Teacher Support Materials

Maths GCE

Paper Reference SS05

Question 1

1 A furnishing company stocks rolls of fabric for making curtains. When there is less than 2 metres of fabric left on a roll, the roll is discarded. The length of fabric left on a discarded roll is X metres, where X has a rectangular distribution over the interval [0, 2].

(a) Find the probability that the length of fabric left on a discarded roll is:

(i) between 0.8 metres and 1.2 metres; *(2 marks)*

(ii) exactly 1 metre. *(1 mark)*

(b) When the length of fabric left on a discarded roll is less than 0.6 metres, the fabric is thrown away; otherwise it is sold as a remnant.

(i) One day, three rolls are discarded. Assuming that the amounts of fabric left on the rolls are independent, find the probability that remnants can be sold from all three rolls. *(3 marks)*

(ii) The fabric left on a particular roll is long enough to be sold as a remnant. Find the probability that this remnant is less than 1 metre long. *(2 marks)*

Student Response

1a)

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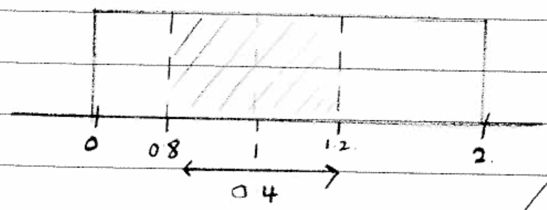


$0.4/0.2 = 0.2$

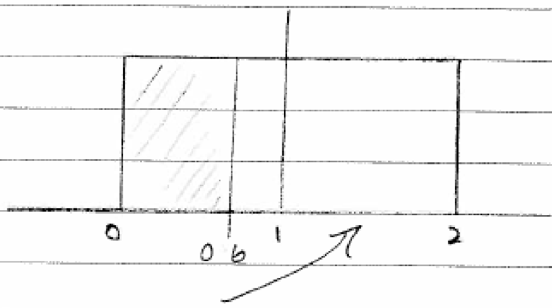
2

Leave blank

ii) $0.8/2 = 0.4 + 1/2(0.2)$

$\therefore = 0.5$ ✗

b) (i)



$0.6/2 = 0.3$

✓ M1

$\therefore (0.3)^3 = 0.027$

1

ii) 0.4 ✗

3

Commentary

Part (a)(i) was very well done and candidates either knew the answer to (a)(ii) or did not. Candidates found part (b) difficult, many missing it out completely. In (b)(i), a reasonable number realised that they needed to find a probability from the rectangular distribution and then raise it to the power of 3. The example shows the most common error. This candidate found the probability that a remnant could **not** be sold from a discarded roll – the opposite of what the question required. As in many cases, there was good use of a diagram to identify the probability to be found, so the error indicates that the question was not read with enough care.

A very small number of candidates recognised what was required in (b)(ii): Good answers were seen using the formula for conditional probability and recognising another rectangular distribution.

Mark scheme

SS05

Q	Solution	Marks	Total	Comments
1(a)(i)	$P(0.8 \le X \le 1.2) = \dfrac{1.2 - 0.8}{2 - 0}$	M1		
	$= \dfrac{0.4}{2} = 0.2$	A1	2	
(a)(ii)	$P(X = 1) = 0$	B1	1	
(b)(i)	$P(X \ge 0.6) = \dfrac{2-0.6}{2} = \dfrac{1.4}{2} = 0.7$	B1		
	P(all three ≥ 0.6)			
	$= (0.7)^3$	M1		probability raised to power of 3
	$= 0.343$	A1	3	CAO
(ii)	P(remnant less than 1 metre long)			
	$= \dfrac{1-0.6}{2-0.6} = \dfrac{0.4}{1.4}$	M1		
	= 0.286 (3 sf)	A1	2	
	or			
	$P(X < 1 \mid X \ge 0.6)$			
	$= \dfrac{P(0.6 \le X < 1)}{P(X \ge 0.6)} = \dfrac{0.2}{0.7}$	(M1)		
	= 0.286	(A1)	(2)	
	Total		**8**	

Question 2

2 Damien has agreed to raise money for charity. He displays a cake and charges entrants to guess its weight. The weights, x grams, guessed by a random sample of 10 entrants are summarised as follows:

$$\bar{x} = 446.9 \qquad s = 13.9$$

(a) Assuming that the weights guessed by all entrants are normally distributed, construct a 95% confidence interval for:

