

0984/01

MATHEMATICS - S2

Statistics

A.M. TUESDAY, 9 June 2015

1 hour 30 minutes plus your additional time allowance

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

a 12 page answer book; a Formula Booklet; a calculator; statistical tables (Murdoch and Barnes or RND/ WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Jim is a tennis player. His serve has a mean speed of 120 miles per hour (mph). He buys a new racket and he wishes to investigate whether or not using this racket changes the mean speed of his serve. He therefore goes to a tennis centre where he hits 10 serves and the measured speeds are as follows (mph).

121-2	119-1	118-3	120-1
117-9	118-3	119-4	119-6
120-3	117-8		

You may assume that this is a random sample from a normal distribution with a standard deviation of 1-2.

(a) State suitable hypotheses for his investigation. [1 mark]

(b) Determine the *p*-value of these results and state your conclusion in context. [8 marks]

- 2. In a certain population, the weights of the men are normally distributed with mean 82 kg and standard deviation 2.5 kg. The weights of the women are normally distributed with mean 65 kg and standard deviation 2 kg.
- (a) Calculate the 95th percentile of the men's weights. [2 marks]
- (b) Determine the probability that the weight of a randomly chosen woman lies between **64 kg** and **68 kg**. [6 marks]
- (c) One morning, 3 men and 4 women hire a boat which has a safety limit of **500 kg**. Calculate the probability that their combined weight exceeds the safety limit. You may assume that the weights of the 3 men and 4 women are independent.

[6 marks]

- 3. When the sugar content of a jar of jam is measured using a certain machine, the reading obtained, in grams, is a normally distributed random variable with mean equal to the actual sugar content and standard deviation 1.5 grams. Successive readings are independent. A shopkeeper sells two varieties of strawberry jam and he wishes to investigate whether or not there is a difference between the sugar contents of the two varieties. He therefore selects 8 jars of Variety A and 8 jars of the same size of Variety B and measures the sugar content of the 16 jars with the results as shown on the opposite page.
- (a) Calculate a 99% confidence interval for the difference between the sugar contents of Variety A and Variety B. [7 marks]
- (b) The shopkeeper's assistant uses the same data to determine another confidence interval for this difference and he obtains [2-19, 4-81]. Determine the confidence level of this interval.

202.6	197-4
200.9	198.6
202.5	197·1
200.6	199.7
200.7	199.1
199.8	196-4
201.8	197-5
203-1	198-2
Variety A	Variety B

4. When Edwin shoots an arrow at a target, he hits it with probability p = 0.4. Successive shots are independent. He attends a course to try to improve his technique and he wishes to test whether or not the course has succeeded in doing that. He defines the following hypotheses.

$$H_0: p = 0.4; \quad H_1: p > 0.4$$

(a) He decides initially to shoot 20 arrows at the target and X denotes the number of these shots which hit the target. Determine the critical region having a significance level closest to 1%.

[3 marks]

(b) He decides to carry out a further test in which he shoots 120 arrows at the target. He finds that 55 of these shots hit the target. Calculate an approximate **p**-value and state your conclusion in context. [7 marks]

5. A fair dice with faces numbered 1, 2, 3, 4, 5 and 6 respectively is thrown 100 times. Use the Central Limit Theorem to calculate, approximately, the probability that the mean of the 100 scores obtained is at least 3.75. [9 marks]

- 6. In a certain factory, the number of breakdowns occurring during a day is a Poisson random variable with mean 1-2. Successive breakdowns are independent. In an attempt to reduce the number of breakdowns, the machines are modified.
- (a) In the 10 days following the modifications, the total number of breakdowns was 9.
 - (i) State suitable hypotheses for testing whether the mean number of breakdowns per day has decreased.
 - (ii) Calculate the **p**-value and state your conclusion in context. [5 marks]
- (b) In the 100 days following the modifications, the total number of breakdowns was 101. Calculate an approximate ρ -value and interpret it in context. [6 marks]

- 7. The continuous random variable X is uniformly distributed on the interval [a, b] where 0 < a < b.
- (a) Let $\mathbf{Y} = \sqrt{\mathbf{X}}$.
 - (i) Find an expression for $P(Y \leqslant y)$ for $\sqrt{a} \leqslant y \leqslant \sqrt{b}$.
 - (ii) Hence find the probability density function of **Y**. [6 marks]
- (b) Given that the mean and the variance of X are 5.5 and 3 respectively, find the values of a and b. [5 marks]

END OF PAPER