frequencies in part (a) were usually correctly evaluated and a sensible attempt at a test statistic with use of Yates' correction was generally seen. Very few applied Yates' correction correctly with the majority finding $(\mathrm{O}-\mathrm{E}-0.5)^{2}$. The solution shown has, in error, used $(\mathrm{O}-\mathrm{E}-0.5)^{2}$ for some elements of the test statistic and has obtained an incorrect answer. The conclusion shown for part (a) is clear, correct and in context.

In part (b) (i) there were a few excellent solutions but many candidates simply carried out a $\chi^{2}$ test for association using the percentages given and made no effort to evaluate the actual frequencies. This is shown in the solution given where expected values of 44, 41 and 15 are seen.
In part (b) (ii) few candidates referred to observed and expected frequencies to identify a source of association. The conclusion seen in the example solution has incorrectly identified acceptance of the null hypothesis as meaning that there is an association but has not made any attempt to identify any source of that association.

## Mark scheme



|  | Expected frequencies |  |  | $\begin{aligned} & \text { M1 } \\ & \text { m1 } \end{aligned}$ |  | For one E correct For all E correct ft if original \% used |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { Drug } \\ & \text { Level } \end{aligned}$ | Standard | New |  |  |  |
|  | Unconscious | 110 | 110 |  |  |  |
|  | Semi- conscious | 102.5 | 102.5 |  |  |  |
|  | Fully <br> conscious | 37.5 | 37.5 |  |  |  |
|  | $\begin{aligned} \mathrm{ts} & =\sum \frac{(O-E)}{E} \\ & =\frac{(130-110}{110} \end{aligned}$ | $-+\frac{(90-}{1}$ | $\frac{0)^{2}}{}+. .$ | M1ft <br> A1 | 10 | ts sum with correct denominators <br> For ts in range $13.0-13.6$ <br> For cv <br> For comparison ts/cv |
|  | $\begin{aligned} & \mathrm{df}=2 \quad 1 \% \\ & \mathrm{ts}>9.21 \end{aligned}$ | $v=9.21$ |  | B1 <br> M1 <br> A1 |  |  |
| (ii) | Sig evidence to suggest an association exists between drug used and level of consciousness - patients given the new drug are far less likely to be unconscious 30 minutes after their operation was completed ( and vice versa). |  |  | $\begin{aligned} & \text { E1 } \\ & \text { E1 } \end{aligned}$ | 2 | Sensible correct interpretation in context. <br> Sources of association identified correctly |

## Question 4

The nicotine content, in milligrams, is measured for a random sample of 16 king-size cigarettes each from a different brand. The brands are either categorised as 'Very Low Tar', 'Low Tar' or no claim is made about tar content.

The results are given in the table.

| Very Low Tar | Low Tar | No Claim Made |
| :---: | :---: | :---: |
| 0.40 | 0.69 | 0.86 |
| 0.67 | 0.96 | 1.06 |
| 0.76 | 1.03 | 1.12 |
| 0.82 | 1.04 | 1.26 |
| 1.01 | 1.08 | 2.03 |
| 1.02 |  |  |
|  |  |  |

Carry out a distribution- free test, using the $5 \%$ significance level, to investigate whether there is any difference in the average nicotine content for cigarette brands categorized as 'Very Low Tar', 'Low Tar' or those for which no claim is made about tar content.

Interpret your conclusion in context.
(13 marks)


## Commentary

Candidates frequently incorrectly stated the hypotheses and, if referring to population medians, failed to mention that the alternative hypothesis should be that at least two of the average nicotine levels from the three cigarette brands differ. The solution shown illustrates this.
The Kruskal Wallis test was carried out successfully by many candidates as seen in the solution considered here but some candidates did not seem to have the confidence to start the test. Most candidates showed their rank values but many made errors in ranking.
Critical values were frequently obtained from $n=16$ rather than $n=3$.
The solution shown has an incorrect cv but one from the correct tables with the correct degrees of freedom so gains a method mark for comparison with the test statistic.
The conclusion was explained well in context and most candidates gained one mark. In this case the candidate has followed through an incorrect conclusion with a correct interpretation.

## Mark scheme



|  | It appears that those king-size cigarettes <br> that have no claim made about tar levels <br> have a significantly higher average <br> nicotine level than those claimed to have <br> 'Very Low Tar'. | E1 |  | claim made and those claimed <br> to have 'Very Low Tar'. |
| :--- | :--- | :--- | :--- | :--- |

## Question 5a

The LDL cholesterol level was measured for each of 16 males living in the USA in 2006.
Of these, 8 had been randomly selected from males aged under 30 years and 8 had been randomly selected from males aged over 50 years.