(i) the mean weight guessed; *(5 marks)*

(ii) the standard deviation of weights guessed. *(5 marks)*

(b) Damien states that, on average, people underestimate the weight of the cake. Its actual weight is 460 grams. Use the appropriate result from part (a) to comment on Damien's statement. *(2 marks)*

(c) Damien suspects that he made a mistake when he wrote down one guess as 350 grams. Use results from **both** the confidence intervals constructed in part (a) to explain why his suspicion is plausible. *(3 marks)*

Student response

2. $\bar{x} = 446.9$ $s = 13.9$ $s^2 = 193.21$

$n = 10$ $x \sim t_9(446.9, 13.9^2)$ $\bar{x} \sim t_9\left(446.9, \frac{13.9^2}{10}\right)$

a) ~~Mean~~ 95% C.I. (i) $446.9 \pm 2.262\sqrt{\frac{13.9^2}{10}}$ ✓ ✓ $[437, 457]$ ✓ 5

~~b)~~ SD 95% (ii) $\frac{(n-1)s^2}{\chi^2_{0.025}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{0.975}} =$ ✓ ✓

$\frac{9 \times 193.21}{2.700} < \sigma^2 < \frac{9 \times 193.21}{19.023} = [9.56, 25.4]$ ✓ 5

$\sqrt{644.03} = [25.4]$ ✓ $\sqrt{91.40987326} = [9.56]$ ✓

b) $\bar{x} = 460$? Damien claim doesn't ~~does~~ lie within the 95% C.I.

Its greater than 457 (upper band). $457 < 460$.

So the claim must be wrong, only based on this sample. ✗ 0

c) The ~~claim~~ claim doesnt fit the mean confience interval, it doesnt lie in between $[437, 457]$. ✗ 0

10

Commentary

Most candidates did very well on part (a) of this question and work was generally presented clearly and concisely. Parts (b) and (c) were much more demanding, though it was interesting to see some good answers from candidates who had made mistakes in constructing the confidence intervals.

This response to part (b) was typical of many candidates who had clearly not completely grasped what the confidence interval represented or possibly what was to be assessed in Damien's claim. The statement that the actual weight of the cake was 460 grams was a fact. It was his claim that, on average, entrants underestimated when they guessed its weight that candidates were asked to assess. Many were apparently interpreting the confidence interval as the range within which the true weight of the cake should lie.

In part (c) there was a common problem, as shown in this example, in recognising that 350 grams was a single value from the underlying population, so there was no reason why it should lie within the confidence interval for the mean. This question was quite demanding as it required candidates to select an appropriate value from each confidence interval and use their knowledge of the properties of a normal distribution to explain why 350 grams was likely to be a mistake.

Mark Scheme

SS05 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$\nu = 9$	B1		here or in (ii)
	$t = \pm 2.262$	B1		$S^2 \times \frac{10}{9}$: withhold last A mark in 1 part
	95% confidence limits for mean are:			
	$446.9 \pm 2.262 \times \frac{13.9}{\sqrt{10}}$	M1 m1		use of formula standard error
	95% confidence interval is: (437, 457) grams	A1	5	(436.9 to 437, 456.8 to 457)
(ii)	$\chi^2 = 2.700,\ 19.023$	B1		both
	95% confidence limits for variance are: $\frac{9\times 13.9^2}{19.023}, \frac{9\times 13.9^2}{2.700}$	M1 A1√		correct values substituted ft on incorrect x^2 values
	(95% CI is (91.410, 644.03)) 95% CI for standard deviation is: $\left(\sqrt{\frac{9\times 13.9^2}{19.023}}, \sqrt{\frac{9\times 13.9^2}{2.700}}\right)$	M1		
	= (9.56, 25.4) grams	A1	5	(9.5 to 9.6, 25.3 to 25.4) CAO
(b)	Damien's claim seems to be correct upper CL for mean is less than 460	B1 E1	2	must say **above** CI
(c)	taking lower CL for mean (437) and upper CL for SD (25.4) 350 is more than 3 SDs below mean making it plausible that Damien made a mistake	E1 E1 E1	3	SC E1 for plausible because 350 well below CI for mean
	Total		**15**	

Question 3

3 Sandeep is training for a marathon. Each weekday he runs the same route from his home in the morning and again in the evening. He records his time for each run. For random samples of 8 morning runs and 10 evening runs, his times, in minutes, are as follows:

