Teacher Support Materials

Maths GCE

Paper Reference SS02
(Q1-5 Only)

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Dr Michael Cresswell, Director General.

Question 1

1 The number of people entering a supermarket may be modelled by a Poisson distribution with mean 2.4 per minute.

(a) Find the probability that, during a particular minute:

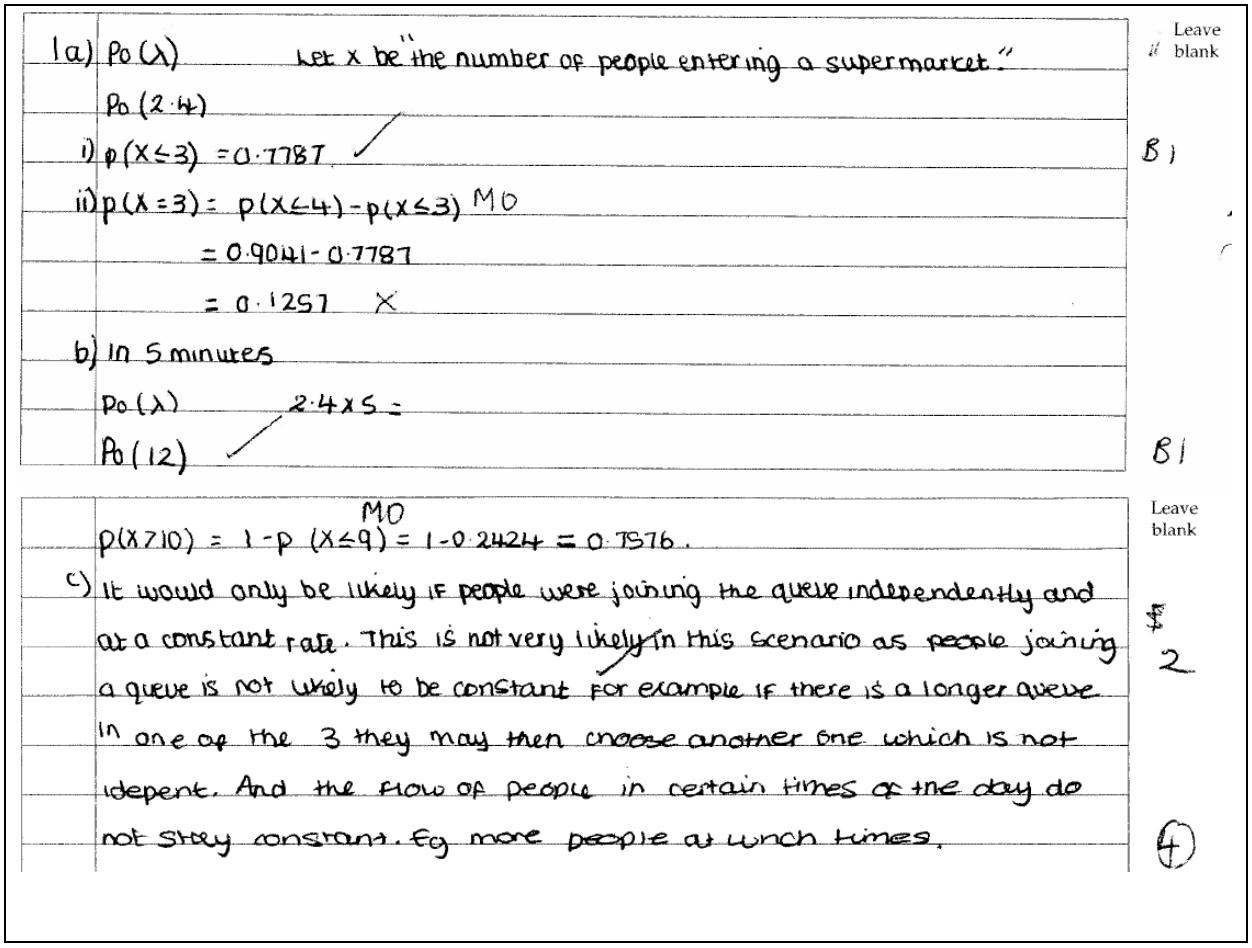
(i) 3 or fewer people enter the supermarket;