The age and the LDL cholesterol level, in $\mathrm{mg} / \mathrm{dl}$, for each male are given in the table.

| Male | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ | $\mathbf{1 5}$ | $\mathbf{1 6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Age | 29 | 18 | 29 | 28 | 23 | 19 | 21 | 27 | 56 | 54 | 51 | 52 | 71 | 65 | 54 | 76 |
| LDL | 121 | 137 | 140 | 159 | 177 | 189 | 191 | 201 | 181 | 196 | 225 | 228 | 234 | 249 | 259 | 339 |

(a) Carry out a Mann-Whitney $U$ test, at the $5 \%$ level of significance, to investigate whether, in the USA, males aged under 30 years have, on average, a lower LDL cholesterol level than those aged over 50 years.
(10 marks)

Student Response



Commentary
In part (a), few students had the confidence to separate the two age groups and carry out the required The Mann-Whitney test on the LDL levels.
Some candidates made an attempt to sort the data into two groups and some made an effort to rank the data as one group but frequently the ages were ranked as well or were ranked as one group with the LDL levels. This is seen in the solution here.
Hypotheses were well worded in most cases as in the given solution but very few totally correct answers were seen.

| 5(a) | $\mathrm{H}_{0}$ Samples are tak populations $\mathrm{H}_{1}$ Samples are not populations (males have lower average <br> 1 tail 5\% | from identical <br> from identical d under 30 years L) | B1 |  | Hypotheses referrring to population averages also acceptable |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Under 30 ranks | Over 50 ranks | M1 |  | Successful separation of age groups |
|  | $\begin{array}{r} 1 \\ \hline 2 \end{array}$ | 6 |  |  |  |
|  | 3 | 11 | M1 |  | Attempt at M-Whitney - ranks as one group |
|  | 4 | 12 |  |  |  |
|  | 5 | 13 |  |  |  |
|  | 8 | 15 |  |  |  |
|  | 10 | 16 |  |  |  |
|  | $\begin{aligned} & \mathrm{T}_{\mathrm{G}}=1+2+\ldots \ldots+10=40 \\ & \mathrm{~T}_{\mathrm{R}}=6+9+\ldots .+16=96 \end{aligned}$ |  | M1 |  | for total attempt |
|  | $\begin{aligned} & \mathrm{U}_{\mathrm{G}}=40-\frac{8 \times 9}{2}=4 \\ & \mathrm{U}_{\mathrm{R}}=96-\frac{8 \times 9}{2}=60 \end{aligned}$ |  | M1 |  | for U formula correct |
|  | Test stat $\mathrm{U}=4$ |  | A1 |  |  |
|  | $\mathrm{cv}=16 \quad n=8 \quad m=$ | 1 tail 5\% | B1 |  | correct/relevant cv used |
|  | $U=4<16$ <br> Reject $\mathrm{H}_{\mathrm{o}}$ <br> Significant evidence at the 5\% level to suggest that the average LDL level is lower for males aged under 30 years. |  | M1 | 10 |  |
|  | Reject $\mathrm{H}_{\text {。 }}$ <br> Significant evidence at the 5\% level to suggest that the average LDL level is lower for males aged under 30 years. |  | A1 |  |  |
|  |  |  | E1 |  | In context |

## Question Sb

The median LDL cholesterol level, for males aged between 35 years and 64 years living in the USA, is known to be $223 \mathrm{mg} / \mathrm{dl}$.
A random sample of 9 males, aged between 35 years and 64 years, living in China, each had their LDL level, in $\mathrm{mg} / \mathrm{dl}$, measured with the following results:
$\begin{array}{lllllllll}158 & 225 & 164 & 178 & 182 & 184 & 191 & 195 & 231\end{array}$
Carry out a sign test, at the $10 \%$ level of significance, to investigate the claim that the median LDL cholesterol level for males aged between 35 years and 64 years is greater for those living in the USA than for those living in China. Interpret your conclusion in context.
(7 marks)

## Student Response



## Commentary

Some excellent solutions were seen in part (b) and the majority of candidates quoted the binomial probability of 0.0898 and showed a comparison with 0.10 . The solution shown gives the correct value of 0.898 from the binomial tables but compares to a 2 tail significance level of $5 \%$.
Candidates lost marks if probabilities from the binomial tables were not stated or a critical region was identified without the relevant probability being quoted.
The hypotheses were frequently stated incorrectly with $\mathrm{H}_{1} \eta>223$ commonly seen as in the solution given here.
Conclusions were often incorrectly stated or poorly worded

## Mark scheme



