# шјес <br> GCE AS/A level cbac 

0984/01

MATHEMATICS - S2

## Statistics

A.M. TUESDAY, 9 June 2015

1 hour 30 minutes plus your additional time allowance

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
a 12 page answer book;
a Formula Booklet;
a calculator;
statistical tables (Murdoch and Barnes or RND/
WJEC Publications).

## INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.
Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Jim is a tennis player. His serve has a mean speed of 120 miles per hour (mph). He buys a new racket and he wishes to investigate whether or not using this racket changes the mean speed of his serve. He therefore goes to a tennis centre where he hits 10 serves and the measured speeds are as follows (mph).
$121-2$
119•1
$118 \cdot 3$
120•1
117.9
$118 \cdot 3$
$119 \cdot 4$
$119 \cdot 6$
$120 \cdot 3$
$117 \cdot 8$

You may assume that this is a random sample from a normal distribution with a standard deviation of 1-2.
(a) State suitable hypotheses for his investigation.
[1 mark]
(b) Determine the $p$-value of these results and state your conclusion in context.
[8 marks]
2. In a certain population, the weights of the men are normally distributed with mean 82 kg and standard deviation 2.5 kg . The weights of the women are normally distributed with mean 65 kg and standard deviation 2 kg .
(a) Calculate the 95th percentile of the men's weights.
[2 marks]
(b) Determine the probability that the weight of a randomly chosen woman lies between 64 kg and 68 kg .
[6 marks]
(c) One morning, 3 men and 4 women hire a boat which has a safety limit of 500 kg . Calculate the probability that their combined weight exceeds the safety limit. You may assume that the weights of the 3 men and 4 women are independent.
[6 marks]
3. When the sugar content of a jar of jam is measured using a certain machine, the reading obtained, in grams, is a normally distributed random variable with mean equal to the actual sugar content and standard deviation 1.5 grams. Successive readings are independent. A shopkeeper sells two varieties of strawberry jam and he wishes to investigate whether or not there is a difference between the sugar contents of the two varieties. He therefore selects 8 jars of Variety A and 8 jars of the same size of Variety $\mathbf{B}$ and measures the sugar content of the 16 jars with the results as shown on the opposite page.
(a) Calculate a $\mathbf{9 9} \%$ confidence interval for the difference between the sugar contents of Variety A and Variety $B$.
[7 marks]
(b) The shopkeeper's assistant uses the same data to determine another confidence interval for this difference and he obtains $[2.19,4.81]$. Determine the confidence level of this interval.

200.9
198.6
$202 \cdot 5$
$197 \cdot 1$
$\begin{array}{ll}0 & \text { N } \\ \dot{O} & \dot{O} \\ \dot{O} & \dot{O} \\ \mathbf{N}\end{array}$
$\begin{array}{ll}N & \text { ㅎ } \\ \dot{O} & \dot{\sigma} \\ \dot{O} & \dot{O} \\ \mathbf{N}\end{array}$
$\begin{array}{ll}\infty & \ddagger \\ \dot{0} & 0 \\ \dot{0} & \text { 0 }\end{array}$

$203 \cdot 1$
$198 \cdot 2$

Variety A
Variety B
4. When Edwin shoots an arrow at a target, he hits it with probability $\rho=0.4$.
Successive shots are independent. He attends a course to try to improve his technique and he wishes to test whether or not the course has succeeded in doing that. He defines the following hypotheses.

$$
H_{0}: p=0.4 ; \quad H_{1}: p>0.4
$$

(a) He decides initially to shoot 20 arrows at the target and $X$ denotes the number of these shots which hit the target. Determine the critical region having a significance level closest to $1 \%$.
[3 marks]
(b) He decides to carry out a further test in which he shoots 120 arrows at the target. He finds that 55 of these shots hit the target. Calculate an approximate $\boldsymbol{\rho}$-value and state your conclusion in context.
[7 marks]
5. A fair dice with faces numbered $1,2,3,4,5$ and 6 respectively is thrown 100 times. Use the Central Limit Theorem to calculate, approximately, the probability that the mean of the 100 scores obtained is at least $3 \cdot 75$.
[9 marks]
6. In a certain factory, the number of breakdowns occurring during a day is a Poisson random variable with mean 1-2. Successive breakdowns are independent. In an attempt to reduce the number of breakdowns, the machines are modified.
(a) In the $\mathbf{1 0}$ days following the modifications, the total number of breakdowns was 9 .
(i) State suitable hypotheses for testing whether the mean number of breakdowns per day has decreased.
(ii) Calculate the $\boldsymbol{\rho}$-value and state your conclusion in context.
[5 marks]
(b) In the 100 days following the modifications, the total number of breakdowns was 101. Calculate an approximate $\boldsymbol{\rho}$-value and interpret it in context.
[6 marks]
7. The continuous random variable $X$ is uniformly distributed on the interval $[a, b]$ where $0<a<b$.
(a) Let $Y=\sqrt{X}$.
(i) Find an expression for $\boldsymbol{P}(\boldsymbol{Y} \leqslant \boldsymbol{y})$ for

$$
\sqrt{a} \leqslant y \leqslant \sqrt{b}
$$

(ii) Hence find the probability density function of $Y$.
[6 marks]
(b) Given that the mean and the variance of $X$ are 5.5 and 3 respectively, find the values of $\mathbf{a}$ and b.
[5 marks]

END OF PAPER