(ii) exactly 3 people enter the supermarket. *(4 marks)*

(b) Find the probability that, during a five-minute interval, more than 10 people enter the supermarket. *(3 marks)*

(c) To pay for their goods, customers must join a queue at one of three checkouts. State, giving a reason, whether it is likely that the number of people per minute joining the queue at a particular checkout may be modelled by a Poisson distribution. *(2 marks)*

Student Response



1a) $Po(\lambda)$ Let x be "the number of people entering a supermarket"

$Po(2.4)$

i) $p(X \leq 3) = 0.7787$ ✓ B1

ii) $p(X = 3) = p(X \leq 4) - p(X \leq 3)$ M0

$= 0.9041 - 0.7787$

$= 0.1257$ ✗

b) In 5 minutes

$Po(\lambda)$ $2.4 \times 5 =$

$Po(12)$ ✓ B1

Leave blank

M0

$p(X \geq 10) = 1 - p(X \leq 9) = 1 - 0.2424 = 0.7576$.

c) It would only be likely if people were joining the queue independently and at a constant rate. This is not very likely in this scenario as people joining a queue is not likely to be constant for example if there is a longer queue in one of the 3 they may then choose another one which is not idepent. And the flow of people in certain times of the day do not stay constant. Eg more people at lunch times. 2 (4)

Leave blank

Commentary

This candidate belonged to the minority who made errors in using the Poisson distribution tables. She did however give a good explanation in part (c).

Mark scheme

SS02

Q	Solution	Marks	Total	Comments
1(a)(i)	P(3 or fewer)=0.779	B1		0.779 (0.778~0.779)
(a)(ii)	P(3)=P(3 or fewer)−P(2 or fewer)	M1		P(3)=P(≤3)−P(≤2)
	=0.7787−0.5679	m1		completely correct method
	=0.209	A1	4	0.209(0.208~0.21)
(b)	Poisson mean 5×2.4=12	B1		Poisson mean 5×2.4
	P(>10)=1−P(10 or fewer)	M1		P(>10)=1−P(10 or fewer)
	=1−0.3472			
	=0.653	A1	3	0.653 (0.652~0.653)
(c)	No, customers are likely to join shortest queue i.e. not at random.	E1 E1	2	No Reason – allow not independent – couple may shop together etc.
	Total		**9**	

Question 2

2 [**Figure 1**, printed on the insert, is provided for use in this question.]

The table shows the expenditure, £ million, of households in the United Kingdom on audio-visual equipment. It also shows the values, y, of an appropriate moving average, and of t, which numbers the values of y from 1 to 10.

Year	2002				2003				2004				2005
Quarter	1	2	3	4	1	2	3	4	1	2	3	4	1
Expenditure	1041	938	982	1526	1065	983	1059	1618	1135	1070	1170	1705	1231
Moving Average, y		1122	1128	1139	1158	1181		1221	1248	1270	1294		
t		1	2	3	4	5	6	7	8	9	10		