Morning:	64.1	69.2	62.1	70.2	65.9	71.8	63.7	64.9		
Evening:	62.8	60.9	65.8	77.6	66.3	75.2	72.7	74.2	65.3	63.5

(a) Sandeep believes that his running times are more variable in the evening than in the morning. Carry out an F-test, at the 5% level of significance, to investigate his belief. Assume that Sandeep's running times for both morning and evening are normally distributed. *(9 marks)*

(b) On Saturdays, Sandeep trains at an athletics stadium. He completes a set number of laps on the running track in the morning and the same number of laps in the afternoon. His running times for both morning and afternoon are normally distributed, each with a standard deviation of 2.1 minutes.

For a random sample of 12 morning runs, Sandeep's mean running time is 61.7 minutes.

For a random sample of 9 afternoon runs, his mean running time is 58.9 minutes.

Sandeep's trainer claims that, on average, Sandeep completes the run **more than one minute** faster in the afternoon than in the morning. Carry out a hypothesis test, at the 5% significance level, to determine whether the evidence supports this claim. *(7 marks)*

Student Response

3 a) $H_0: \sigma^2_E = \sigma^2_m$ 1 tail B1

$H_1: \sigma^2_E > \sigma^2_m$ $\alpha = 0.05$

$S_E = 5.9198 \quad \therefore S^2_E = 35.044$ ✓ B1

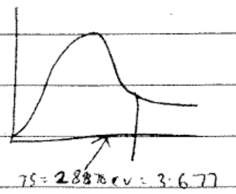
$S_m = 3.4836 \quad \therefore S^2_m = 12.135$ ✓ B1

$v_1\ n_m - 1 = 8 - 1 = 7$ ✓ $\quad \therefore CV = 3.677$ ✓ B1

$n_E - 1 = 10 - 1 = 9$ ✓ B1

$T.S = \dfrac{S^2_E}{S^2_m} = \dfrac{35.044}{12.135} = 2.8878$ ✓ A1

M1



Accept Ho ✓ no significant evidence to support Sandeep's belief that his running times are more variable in the morning. ✓ A1

8

Question number

Leave blank

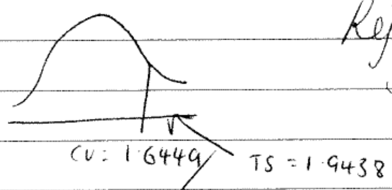
b) $H_0: M_A - M_m = 1$ } 1 tail

$H_1: M_A - M_m > 1$ } $\alpha = 0.05$ B1

$CV = 1.6449$.

$T.S = \dfrac{(61.7 - 58.9) - 1}{\sqrt{\dfrac{2.1^2}{12} + \dfrac{2.1^2}{9}}} = \dfrac{1.8}{\sqrt{0.8575}} = 1.9438$ (5SF)

M1 m1 A1



B1

Reject H_0, evidence that Sandeep completes the run more than one minute faster in the afternoon than the morning. A0

(Wrong Hypotheses)

5

13

Commentary

Part (a) of this question produced many excellent solutions, a large proportion scoring full marks.

There were many candidates who knew how to carry out the kind of hypothesis test examined in part (b), but did not set up the hypotheses correctly in the first place. This example shows a very common error in which 'faster in the afternoon than in the morning' was interpreted as 'mean running time in the afternoon greater than in the morning'. The test was then applied correctly, but the conclusion drawn was not consistent with the stated hypotheses. Some candidates attempted to substitute into the formula consistently with their hypotheses and obtained a negative test statistic which should have set alarm bells ringing, given the > sign in the alternative hypothesis.

