WJEC CBAC

GCE AS/A level

0975/01

MATHEMATICS – C3

Pure Mathematics

A.M. FRIDAY, 24 May 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_1^3 \ln\left(x^3 + 6\right) \mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

(b) Use your answer to part *(a)* to deduce an approximate value for the integral

$$\int_1^3 \ln\sqrt{x^3 + 6}\, \mathrm{d}x.$$ [1]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$4\cot^2\theta - 8 = 2\,\mathrm{cosec}^2\theta - 5\,\mathrm{cosec}\,\theta.$$ [6]

(b) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$\sec\phi + 2\tan\phi = 0.$$ [3]

3. The curve C is defined by

$$x^3y^2 = 128.$$

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [3]

The point P lies on C and has coordinates (a, b).

(b) Given that the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at the point P is 3,

(i) show that $b = -2a$,

(ii) find the value of a and the value of b. [4]

4. Given that, for $t > 0$,

$$x = \ln t,\ y = 5t^4,$$

(a) find and simplify an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, [4]

(b) find the value of t for which $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0{\cdot}648$. [4]

5. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $(7 - 9x^2)^5$ *(b)* $\tan^{-1} 6x$ [2], [2]

(c) $e^{4x}\tan 2x$ *(d)* $\dfrac{3 + \sin x}{2 + \cos x}$ [3], [3]

6. *(a)* Find

(i) $\displaystyle\int \cos\left(3x + \frac{\pi}{2}\right) \mathrm{d}x$, (ii) $\displaystyle\int e^{3-4x}\, \mathrm{d}x$,

(iii) $\displaystyle\int \frac{7}{8x + 5}\, \mathrm{d}x$. [6]

(b) Evaluate $\displaystyle\int_1^2 \frac{9}{(2x-1)^4}\, \mathrm{d}x$. [4]

7. *(a)* Show, by counter-example, that the statement

'If $|a + 1| = |b + 1|$, then $a = b$'

is false. [2]

(b) Solve the inequality

$$|x^2 - 10| \leqslant 6.$$ [4]

8. **You may assume** that the equation

$$x^2 + e^x - 3 = 0$$

has a root α between -2 and -1.

The recurrence relation

$$x_{n+1} = -\left(3 - e^{x_n}\right)^{\frac{1}{2}}$$

with $x_0 = -1{\cdot}5$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

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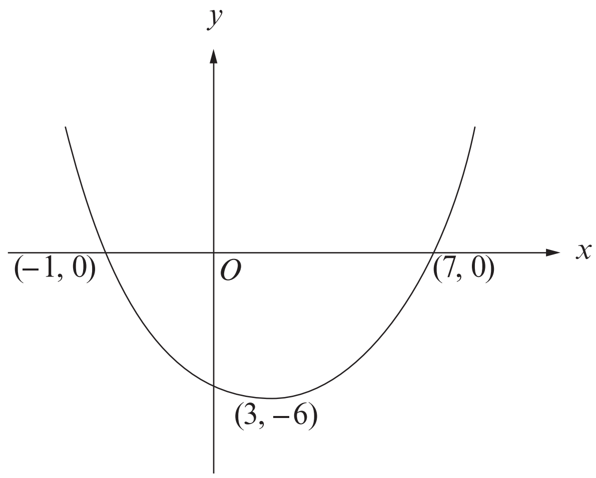
9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-1, 0)$ and $(7, 0)$ and has a minimum point at $(3, -6)$.



Sketch the graph of $y = -\frac{2}{3}f(x+4)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

10. The function f has domain $(-\infty, 10]$ and is defined by

$$f(x) = e^{5-\frac{x}{2}} + 6.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

11. The functions f and g have domains $(0, \infty)$ and $\left(0, \frac{\pi}{4}\right]$ respectively and are defined by

$$f(x) = \ln x,$$
$$g(x) = \tan x.$$

(a) (i) Write down the domain of fg.

(ii) Write down the range of fg. [3]

(b) (i) Solve the equation $fg(x) = -0{\cdot}4$. Give your answer correct to two decimal places.

(ii) Giving a reason, write down a value for k so that $fg(x) = k$ has no solution. [3]