Source: *Consumer Trends*, Office for National Statistics, Quarter 2, 2005

(a) Calculate the value of the missing moving average. *(2 marks)*

(b) Plot the values of the moving average on **Figure 1**. *(2 marks)*

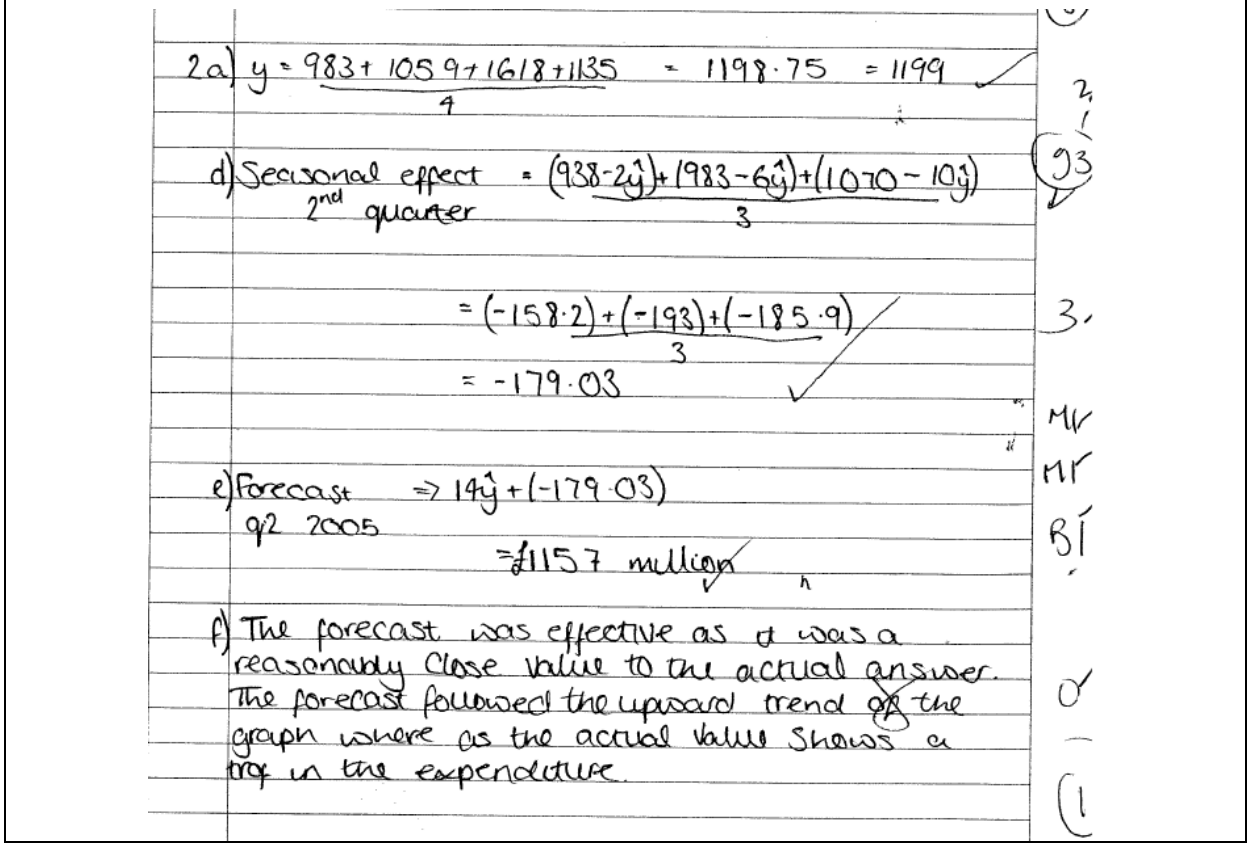
(c) The equation of the regression line of y on t is $y = 1086 + 19.96t$. Add this line to **Figure 1**. *(2 marks)*

(d) Estimate the second quarter seasonal effect. *(3 marks)*

(e) Forecast the expenditure for quarter 2 of 2005. Indicate the method used and give your answer to an appropriate degree of accuracy. *(4 marks)*

(f) The actual expenditure on audio-visual equipment in quarter 2 of 2005 was £1065 million. Comment on this value and on the effectiveness of your method of forecasting. *(2 marks)*

Student response



2a) $y = \frac{983 + 1059 + 1618 + 1135}{4} = 1198.75 = 1199$ ✓

d) Seasonal effect 2nd quarter $= \frac{(938 - 2\hat{y}) + (983 - 6\hat{y}) + (1070 - 10\hat{y})}{3}$

$= \frac{(-158.2) + (-193) + (-185.9)}{3}$ ✓

$= -179.03$ ✓

e) Forecast q2 2005 $\Rightarrow 14\hat{y} + (-179.03)$

$= £1157$ million ✓

f) The forecast was effective as it was a reasonably close value to the actual answer. The forecast followed the upward trend of the graph where as the actual value shows a drop in the expenditure.

2, 1, 93, 3, M1, M1, B1, 0, (1

Commentary

In part (c) this candidate evaluated three points and plotted them with a dot on the graph. Then, surprisingly, he drew a line which did not pass through the points.

Like a number of other candidates he did not know what was required in part (d) butiimplied the answer to (d) in part (e). This received 2 marks out of 3.