Mark Scheme (next page)

SS05 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Morning: $s_X^2 = 12.136$ or $s_X = 3.48$	B1		12.1 to 12.2; AWRT
	Evening: $s_Y^2 = 35.045$ or $s_Y = 5.92$	B1		35.0 to 35.1; AWRT
	H_0: $\sigma_X^2 = \sigma_Y^2$	B1		or equivalent
	H_1: $\sigma_X^2 < \sigma_Y^2$	B1		
	Ratio of variances $= \dfrac{35.045}{12.136}$	M1		
	$= 2.89$ (or 0.346)	A1√		2.86 to 2.89 (0.344 to 0.349) ft on sample variances
	$\nu_1 = 9$; $\nu_2 = 7$	B1		both, either way round
	Critical value of $F = 3.677$	B1		accept 0.368 (0.271 to 0.272)
	(or $\dfrac{1}{3.677} = 0.272$)			if used H_1 with $\neq$ must have $F = 4.823$
	$2.89 < 3.677$ (or $0.346 > 0.272$) There is not sufficient evidence at the 5% level to support Sandeep's belief	A1√	9	ft on variance ratio and CV
(b)				μ_M, μ_A reversed, lose first B1 and last A1
	H_0: $\mu_M - \mu_A = 1$	B1		or equivalent
	H_1: $\mu_M - \mu_A > 1$	B1		
	CV of $z = 1.6449$	B1		If $H_1 \neq$ must have 1.96 accept 1.64, 1.645 or $P(Z>1.94) = 0.2619$
	sample value of $z = \dfrac{(61.7-58.9)-1}{2.1\sqrt{\dfrac{1}{9}+\dfrac{1}{12}}}$	M1		difference of means over sd
		m1		correct form of sd
	= 1.94	A1		CAO; AWRT
	$1.94 > 1.6449$ so reject H_0. There is sufficient evidence at the 5% level to support the trainer's claim	A1√	7	ft on sample value and CV
	Total		**16**	

Question 4

4 Adrian is a skilful badminton player. When he serves low, the height, in centimetres, at which the shuttle crosses over the top of the net may be modelled by an exponential distribution with parameter $\lambda = 0.4$.

(a) For one of Adrian's low serves, find the probability that the height at which the shuttle crosses over the top of the net is:

(i) less than 2 cm; *(2 marks)*

(ii) between 2 cm and 5 cm. *(2 marks)*

(b) Verify that, for Adrian's low serves, the median height at which the shuttle crosses over the top of the net is between 1.7 cm and 1.8 cm. *(4 marks)*

Student Response

1

4) $\lambda = 0 \cdot 4$ mean $= \frac{1}{0\cdot 4} = 2\cdot 5$ ✓

$p(x<2)$ $1 - e^{-\lambda x}$

~~$p(x \leq 1)$~~ $1 - e^{-(2\cdot5)\cdot 2} = 0\cdot 8358 \Rightarrow 0\cdot 84$ ✗ 0

(ii) $p(2<x<5)$

$1 - e^{-2\cdot5\cdot5} - 1 - e^{-2\cdot5\cdot2}$ M1 1

$= 0\cdot 84 - 0\cdot 59 = 0\cdot 25$ ✗

b) $e^{-\lambda}\frac{\lambda x}{x!}$ $1\cdot8 - 1\cdot7 = 0\cdot1$ $1 - e^{-2\cdot5\cdot0\cdot1} = 0\cdot99$ ✗ 0

1

Commentary

Although most candidates answered part (a) of this question very well, a significant minority were confused about the roles of the parameter λ and the mean 1/λ in the formula for probability. The example shows the candidate calculating the mean correctly but substituting it into the formula as if it was the parameter, in spite of writing down the formula correctly in the first place. It can be seen how a candidate who knows what is required and the basic method to be used can still lose a number of marks through a fundamental error like this.

In part (b), few candidates showed evidence that they knew how to relate the median value of a distribution to a probability. Those who recognised that a probability of 0.5 was required usually went on to solve the problem correctly.

Mark Scheme (next page)

SS05 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\mathrm{P}(X<2)=1-\mathrm{e}^{-0.4\times 2}$	M1		or by integration
	$=1-\mathrm{e}^{-0.8}=0.551$	A1	2	AWRT
(ii)	$\mathrm{P}(2 \le X \le 5)=\mathrm{F}(5)-\mathrm{F}(2)$			
	$=(1-\mathrm{e}^{-2})-(1-\mathrm{e}^{-0.8})$	M1		or by integration
	$=0.314$	A1	2	AWRT
(b)	for median m, $\mathrm{F}(m) = 0.5$ $(= 1 - \mathrm{F}(m))$	B1		may be implied
	$\mathrm{F}(1.7)=1-\mathrm{e}^{-0.68}=0.493$	B1		
	$\left(\mathrm{e}^{-0.68}=0.507\right)$			
	$\mathrm{F}(1.8)=1-\mathrm{e}^{-0.72}=0.513$	B1		
	$\left(\mathrm{e}^{-0.72}=0.487\right)$			
	0.5 lies between 0.493 and 0.513 so median lies between 1.7 and 1.8	E1	4	
	or			
	$\mathrm{e}^{-0.4m}=0.5$	(M1)		equation of correct form
	$-0.4m=\ln(0.5)$	(m1)		attempt to solve using logs
	$m=\dfrac{0.693}{0.4}=1.73$	(A1)		
	so median lies between 1.7 and 1.8	(E1)		solution used to answer question
	Total		**8**	

