GCE AS/A level

0983/01

MATHEMATICS – S1

Statistics

A.M. TUESDAY, 9 June 2015

1 hour 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

a 12 page answer book;

a Formula Booklet;
a calculator;

statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The random variable X has the binomial distribution $B(10, 0{\cdot}3)$ and $Y = 2X + 1$.

Calculate

(a) the mean and the variance of Y [5 marks]

(b) $P(Y = 7)$. [3 marks]

2. **The events A and B are such that**

$$P(A) = 0{\cdot}4, \quad P(B) = 0{\cdot}5$$

and $P(A \cup B) = 2 \times P(A \cap B)$

(a) **Show that** $P(A \cap B) = 0{\cdot}3$ **[2 marks]**

(b) **Evaluate** $P(A|B)$ **[2 marks]**

(c) **Evaluate** $P(B|A')$ **[3 marks]**

3. A bag contains 10 sweets of which 5 are red, 3 are green and 2 are yellow. Ann chooses a sweet at random from the bag and immediately puts it in her pocket so that nobody can see what colour it is. Bethan then chooses a sweet at random from the remaining 9 sweets.
Calculate the probability that

(a) Ann chooses a green sweet, [1 mark]

(b) Bethan chooses a yellow sweet, [3 marks]

(c) Ann and Bethan choose sweets of different colours. [3 marks]

4. The number of members, X, of a club attending a monthly meeting is modelled by a Poisson distribution with mean 10.

(a) Determine the probability that, at a randomly chosen meeting, the number of members attending is

(i) exactly 9,

(ii) less than 12. [4 marks]

(b) Determine the smallest value of n such that $P(X \geqslant n)$ is less than $0{\cdot}01$. [2 marks]

5. At a certain university, 60% of the students are male and 40% are female. It is known that 75% of the male students own a bicycle and 30% of the female students own a bicycle. One of the students is selected at random.

(a) Calculate the probability that the selected student

(i) is a male student who owns a bicycle,

(ii) owns a bicycle. [5 marks]

(b) Given that the selected student owns a bicycle, calculate the probability that this student is female. [3 marks]

6(a) A factory manufactures cups. The manager knows from past experience that 5% of the cups produced are defective. Given a random sample of 50 of these cups, determine the probability that the number of defective cups in this sample is

(i) exactly 2,

(ii) between 3 and 8 (both inclusive).

[6 marks]

(b) The factory also manufactures plates. The manager knows that 1·5% of the plates produced are defective. Use an appropriate Poisson distribution to find, approximately, the probability that a random sample of 250 of these plates contains exactly 4 defective plates.

[3 marks]

7. The discrete random variable X has the following probability distribution.

$$P(X = x) = \frac{k}{x} \quad \text{for } x = 2, 3, 4, 6,$$

$$P(X = x) = 0 \quad \text{otherwise.}$$

(a) Show that $k = \frac{4}{5}$. [2 marks]

(b) Determine $E(X)$. [2 marks]

(c) Given that X_1 and X_2 are independent observations from the distribution of X, determine the probability that the product X_1X_2 is equal to 12. [4 marks]

8. Fred is a cricket player. When he throws a ball at the wicket from a point P, he hits it with probability $0 \cdot 3$. You may assume that successive throws are independent.

(a) One morning, he goes out to practise his throwing from the point P. Calculate the probability that he hits the wicket for the first time with his third throw. [2 marks]

George is also a cricket player. When he throws a ball at the wicket from the point P, he hits it with probability $0 \cdot 2$. You may again assume that successive throws are independent.

(b) On another morning, Fred and George decide to play a game in which they throw balls, alternately, at the wicket from the point P. The winner is the player who is first to hit the wicket. Given that George throws first, calculate the probability that Fred

(i) wins the game with his first throw,

(ii) wins the game with his second throw,

(iii) wins the game. [7 marks]

9. The continuous random variable X has probability density function f given by

$$f(x) = \frac{4}{9}(4x - x^3) \quad \text{for } 1 \leqslant x \leqslant 2,$$

$$f(x) = 0 \quad \text{otherwise.}$$

(a) Determine $E\left(\frac{1}{X}\right)$ [4 marks]

(b) (i) Find an expression for $F(x)$, valid for $1 \leqslant x \leqslant 2$, where F denotes the cumulative distribution function of X.

(ii) Hence calculate

$$P(1{\cdot}25 \leqslant X \leqslant 1{\cdot}75).$$

(iii) Calculate the median of X. [9 marks]

END OF PAPER