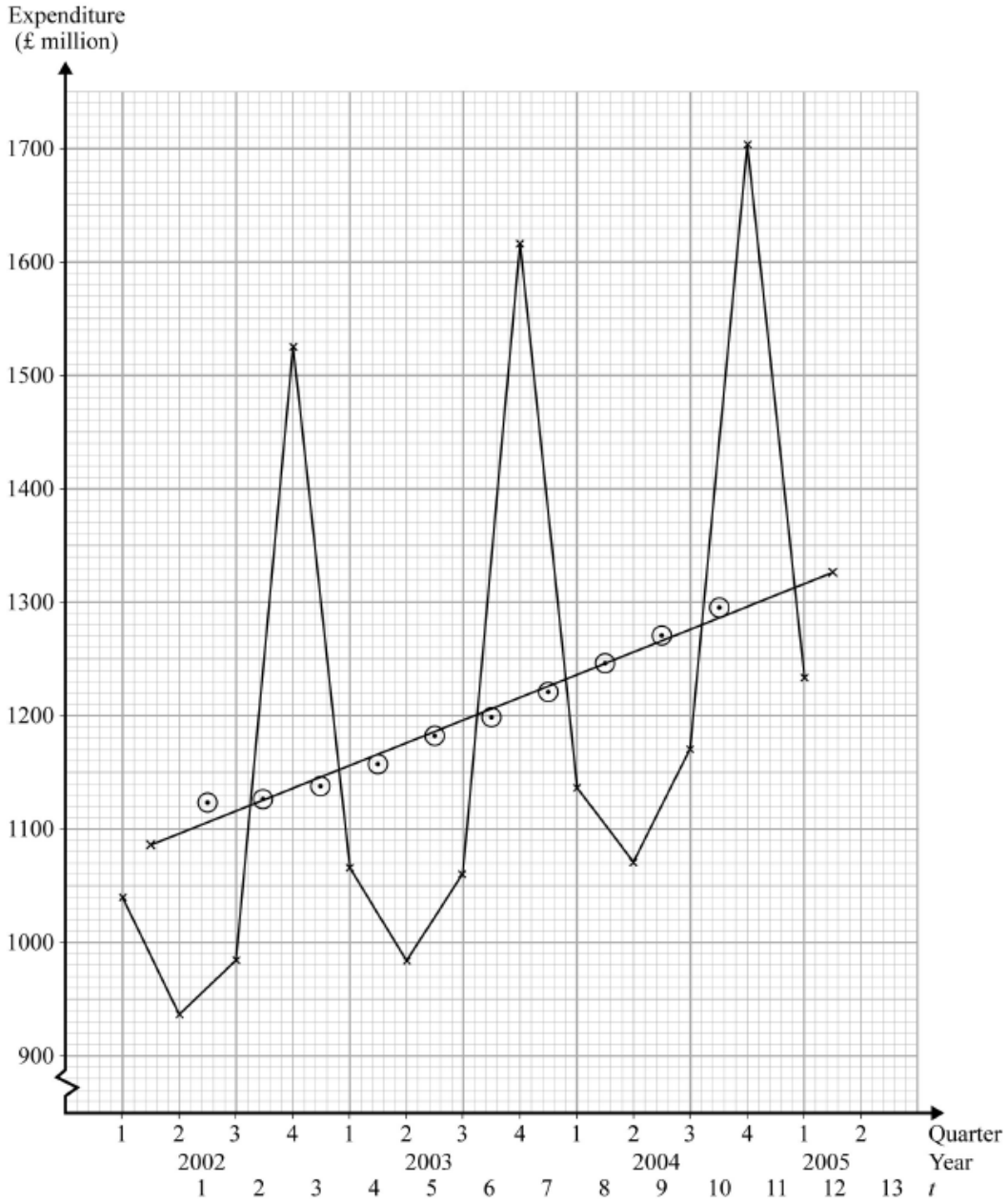
In part (e) the question asked for 'an appropriate degree of accuracy' and so the candiate was penalised for giving 6 significant figures.

A good attempt at part (f) but to gain both marks he needed to point out that quarter 2 of 2005 did not follow the previous pattern and so could not be successfully forecast by a purely statistical method.