Question 5

5 (a) The continuous random variable X has a normal distribution with mean 310 and standard deviation 4. The following table shows probabilities for ranges of values of X.

Range of values	**Probability**
$X < 304$	a
$304 \leqslant X < 306$	b
$306 \leqslant X < 308$	0.1499
$308 \leqslant X < 310$	0.1915
$310 \leqslant X < 312$	0.1915
$312 \leqslant X < 314$	0.1499
$314 \leqslant X < 316$	c
$X \geqslant 316$	d

Calculate the values of a, b, c and d, giving your answers to four decimal places.
(4 marks)

(b) Mollie sells raspberries in small punnets. She weighs the contents of each of 100 punnets and obtains the following results.

Weight of raspberries (grams)	**Number of punnets**
Less than 304	5
304 –	13
306 –	10
308 –	18
310 –	25
312 –	20
314 –	5
316 or more	4

Mollie plans to model the weight, in grams, of raspberries per punnet by a normal distribution with mean 310 and standard deviation 4. Carry out a goodness of fit test, at the 10% significance level, to investigate whether her proposed model is suitable.
(8 marks)

(c) Mollie considers three possibilities for labelling a punnet of raspberries:

1 Average contents 310 grams

2 Minimum contents 300 grams

3 Minimum contents 305 grams

Assuming that the model proposed in part (b) is suitable, comment on **each** of these three possibilities. You should refer to both the given data and the proposed model where appropriate.
(4 marks)

Student Response

Question number

Leave blank

5. (a)

~~Range of value~~	~~z~~	~~P(≤z)~~	~~Probability~~
~~X<304~~	~~-3~~	~~0.00135~~	~~[illegible]~~ ~~0.0014~~
~~304≤X<306~~	~~-2~~	~~0.02275~~	~~[illegible] = 0.0214~~
~~306≤X<308~~	~~-1~~	~~0.15866~~	~~[illegible]~~
~~308≤X<310~~	~~0~~	~~0.50000~~	
~~310≤X<312~~	~~1~~	~~0.84134~~	
~~312≤X<314~~	~~2~~	~~0.97725~~	
~~314≤X<316~~	~~3~~	~~0.99865~~	
~~X≥316~~	~~3~~	~~[illegible]~~	

(a)

Range of value	z	P(≤z)	Probability
X<304	-1.5	0.06681	0.0668 ✓
304≤X<306	-1	0.15866	0.0919 ✓
306≤X<308	-0.5	0.30854	0.1499
308≤X<310	0	0.50000	0.1915
310≤X<312	0.5	0.69146	0.1915
312≤X<314	1	0.84134	0.1499
314≤X<316	1.5	0.93319	0.0919 ✓
X≥316	1.5	0.93319	0.0666 ✓

4

(b)

H_0: the data is a random sample from normal distribution

H_1: the data is not a random sample from normal distribution

Contd below

Leave blank

(b)

H_0: the data is a random sample from normal distribution

H_1: the data is not a random sample from normal distribution

Weight of ~~…~~	z	P(<z)	Probability	O	E
Less than 304	-1.5	0.06681	0.06681	5	6.681
304 -	-1	0.15866	0.09185	13	9.185
306 -	-0.5	0.30854	0.14988	10	14.988
308 -	0	0.50000	0.19146	18	19.146
310 -	0.5	0.69146	0.19146	25	19.146
312 -	1	0.84134	0.14988	20	14.988
314 -	1.5	0.93319	0.09185	5	9.185
316 or more	1.5	0.93319	0.06681	4	6.681

M1

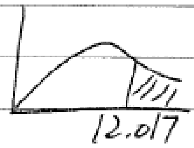
$\chi^2 = [(5-6.681)^2/6.681] + [(13-9.185)^2/9.185] + [(10-14.988)^2/14.988] + [(18-19.146)^2/19.146] + [(25-19.146)^2/19.146] + [(20-14.988)^2/14.988] + [(5-9.185)^2/9.185] + [(4-6.681)^2/6.681]$ M1 A1

