GCE AS/A level

0976/01

MATHEMATICS – C4

Pure Mathematics

A.M. MONDAY, 16 June 2014

1 hour 30 minutes plus your additional time allowance

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

a 12 page answer book;
a Formula Booklet;
a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink, black ball-point pen or your usual method.

Answer ALL questions.

Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The curve C is defined by

$$3x^3 - 5xy^2 + 2y^4 = 15$$

The point P has coordinates $(1, 2)$ and lies on C
Find the equation of the NORMAL to C at P

[5 marks]

2(a) Express $\dfrac{5x^2 + 7x + 17}{(x + 1)^2 (x - 4)}$ in terms of partial fractions. [4 marks]

(b) USE YOUR ANSWER TO PART (a) to express

$$\frac{5x^2 + 9x + 9}{(x + 1)^2 (x - 4)}$$

in terms of partial fractions. [2 marks]

3(a) Find all values of x in the range

$0^\circ \leqslant x \leqslant 180^\circ$ satisfying

$\tan 2x = 3\cot x$ [4 marks]

(b) (i) Express $21\sin\theta - 20\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$

(ii) Use your results to part (i) to find the greatest value of

$$\frac{1}{21\sin\theta - 20\cos\theta + 31}$$

Write down a value for θ for which this greatest value occurs. [6 marks]

4. The region R is bounded by the curve

$y = 3 + 2\sin x$, the x-axis and the lines

$x = 0,\ x = \dfrac{\pi}{4}$

Find the volume of the solid generated when R is rotated through four right angles about the x-axis. Give your answer correct to the nearest integer. [6 marks]

5. Expand

$$6\sqrt{1 - 2x} - \frac{1}{1 + 4x}$$

in ascending powers of x up to and including the term in x^2

State the range of values of x for which your expansion is valid. [7 marks]

6. The curve C has the parametric equations

$$x = 2t,\ y = 5t^3$$

The point P lies on C and has parameter p

(a) Show that the equation of the tangent to C at the point P is

$$2y = 15p^2x - 20p^3$$

[4 marks]

(b) The tangent to C at the point P intersects C again at the point $Q\,(2q,\ 5q^3)$

Given that

$p = 1$, show that q satisfies the equation

$$q^3 - 3q + 2 = 0$$

Hence find the value of q [5 marks]

7. (a) Find $\int x^4 \ln 2x \, dx$ [4 marks]

(b) Use the substitution

$u = 10\cos x - 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \sqrt{(10\cos x - 1)} \sin x \, dx$$

[4 marks]

8. The value $£V$ of a long term investment may be modelled as a continuous variable.
At time t years, the rate of increase of V is directly proportional to the value of V

(a) Write down a differential equation satisfied by V

[1 mark]

(b) Show that $V = Ae^{kt}$, where A and k are constants.

[3 marks]

(c) The value of the investment after 2 years is $£292$ and its value after 28 years is $£637$

(i) Show that $k = 0{\cdot}03$, correct to two decimal places.

(ii) Find the value of A correct to the nearest integer.

(iii) Find the initial value of the investment. Give your answer correct to the nearest pound.

[6 marks]

9(a) The vectors $\underline{p}$ and $\underline{q}$ are given by

$$\underline{p} = 2\underline{i} - \underline{j} + 3\underline{k}$$

and $$\underline{q} = 5\underline{i} + 4\underline{j} - 8\underline{k}$$

Find the angle between $\underline{p}$ and $\underline{q}$

[4 marks]

9(b) In the diagram opposite, the points O, A, B, C and D are such that A is the mid-point of OD and C is the mid-point of OB

Taking O as the origin, the position vectors of A and B are denoted by $\underline{a}$ and $\underline{b}$ respectively.