Mark Scheme

2(a)	$\frac{983+1059+1618+1135}{4}=1199$	M1 A1	 2	method 1199 (1198~1200)
(b)	on next page	M1 A1	 2	m.a. in correct position Accurate plot – by eye – allow 1 small slip
(c)	$t=0 \quad y=1086$ $t=12 \quad y=1326$ + line	M1 A1	 2	method for line accurate line drawn
(d)	residuals for Q2 $-158, -193, -196$ mean = –179	M1 m1 A1	 3	method for residual – allow from graph – ignore sign – their line method for seasonal effect – ignore sign – allow omission of Q2,2002 –179(–170~–200) **2 maximum if answered in (e)**
(e)	$1086+12.5\times19.96-179$ =1335.5–179 =1156.5 forecast £1160 million	M1 M1 B1 B1√	 4	method for trend – allow them from graph – their line method for including their negative seasonal effect – their trend (1130~1170) allow 1100 or 1200 2 or 3 sf and £m **Allow 3 maximum if method is not clear or based on Q2 results only**
(f)	this is a poor forecast but no purely numerical method could have predicted Q2,2005 would be less than Q2,2004	E1 E1	 2	poor forecast / ineffective method no numerical method could have forecast this result / extrapolation is inherently unreliable
	Total		**15**	

SS02 (cont)


Expenditure
(£ million)
1700
1600
1500
1400
1300
1200
1100
1000
900
1 2 3 4 1 2 3 4 1 2 3 4 1 2
Quarter
2002 2003 2004 2005
Year
1 2 3 4 5 6 7 8 9 10 11 12 13
t

Question 3

3 Imran wishes to buy a house in Cheadleville. The number of houses, X, in Cheadleville advertised for sale in a copy of the *Cheshire Weekly Sentinel* may be modelled by the following probability distribution.

x	0	1	2	3	4	5
$P(X=x)$	0.32	0.25	0.19	0.12	0.09	0.03

(a) Find the mean and the standard deviation of X. *(5 marks)*

(b) The number of houses in Cheadleville advertised for sale in a copy of the *Cheshire Weekly Clarion* may be modelled by the random variable Y.

Given that $E(Y)=2.5$

and $E[(Y-2.5)^2]=2.2$:

(i) evaluate the standard deviation of Y; *(2 marks)*

(ii) compare the number of houses in Cheadleville advertised for sale in the *Cheshire Weekly Sentinel* with that in the *Cheshire Weekly Clarion*. *(2 marks)*

(c) Imran intends to subscribe to one of the two papers. Advise him which one to choose, justifying your answer. *(2 marks)*

Student Response

3a) mean = $E(X) = (0\times0.32)+(1\times0.25)+(2\times0.19)+(3\times0.12)+(4\times0.09)+(5\times0.03)$

$= 1.5$ ✓

s.d = $\sqrt{E(X^2)-[E(X)]^2}$

$= E(X^2) = (0^2\times0.32)+(1^2\times0.25)+(2^2\times0.19)+(3^2\times0.12)+(4^2\times0.09)+(5^2\times0.03)$

$= 4.28$ ✓ 5

$4.28 - 1.5^2 = 2.03$ ✓

$\sqrt{2.03} = 1.425$ (3dp) 2

b) i) s.d = $\sqrt{2.2} = 1.483$ (3dp) ✓

ii) The mean n° of houses advertised in the Sentinel is less than that in the Clarion and the s.d are pretty similar E1 E0

c) He should subscribe to the Sentinel because there are less houses advertised in this so there is a higher chance of his house getting seen due to less competition. B0 B0 (8)

Commentary

A good answer although all that was needed in part (b)(i) was to evaluate the square-root of 2.2. The candidates method was correct but more complicated than necessary.

For full marks in part (b)(ii) it was necessary make a judgement and to give some interpretation to the standard deviation i.e. variability similar or variability slightly higher for Clarion

Mark Scheme

SS02 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathrm{E}(X) = 0\times0.32 + 1\times0.25 + 2\times0.19 + 3\times0.12 + 4\times0.09 + 5\times0.03$	M1		method for E(X)
	$= 1.5$	A1		1.5 CAO
	$\mathrm{E}(X^2) = 0^2\times0.32 + 1^2\times0.25 + 2^2\times0.19 + 3^2\times0.12 + 4^2\times0.09 + 5^2\times0.03$	M1		method for E(X^2) – may be implied
	$= 4.28$			
	$\mathrm{Var}(X) = 4.28 - 1.5^2 = 2.03$	m1		method for s.d.; allow for variance = 2.03
	s.d. $= \sqrt{2.03}$			
	$= 1.42$	A1	5	1.42(1.41~1.43)
(b)(i)	s.d. $= \sqrt{2.2}$	M1		method
	$= 1.48$	A1	2	1.48(1.48~1.49)
(b)(ii)	more houses in Cheadleville are advertised in the Clarion than in the Sentinel. The week to week variability is similar	E1√		Clarion higher average
		E1	2	variability similar
(c)	choose Clarion – since more houses in Cheadleville advertised on average	B1√		Clarion
		B1	2	higher mean
	Total		**11**	

Question 4

4 [A sheet of graph paper is provided for use in this question.]