$= 10.1847$ A1

$\alpha = 10\%$

$v = k-1 = 8-1 = 7$ B1

$c.v. = 12.017$ B1



$\chi^2 < 12.017$ so we accept H_0 and conclude that the data is a random sample from normal distribution, so the model is suitable. A1

7

(c)

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Commentary

Many candidates did very well on this question with concise and accurate solutions to part (a) and efficient applications of the goodness of fit test. A common problem was to include too much detail of working and to carry out unnecessary calculations. In this example, the candidate worked out the four missing probabilities and presented the results concisely, but also recalculated the probabilities given in the question.

There were similar problems in part (b): one table with columns for O, E and $(O – E)^2/E$ was sufficient to indicate method and record enough results to obtain all the marks for calculating the test statistic. The example shows one of several ways of doing more than was needed, with recalculation of the probabilities found in part (a) and excessive detail of applying the formula. This candidate scored almost full marks for parts (a) and (b), but must have spent a great deal more time than was needed.

Candidates were able to score full marks on part (c) even if they had gone wrong in earlier parts, but few were able to use both the data and the model effectively to explain their assessments of the proposed labels for punnets of raspberries.

Mark Scheme

SS05 (cont)

<table>
<tr><th>Q</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>5(a)</td><td>$\mathrm{P}(X < 304) = \Phi\left(\frac{304-310}{4}\right)$</td><td>M1</td><td></td><td>attempt to find a probability</td></tr>
<tr><td></td><td>$a = \Phi(-1.5) = 0.0668$ (or 0.0667)</td><td>A1</td><td></td><td>one missing value found</td></tr>
<tr><td></td><td>$b = 0.0918$ (or 0.0919)
$c = 0.0918$ (or 0.0919)</td><td>B1</td><td></td><td>second value found by any method</td></tr>
<tr><td></td><td>$d = 0.0668$ (or 0.0667)</td><td>B1</td><td>4</td><td>remaining values correct</td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th>O</th><th>E</th><th>$\frac{(O-E)^2}{E}$</th></tr>
<tr><td>5</td><td>6.68</td><td>0.42</td></tr>
<tr><td>13</td><td>9.19</td><td>1.58</td></tr>
<tr><td>10</td><td>14.99</td><td>1.66</td></tr>
<tr><td>18</td><td>19.15</td><td>0.07</td></tr>
<tr><td>25</td><td>19.15</td><td>1.79</td></tr>
<tr><td>20</td><td>14.99</td><td>1.68</td></tr>
<tr><td>5</td><td>9.19</td><td>1.91</td></tr>
<tr><td>4</td><td>6.68</td><td>1.08</td></tr>
<tr><td></td><td></td><td>10.2</td></tr>
</table>
</td><td>
M1
M1
A1

A1</td><td></td><td>If $E = 12.5$ throughout, just second M1 available

probabilities $\times 100$
use of formula
at least 4 values correct (AWRT)

$\sum E \neq 100$: lose this and final A1
total correct; AWRT</td></tr>
<tr><td></td><td>H_0: can be modelled by $\mathrm{N}(310, 4^2)$
H_1: Not H_0</td><td>B1</td><td></td><td>both</td></tr>
<tr><td></td><td>$\nu = 8 - 1 = 7$</td><td>B1</td><td></td><td></td></tr>
<tr><td></td><td>$\chi^2_{10\%} = 12.017$</td><td>B1</td><td></td><td>any grouping of categories: lose final A1</td></tr>
<tr><td></td><td>$10.2 < 12.017$
Accept H_0 at 10% level. There is not sufficient evidence to reject the model</td><td>A1√</td><td>8</td><td>ft on calculated value and cv</td></tr>
<tr><td>(c)</td><td>(1) Reasonable claim as model has mean 310. (Does not say much about one punnet)</td><td>E1</td><td></td><td></td></tr>
<tr><td></td><td>(2) Looks a safe claim. Only 5 punnets in sample < 304g; shape of normal distribution suggests few, if any, will</td><td>E1</td><td></td><td>reference to relevant figure from sample in (2) or (3)</td></tr>
<tr><td></td><td>be < 300g
(3) At least 5 punnets in sample < 305g and shape suggests claim could be</td><td>E1</td><td></td><td>reference to property of normal in (2) or (3)</td></tr>
<tr><td></td><td>wrong for about 10% of punnets</td><td>E1</td><td>4</td><td>appropriate assessment of possibilities must use data and model for E4</td></tr>
<tr><td></td><td>Total</td><td></td><td>16</td><td></td></tr>
</table>

