GCE AS/A level

0974/01

MATHEMATICS – C2
Pure Mathematics

P.M. FRIDAY, 18 January 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^2 \sqrt{10 - x^3}\, \mathrm{d}x.$$

Show your working and give your answer correct to four decimal places. [4]

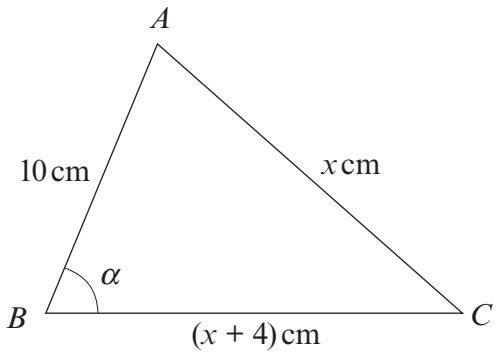
2. *(a)* Find all values of θ between 0° and 360° satisfying

$$7\sin^2\theta - \sin\theta = 3\cos^2\theta.$$ [6]

(b) Find all values of x between 0° and 180° satisfying

$$\tan(3x - 20°) = 1{\cdot}28.$$ [4]

3. The diagram below shows a sketch of the triangle ABC with $AB = 10\,\text{cm}$, $AC = x\,\text{cm}$, $BC = (x + 4)\,\text{cm}$ and $A\hat{B}C = \alpha$, where $\cos\alpha = \frac{3}{5}$.



(a) Write down and simplify an equation satisfied by x. Hence, evaluate x. [3]

(b) Find the exact value of the area of triangle ABC. [3]

4. *(a)* The first term of an arithmetic series is 1 and the common difference is 4.

(i) Show that the nth term of the arithmetic series is $4n - 3$.

(ii) The sum of the first n terms of this series is given by

$$S_n = 1 + 5 + \ldots + (4n - 7) + (4n - 3).$$

Prove from first principles, without using the formula for the sum of the first n terms, that

$$S_n = n(2n - 1).$$ [4]

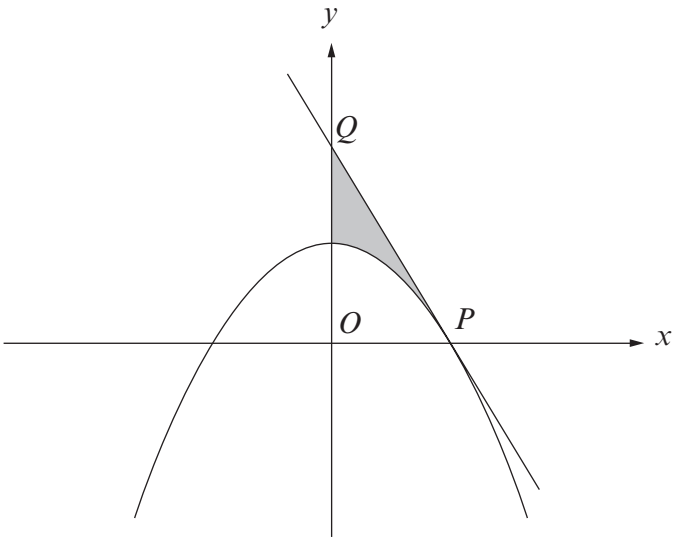
(b) The sum of the first ten terms of another arithmetic series is 55. The sum of the fourth, seventh and ninth terms of the series is 27. Find the first term and the common difference of this arithmetic series. [5]

5. *(a)* The pth term of a geometric series is 16. The $(p + 1)$th term of this series is 24. Find the $(p + 4)$th term of the series. [3]

(b) The sum of the first three terms of another geometric series is 22·8. The sum to infinity of the series is 18·75. Find the common ratio and the first term of this geometric series. [6]

6. *(a)* Find $\int\left(\frac{5}{x^4} - 7x^{\frac{2}{3}}\right)\mathrm{d}x$. [2]

(b)



The diagram shows a sketch of the curve $y = 9 - x^2$ which intersects the positive x-axis at the point $P(a, 0)$.

(i) Find the value of a.

The tangent to the curve at P intersects the y-axis at the point $Q(0, b)$.

(ii) Show that $b = 18$.

(iii) Find the area of the shaded region. [10]

7. *(a)* Given that $x > 0$, $y > 0$, show that

$$\log_a \frac{x}{y} = \log_a x - \log_a y.$$ [3]

(b) Solve the equation

$$6^{2x + 5} = 7.$$

Show your working and give your answer correct to three decimal places. [3]

TURN OVER

8. The circle C has centre A and equation

$$x^2 + y^2 + 6x - 10y + 14 = 0.$$

(a) (i) Find the coordinates of A and the radius of C.

(ii) The point P has coordinates $(-6, 2)$. Determine whether P lies inside C, on C or outside C. [5]

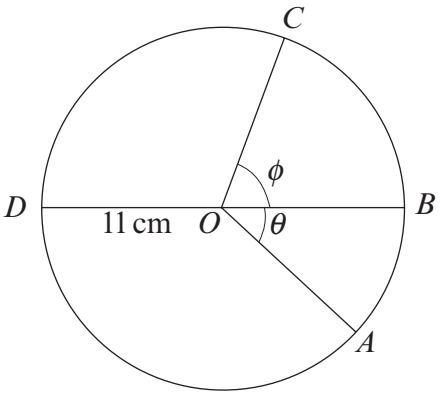
(b) The line L has equation

$$y = 2x + 1.$$

(i) Show that L is a tangent to the circle C and find the coordinates of Q, the point of contact of L and C.

(ii) The point R has coordinates $(4, 9)$ and R lies on L. Find $A\hat{R}Q$. [8]

9.



The diagram shows a sketch of a circle with centre O and radius 11 cm. Four points A, B, C and D lie on the circle. The line BD is a diameter of the circle, $A\hat{O}B = \theta$ radians and $B\hat{O}C = \phi$ radians.

(a) The area of sector AOB is $43{\cdot}56\,\text{cm}^2$. Find the value of θ. [2]

(b) The length of the arc BC is 13 cm less than the length of the arc CD.
Find the value of ϕ, giving your answer correct to two decimal places. [4]