Table 1 shows details of the numbers joining, and **Table 2** shows details of the numbers leaving, the United Kingdom armed forces between 1993 and 2004.

Table 1

Intake of UK regular forces from civilian life: by service

	1993 /94	1994 /95	1995 /96	1996 /97	1997 /98	1998 /99	1999 /00	2000 /01	2001 /02	2002 /03	2003 /04
All Services:											
Male	10 620	11 150	15 500	19 230	19 740	22 560	22 390	20 410	20 950	23 040	20 760
Female	1 330	1 850	2 180	2 940	3 220	3 440	3 160	2 610	2 700	3 240	2 710
Total	11 950	13 010	17 670	22 160	22 960	26 000	25 550	23 020	23 650	26 280	23 470
Naval Service:											
Male	1 280	960	2 010	3 400	3 540	4 110	4 250	3 990	4 270	4 420	3 530
Female	260	340	350	560	570	660	700	630	740	800	580
Total	1 540	1 300	2 360	3 960	4 110	4 770	4 950	4 620	5 010	5 220	4 120
Army:											
Male	8 760	9 490	11 510	13 580	13 500	15 010	14 750	13 450	13 620	15 060	13 930
Female	810	1 190	1 380	1 940	1 970	1 980	1 750	1 320	1 240	1 550	1 260
Total	9 580	10 680	12 890	15 520	15 470	16 990	16 500	14 770	14 850	16 610	15 190
Royal Air Force:											
Male	580	700	1 980	2 250	2 700	3 450	3 380	2 980	3 070	3 550	3 290
Female	260	320	450	430	680	800	710	660	720	890	870
Total	840	1 020	2 420	2 680	3 380	4 250	4 100	3 630	3 780	4 450	4 160

Source: *Annual Abstract of Statistics*, Office for National Statistics, 2005

Table 2

Outflow of UK regular forces: by service

	1993 /94	1994 /95	1995 /96	1996 /97	1997 /98	1998 /99	1999 /00	2000 /01	2001 /02	2002 /03	2003 /04
All Services:											
Male	29 700	31 050	25 750	29 320	21 860	24 500	23 870	22 520	22 360	21 770	21 200
Female	2 430	2 990	3 120	3 680	2 490	2 970	2 750	2 430	2 350	2 340	2 200
Total	32 130	34 040	28 860	33 000	24 350	27 470	26 620	24 950	24 710	24 100	23 400
Naval Service:											
Male	4 610	5 500	4 310	6 190	4 650	4 920	5 160	4 480	5 110	4 680	4 230
Female	490	680	630	940	620	610	630	550	690	620	540
Total	5 110	6 180	4 940	7 130	5 270	5 530	5 800	5 040	5 800	5 300	4 770
Army:											
Male	19 630	20 230	13 940	13 760	13 190	15 320	14 620	13 900	13 290	13 420	13 500
Female	1 290	1 650	1 510	1 600	1 280	1 730	1 580	1 330	1 090	1 140	1 090
Total	20 920	21 880	15 440	15 350	14 470	17 050	16 200	15 230	14 380	14 560	14 600
Royal Air Force:											
Male	5 450	5 310	7 500	9 380	4 020	4 250	4 080	4 140	3 960	3 670	3 470
Female	650	660	980	1 140	590	640	540	540	570	580	570
Total	6 100	5 970	8 480	10 520	4 610	4 890	4 620	4 680	4 530	4 250	4 040

Source: *Annual Abstract of Statistics*, Office for National Statistics, 2005

(a) How many males left the Army during 1998/99? *(1 mark)*

(b) How many more females joined than left the Royal Air Force during 2002/03? *(2 marks)*

(c) During which of the years shown did the number of females joining the Naval Service exceed the number leaving? *(2 marks)*