(i) Show that $\underline{CD} = 2\underline{a} - \frac{1}{2}\underline{b}$

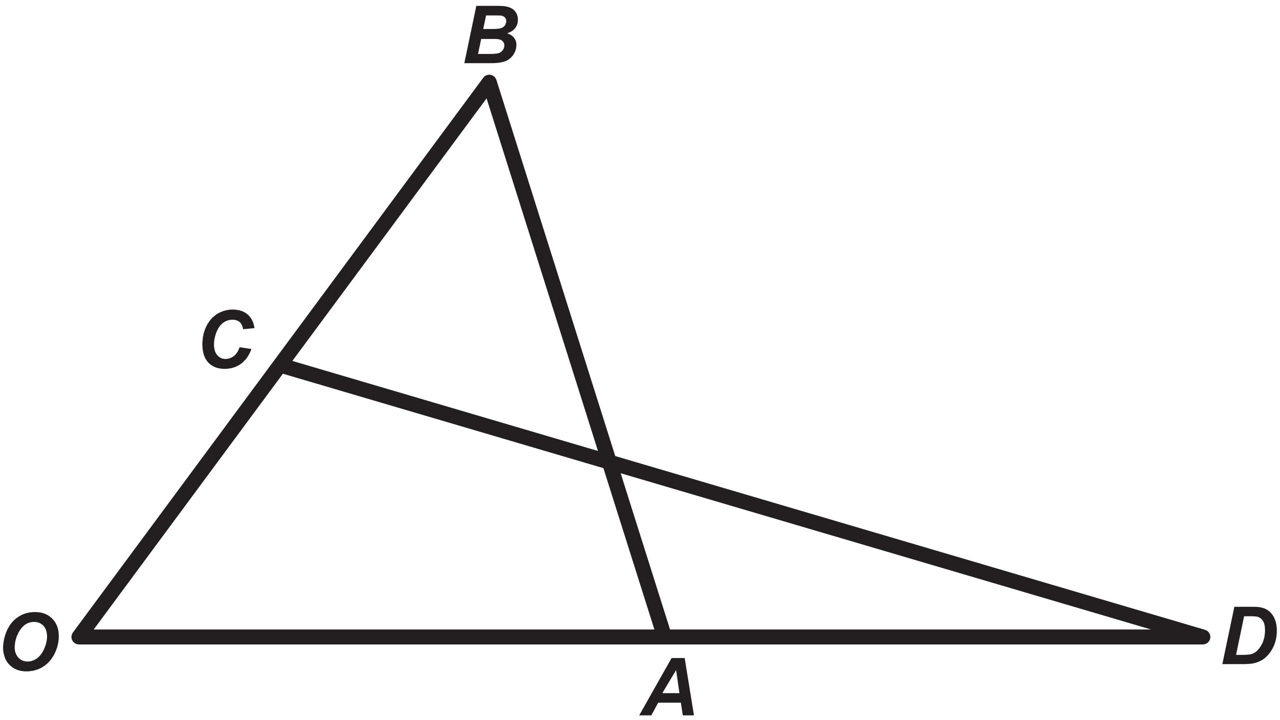
Hence show that the vector equation of the line CD may be expressed in the form

$$\underline{r} = 2\lambda\underline{a} + \frac{1}{2}(1 - \lambda)\underline{b}$$

The vector equation of the line L may be expressed in the form

$$\underline{r} = \frac{1}{3}\mu\underline{a} + \frac{1}{3}(\mu - 1)\underline{b}$$

The lines CD and L intersect at the point E.


B
C
O
A
D

9(b)(ii) By giving λ and μ appropriate values, or otherwise, show that E has position vector $\frac{2}{3}\underline{a} + \frac{1}{3}\underline{b}$

(iii) Give a geometrical interpretation of the fact that E has position vector $\frac{2}{3}\underline{a} + \frac{1}{3}\underline{b}$ [7 marks]

10. Complete the following proof by contradiction to show that

$$\sin\theta + \cos\theta \leqslant \sqrt{2}$$

for all values of θ.

[3 marks]

Assume that there is a value of θ for which

$$\sin\theta + \cos\theta > \sqrt{2}$$

Then squaring both sides, we have:

END OF PAPER