Question 6

6 Amy and Ben have part-time jobs making sandwiches, which are then packaged for sale to local businesses. When making cheese sandwiches, they have to judge by eye the amount of grated cheese to put in a sandwich. During an informal assessment, their supervisor concluded that the mean amounts of grated cheese put into sandwiches by Amy and Ben were about the same.

Later, as part of a formal assessment, the supervisor weighs the amount of grated cheese, x grams, selected by Amy for a sandwich on each of 11 randomly chosen occasions, and the amount of grated cheese, y grams, selected by Ben on each of 9 randomly chosen occasions. She then calculates the means, $\overline{x}$ and $\overline{y}$, and the standard deviations, s_x and s_y, for the two samples of weights.

	Mean	Standard deviation
Amy	$\overline{x} = 41.6$	$s_x = 3.24$
Ben	$\overline{y} = 38.4$	$s_y = 2.71$

Stating **two** necessary assumptions, carry out a hypothesis test, at the 1% level of significance, to investigate whether the supervisor's earlier assessment was correct.

(12 marks)

Student Response

Question number

6) One assumption is that the underlying population is normally distributed. B1
One other assumption is that the weights of the cheese are random.

Given

$H_0 : \mu_x = \mu_y$ x = Amy $\overline{x} = 41.6$ $n_x =$
$H_1 : \mu_x \neq \mu_y$ B1 y = Ben $\overline{y} = 38.4$ $n_y =$
$S_x = 3.24$
$S_y = 2.71$

Test Statistic $t = \dfrac{(\overline{x} - \overline{y})}{\sqrt{\dfrac{\sigma^2}{n_x} + \dfrac{\sigma^2}{n_y}}}$

$= \dfrac{(41.6 - 38.4)}{\sqrt{\dfrac{3.24^2}{11} + \dfrac{2.71^2}{9}}}$ M1

M1 (test choice)

$= \dfrac{3.2}{1.330540636} = 2.41$ X

Critical Value $t = 2.878$ B1

$V = 11 + 9 - 2 = 20 - 2 = 18$ B1

1% sig leve 2 tailed

2.41 < 2.878. Accept Ho. No sufficient evidence to suggest that the mean amounts of cheese put into sandwiches by Amy and Ben are different. A1

Leave blank

6

7

Commentary

In general this question was well answered, with many candidates scoring full marks. The most common mistake was to confuse this topic, testing for equality of means for two normal distributions with unknown but equal variances, with a similar one on the specification, testing for equality of means for normal distributions with known variances. Failure to find a pooled estimate for variance inevitably led to an incorrect formula for the test statistic and the loss of at least four marks. In the example shown, the candidate gained all the other available marks by using the t-test with the correct degrees of freedom and drawing a conclusion consistent with the earlier work. Many candidates who made the same basic error found their critical value from normal distribution tables and so scored very few marks

Mark Scheme

SS05 (cont)

Q	Solution	Marks	Total	Comments
6	assume weights selected by Amy and Ben are normally distributed with common variance independence between samples	B1 B1		any two
		M1		attempt to use t-test for difference of means
	H$_0$: $\mu_A = \mu_B$ H$_1$: $\mu_A \neq \mu_B$	B1		both
	pooled estimate of variance			
	$=\frac{(10\times3.24^2)+(8\times2.71^2)}{10+8}$	M1		
	$=9.096$	A1		accept 9.09 to 9.10
	$v=18$	B1		
	$t=\pm2.878$	B1		
	sample statistic $=\frac{41.6-38.4}{\sqrt{9.096\left(\frac{1}{11}+\frac{1}{9}\right)}}$	M1 A1		correct values substituted
	$=2.36$	A1√		ft on standard error; AWRT
	$2.36<2.878$ so accept H$_0$			
	There is not enough evidence at the 1% level to say that the earlier assessment was wrong	A1√	12	ft on sample statistic and t depends on first and last M1
	Total		**12**	
	TOTAL		**75**	