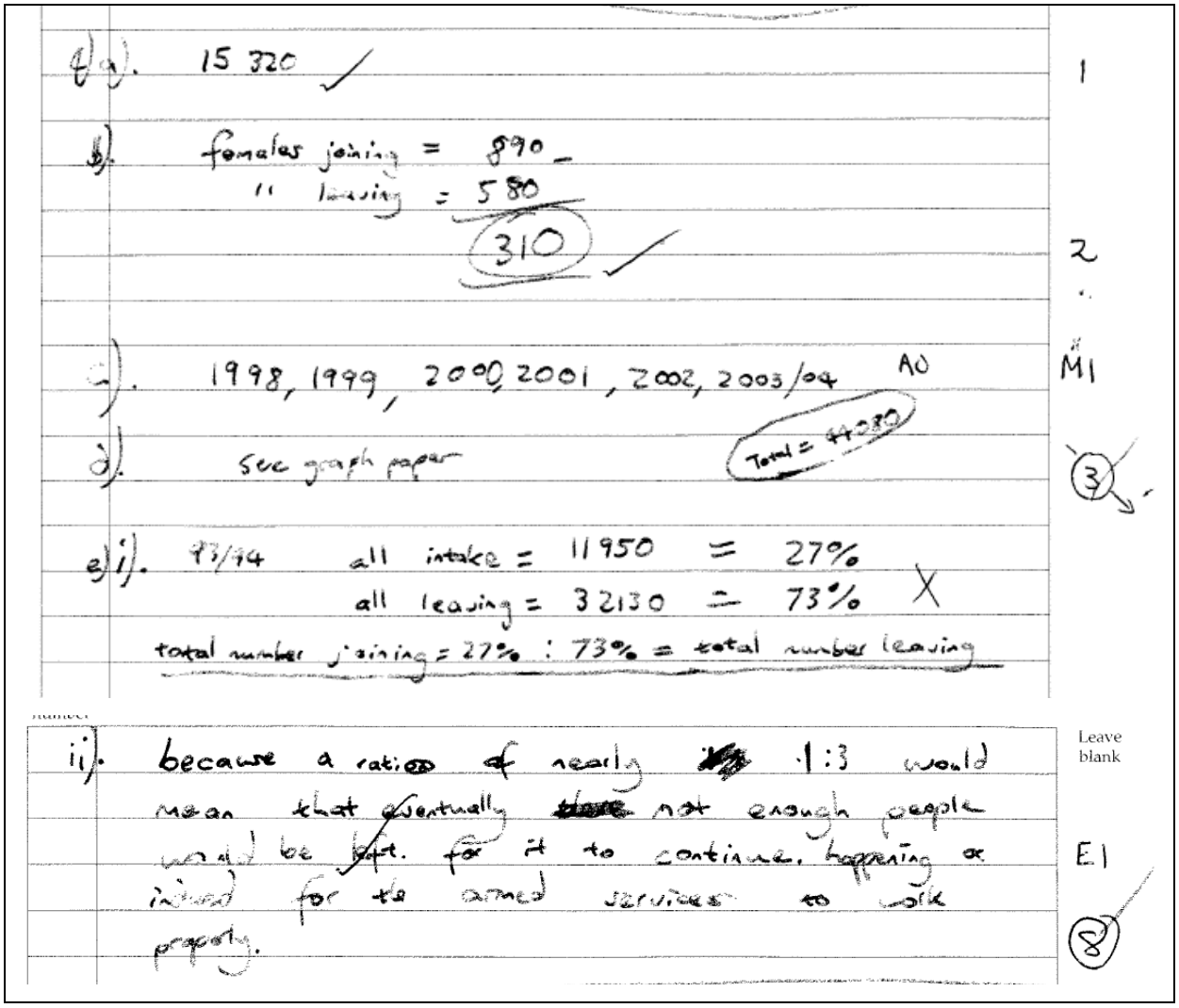
(d) Draw a line diagram to compare the numbers of females leaving the Naval Service, the Army and the Royal Air Force during 2003/04. *(3 marks)*

(e) For 1993/94:

(i) calculate, as a percentage, the ratio of the total number joining All Services to the total number leaving All Services;

(ii) explain why this ratio could not continue in the long term. *(3 marks)*

Student Response

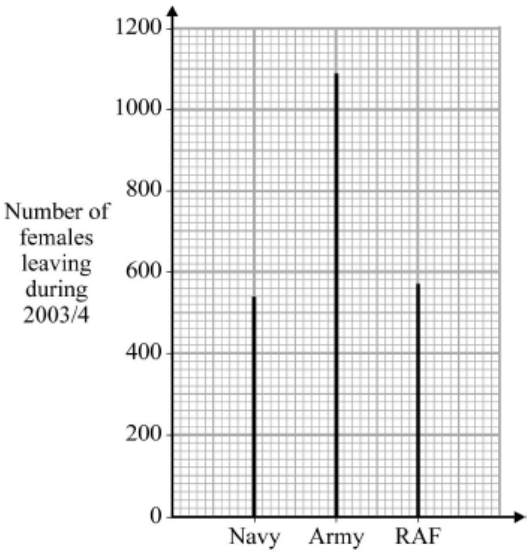


Commentary

A good answer apart from breaking the vertical scale for the line diagram in part (d)

4(a)	15320	B1	1	15320 or 15300
(b)	890 – 580 = 310	M1 A1	 2	method 310 CAO
(c)	1998/9 to 2003/4	M1 A1	 2	method – allow small slip 1998/9 to 2003/4 CAO
(d)	on next page	M1 B1 A1	 3	method – allow horizontal – allow bars instead of lines but not if joined – disallow broken scale axes labelled – generous accurate plot by eye
(e)(i)	$\frac{11950}{32130}\times 100 = 37.2\%$	M1 A1	 	method for ratio 37.2 (37~37.3)
(e)(ii)	more leaving than joining – in the long run this would lead to no one being left to leave	E1	3	explanation
	Total		**11**	

SS02 (cont)



Question 5

5 A company, with 9320 employees, provides refuse collection services for 47 councils in the United Kingdom. The company asks a market research firm to carry out an opinion poll of its employees concerning union membership.

(a) Describe how the market research firm could obtain a simple random sample of size 120 from the 9320 employees. *(4 marks)*

(b) The market research firm selects 4 of the 47 councils at random.

(i) What further step(s) would be necessary to obtain a cluster sample of size 120 from the 9320 employees? *(2 marks)*

(ii) Give a reason why the market research firm might prefer a cluster sample to a random sample. *(2 marks)*

(c) It is proposed that a stratified sample be used.

(i) Suggest two factors which could be used to stratify the sample. *(2 marks)*

(ii) Suggest a reason why a stratified sample might be preferred to a cluster sample. *(1 mark)*

Student Response

number

Leave blank

(5a) Number all the employees from 000 Assign a 4 number digget to all the employees ranging from 0000 – 9219. ✓ Using random numbers from a random number chart pick 4 digit random numbers ranging from 0000 – 9219 ✓. Choos 120 set of numbers then choice corrospanding employe. Ignore any repeats and any numbers above 9219 4

(b)(i) At random pick out 30 employees from each ~~council~~ of the four councils 2

(ii) This is more convient less traveling required and therefore it's less expensive ✗2

(c)(i) ~~large number of employees and different~~ different concils have different number of employees not enough ✓

(ii) They could be a more variety of opinions E1

9

Commentary

This candidate got off to a bad start by confusing the sample size with the population size.

Like many candidates he answered a different question than the one which was asked in part (c)(i).

Mark Scheme

SS02 (cont)

Q	Solution	Marks	Total	Comments
5(a)	number employees 0000 to 9319 select 4-digit random numbers ignore repeats and >9319 continue until 120 numbers obtained select corresponding employees	E1 E1 E1 E1	4	any valid numbering select 4-digit random numbers ignore repeats and >9319 (must be consistent in numbering) continue until 120 numbers obtained
(b)(i)	from each of the 4 chosen councils select a random sample of 30 employees	E1 E1	2	select a sample from each of the 4 councils of size 30
(b)(ii)	employees to be interviewed would be geographically localised / easier / cheaper	E2,1	2	reason – easier/cheaper without further explanation gets E1
(c)(i)	council / age / sex / length of service	B1B1	2	any sensible suggestion; B1 for each
(c)(ii)	More representative of population	E1	1	more representative allow all have equal chance
	Total		**11**	